Some remarks are required on the comment "Non-objectivity of the M function and other thoughts" by G. Haller, as it contains several inaccurate and misleading statements. The comment argues that the M-function is non-objective because it provides different outputs for dynamical systems that are related by a Galilean coordinate transformation. In particular the comment discusses in detail a particular example for which claims that the output of M in one of the frames is wrong. The comment considers two 2D autonomous dynamical systems that are related by a Galilean transformation. The quoted example is described in fact in the basic dynamical system book *Ordinary Differential Equations* by Arnold (p. 44, Problem 3 and p. 45, Fig. 47) and for the system in which M is claimed to provide the 'wrong' result, M provides in reality exactly the same description which is found in this book and never questioned until the above comment. Despite being related by a Galilean transformation the phase space structure of the two dynamical systems is very different. Therefore the M function should provide different information in each case–information that reflects the phase space structure for the particular dynamical system. More details for this example are given next.

In particular the comment considers the following dynamical system:

$$\dot{\mathbf{x}} = 0, \qquad \text{where} \quad \mathbf{x} \in \mathrm{IR}^2 \tag{1}$$

We subject this vector field to a Galilean transformation, i.e., a rotation $\mathbf{y} = R(t)^T\mathbf{x}$, such that in the rotating frame (1) has the form:

$$\begin{aligned} \dot{y_1} &= y_2 \\ \dot{y_2} &= -y_1. \end{aligned} \tag{2}$$

Here $R(t)^T$ is the transpose of the orthogonal matrix:

$$R(t) = \begin{pmatrix} \cos t & -\sin t \\ \sin t & \cos t \end{pmatrix}$$

The phase portrait of (1) consists entirely of fixed points. The phase portrait of (2) consists of a one-parameter family of invariant circles. It will be convenient to express (2) in action angle variables (ρ, θ) as follows:

$$\begin{aligned} \dot{\rho} &= 0 \\ \dot{\theta} &= -1 \end{aligned} \tag{3}$$

where $y_1 = \rho \cos\theta$ and $y_2 = \rho \sin\theta$. H is the Hamiltonian in action-angle variables: $H(\rho, \theta) = \rho$. From these expressions it is clear that $(\rho = \rho_0, \theta(t) = -t + \theta_0)$ are solutions to the system (3) which correspond to invariant 1-tori.

Now we apply the M function to (1) and (3), where M measures the arclength of a trajectory through an initial condition in both forward and backward time. Clearly, for (1) M is zero for all initial conditions since every point is a fixed point (see Madrid and Mancho (2009)). For (3) M=$(2\tau)\rho$ where 2τ is the forwards-backwards time interval length. Hence the contours of M are in 1-1 correspondence with the trajectories of (3) (see Mezic and Wiggins (1999) and Susuki and Mezic (2009)). Hence M recovers the correct phase space structure for both (1) and (3). If M were the same for both of these vector fields it would not accurately recover the phase space structure for each vector field, as is the case of the Lagrangian-averaged vorticity deviation (LAVD).