Referee Report on "Detecting and tracking eddies in oceanic flow fields: A vorticity based Euler-Lagrange method" by R. Vortmeyer-Kley, U. Grawe, and U. Feudel

Potentially this paper contains some results that may be of interest to NPG, but not in its present form. The main issue is that the authors describe the method of Lagrangian descriptors in a variety of misleading and wrong ways, and they also have neglected to mention a large amount of relevant literature.

The authors develop a new Lagrangian descriptor based on integrating the magnitude of the vorticity (an Eulerian quantity) along trajectories. They claim this is a new "Eulerian-Lagrangian descriptor". However, their approach follows exactly the methodology for Lagrangian descriptors described in the following paper:

> A. M. Mancho, S. Wiggins, J. Curbelo, C. Mendoza Lagrangian descriptors: A method for revealing phase space structures of general time dependent dynamical systems, Communications in Nonlinear Science and Numerical Simulation, 18(12), 3530 3557 (2013)

At the bottom of page 3532 of this paper it is stated that Lagrangian descriptors can be constructed by integrating any "bounded positive intrinsic physical or geometric property of the velocity field...". Certainly the magnitude of the vorticity satisfies that criteria, but also the magnitude of the velocity field, which is a common Lagrangian descriptor used in the abovementioned paper (henceforth, Mancho et al 2013). It just so happens that the integral of the square root of the magnitude of the velocity field along trajectories has the interpretation of arclength, but it is still of the same character as the quantity studied by the authors who wish to rename the quantity "Eulerian-Lagrangian descriptor". This is completely misleading and contrary to the methodology introduced in Mancho et al 2013. Indeed, all of the quantities in Mancho et al. 2013 would then be ''Eulerian-Lagrangian descriptors" in the terminology of the authors of the paper under review as all of the quantities of Table 1 in Mancho et al. 2013 proposed for the construction of Lagrangian descriptors are Eulerian quantities. In this sense *all* Lagrangian descriptors are constructed from Eulerian and Lagrangian quantities, with the purpose of providing Lagrangian transport information. So there is nothing new "methodologically" in this paper.

The authors claim that Lagrangian descriptors are not objective, and justify this claim by referring to reference [15]. However, we have looked at reference [15], and I cannot find any proof of non-objectivity for Lagrangian descriptors in that reference. If the authors are going to make such a strong claim, then they must provide a reference to a proof of their claim. With respect to objectivity, on page 18 the authors state:

... it is a heuristic method that lacks objectivity. This can be problematic since it might lead to failure in the detection of some eddies.

I do not believe this. Can the authors provide *one* example where *lack of objectivity* can lead to non-detection of eddies? Which eddies? As it stands, the authors are making an unsupported statement. This statement of the authors illustrates the fact that the notion of ''objectivity" is a total red herring for this type of analysis. This is discussed in Section 4.3 of C. Mendoza and A. M. Mancho, The Lagrangian description of aperiodic flows: a case study of the Kuroshio Current, Nonlinear Processes in Geophysics 19 (4) (2012) 449-472. Moreover, in the paper by K. Ide, D. Small, and S. Wiggins, Distinguished hyperbolic trajectories in time dependent fluid flows: analytical and computational approach for velocity fields defined as data sets, Nonlinear Processes in Geophysics, 9(3-4), 237 ? 263 (2002) there is a proof in the appendix that hyperbolic trajectories are preserved under time-dependent transformations that grow "sub-exponentially" in time (obviously, if the time dependence of the transformation is "strongly exponential" it can "cancel" the hyperbolicity).

The authors are proposing what they refer to as a new characterization of eddies based on an elliptic region bounded by segments of stable and unstable manifolds of a hyperbolic trajectory. This allows the lobe dynamics mechanism to control transport in and out of the elliptic region. However, a careful development of eddies from this point of view has already been given in the following references:

M. Branicki and S. Wiggins, Finite-time Lagrangian transport analysis: stable and unstable manifolds of hyperbolic trajectories and finite-time Lyapunov exponents, Nonlin. Processes Geophys., 17, 136 (2010).

M. Branicki, A. M. Mancho, and S. Wiggins, A Lagrangian description of transport associated with a front-eddy interaction: Application to data from the North-Western Mediterranean Sea, Physica D, 240(3), 282-304, (2011).

Moreover, besides eddies defined in this way using data sets, kinematic models possessing transient eddies are also developed. These have the advantage of allowing detailed study and precise investigations. It is also shown in the Branicki and Wiggins paper that FTLE will often fail to capture the detailed structure of eddies defined in this way.

The authors claim that Lagrangian descriptors cannot detect eddies. I find this to be a very surprising statement based on several papers in the literature that characterize eddies in the Gulf stream and Gulf of Mexico, for example, in terms of Lagrangian descriptors. See, for example:

C. Mendoza, A. M. Mancho, M.-H. Rio. The turnstile mechanism across the Kuroshio current: analysis of dynamics in altimeter velocity fields. Nonlinear Proc. Geoph 17 (2010), 2, 103-111.

C. Mendoza, A. M. Mancho. The Lagrangian description of aperiodic flows: a case study of the Kuroshio Current. Nonlinear Processes in Geophysics 19 (4) (2012) 449-472.

C. Mendoza, A. M. Mancho, S. Wiggins. Lagrangian Descriptors and the Assesment of the Predictive Capacity of Oceanic Data Sets. Nonlinear Processes in Geophysics 21 (2014) 677-689.

V. J. Garcia-Garrido, A. M. Mancho, S. Wiggins, C. Mendoza. A dynamical systems approach to the surface search for debris associated with the disappearance of flight MH370. Nonlinear Processes in Geophysics 22 (6) (2015) 701-712.

There has been a literature developed in recent years related to finding Eulerian quantities that allow one to make conclusions about certain Lagrangian transport phenomena.

R. Sturman and S. Wiggins, Eulerian indicators for predicting and optimizing mixing quality, New J. Phys., 11, 075031 (2009).

K. L. McIlhany, D. Mott, E. Oran and S. Wiggins, Optimizing mixing in lid-driven flow designs through predictions from Eulerian indicators. Physics of Fluids, 23(2), 082005 (2011).

K. L. McIlhany and S. Wiggins, Eulerian indicators under continuously varying conditions. Physics of Fluids, 24, 073601 (2012).

K. L. McIlhany, S. Guth, and S. Wiggins, Lagrangian and Eulerian analysis of transport and mixing in the three dimensional, time dependent Hill's spherical vortex, Physics of Fluids, 27, 063603 (2015).

On page 2 the authors state that:

Another more heuristic approach is the computation of distinguished hyperbolic trajectories (DHT) and their stable and unstable manifolds to identify Lagrangian coherent structures in a flow.

The word "heuristic" and the phrase "to identify Lagrangian coherent structures in a flow" are used very bizarrely, and incorrectly, here. First, there is nothing "heuristic" about this approach. A hyperbolic trajectory is a trajectory of the fluid flow having stable and unstable manifolds. The stable and unstable manifolds are made up of trajectories, this is why trajectories cannot cross them. They ARE flow barriers by construction (they do not have to be "identified"). In other words, they are the direct construction of Lagrangian coherent structures. FTLEs and Lagrangian descriptors are methods to detect these structures (not to construct them).

In several places the authors use the word "ridges" to refer to some property of Lagrangian descriptors. It is not clear what they mean by this since, to my knowledge, it has not been used in the literature to refer to any property of Lagrangian descriptors. It was used in the original Shadden and Marsden paper to refer to a feature of FTLE fields, and in that paper it was given a precise mathematical meaning. However, that meaning appears to have largely been "lost" as people now tend to throw around the term rather cavalierly (as

these authors have done) without providing an understanding of its context and mathematical definition in the situation in which they are writing.

In Section 4.2 the authors add noise to their velocity field. The details of this are not clear, especially if, and how, Lagrangian descriptors would fit into this framework. Lagrangian descriptors are integrations of positive quantities over trajectories. Noisy velocity fields give rise to stochastic ODEs, whose solutions are stochastic processes, not trajectories.

In summary, this paper should not be published. The authors do not exhibit a clear understanding of the topics on which they are writing. The paper contains fundamental misunderstandings, and it displays a profound lack of knowledge of the relevant literature. The section on the lifetimes of eddies was interesting. However, because of the way in which the paper was written in is not clear what of the rest of the paper is required for that work.