Nonlin. Processes Geophys. Discuss.,
doi:10.5194/npg-2016-16-SC2, 2016

Nonlinear Processes in Geophysics
Discussions
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Interactive comment on "Detecting and tracking eddies in oceanic flow fields: A vorticity based Euler-Lagrangian method" *by* R. Vortmeyer-Kley et al.

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Received and published: 10 March 2016

A scalar field is objective if it is independent of the observer (see introductory books on continuum mechanics). Specifically, an objective scalar field remains unchanged under all Euclidian coordinate changes of the form

x=Q(t)y+b(t), (*)

where x are the coordinates in the present frame, y are the coordinates in the new frame, Q(t) is a proper orthogonal tensor and b(t) is a vector. Both Q and b should be continuously differentiable in time, but otherwise arbitrary. A weaker notion of frame independence is Galilean invariance, which only requires the above invariance for

Q(t)\equiv 0, i.e., only under translations of the frame but not under rotations.

An axiom of continuum mechanics is that material response (including material transport) is an objective phenomenon and hence can only be described self-consistently via objective quantities. Indeed, the assessment of whether a fluid mass travels coherently or mixes with its environment should not be dependent on coordinates (I've just described it without coordinates). A person on a boat, a circling airplane or a ship should invariably reach the same conclusion in this regard. For this reason, non-objective coherent structure detection methods cannot capture material transport self-consistently.

The M-function (the length of trajectories in a velocity field) is not objective, given that the trajectory length depends on the observer. For instance, for an observer traveling with any given trajectory of a vector field, the length of that trajectory is zero. So, by a simple Galilean change of the frame, the M function can be made zero at any desired location. This simple argument proves that the M-function is not even Galilean invariant, let alone objective.

One might still contend, however, that at least the topological features (say, maxima and minima) of the M function are objectively defined, even if its values change from one frame to the other. That is not true either, unfortunately.

Indeed, take a trivial flow that is just full of fixed points (a fluid at rest) in the x-frame. The M function is, therefore, identically zero, suggesting (correctly) that there is no vortex in this steady flow. Pick now an arbitrary constant vector b(t) \equiv b_0, and any rotation matrix Q(t). Then in the y-frame defined by the coordinate change (*), the location y=0 (formerly x=b_0) has zero trajectory length, whereas all other points in the y-frame will be rotating around b_0, accumulating nonzero trajectory length. The angular velocity of this rotation is the same for all these points (governed by Q(t)), so the further these points are from b_0, the larger arclength they cover in a given period of time. Consequently, in the y frame, the M function has a global minimum around the point x=b_0 that I have just arbitrarily selected above.

Before anyone says "... but it did give the right answer in the original frame!", consider that you would not know which of the two different answers to trust, had I not given you the answer in advance. I could have started by giving you the velocity in the y frame, and you would have given me the wrong answer in the original frame. In a truly unsteady flow, there is no distinguished frame [Lugt, 1979].

Therefore, the topology of the M function (including the locations of minima) is not objective either. Anyone thinking of defining a vortex/eddy as a region filling a valley surrounding a local minimum of the M function should keep the above example in mind.

Without doubt, under Lagrangian advection, any scalar quantity will create patterns when plotted over initial conditions. That's precisely why coherent structures are important: they tend to create coherent pattens in everything advected by the flow. But this does not imply that the advected quantity (including the creamer in one's stirred coffee or a piece of fishnet in the ocean) has any deep, intrinsic meaning for coherent structures in the carrier fluid. More pseudo-mathematically speaking, one can certainly integrate x_1ˆ3+15.4*x_2 over trajectories and plot the results over the initial conditions of those trajectories in the (x_1,x_2) plane. One might then ponder about the deep meaning of this cubic polynomial for fluid transport, encouraged by the various patterns that will undoubtedly emerge (unless the flow is a parallel shear flow in the x_2 direction).

Yet perhaps most would agree at this point: this cubic polynomial has no meaning for fluid mixing. To make this point, one does not need to engage in an endless argument with the proud inventor of this cubic diagnostic (that would be myself), who will no doubt defend this great tool, saying that critiques simply do not understand the method. Instead, one can simply point out that this diagnostic is not objective and hence cannot possibly capture anything intrinsic about material transport. End of discussion.

The continuum mechanics community went through the same deliberation a long time

ago. At some point, they stopped even considering newly proposed heuristic constitutive laws if those laws were not objective. It might be time for this to happen in the coherent structure detection industry as well.

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