Response to Reviewer #1

We would like to thank the reviewer for his critical assessment of our manuscript and the suggestions to improve it, which we have taken into account. In the following we respond to all the concerns of the reviewer and indicate the changes in the manuscript:

The authors develop a new Lagrangian descriptor based on integrating the magnitude of the vorticity (an Eulerian quantity) along trajectories. They claim this is a new Eulerian-Lagrangian descriptor". However, their approach follows exactly the methodology for Lagrangian descriptors described in the following
paper:
A. M. Mancho, S. Wiggins, J. Curbelo, C. Mendoza Lagrangian descriptors: A method for revealing phase space structures of general time dependent dynamical systems, Communications in Nonlinear Science and Numerical Simulation, 18(12), 3530 3557 (2013)
At the bottom of page 3532 of this paper it is stated that Lagrangian descriptors can be constructed by integrating any bounded positive intrinsic physical or geometric property of the velocity field...". Certainly the magnitude of the vorticity satisfies that criteria, but also the magnitude of the velocity field, whichis a common Lagrangian descriptor used in the abovementioned paper (henceforth, Mancho et al 2013). It just so happens that the integral of the square root of the magnitude of the velocity field along trajectories has the interpretation of arclength, but it is still of the same character as the quantity studied bythe authors who wish to rename the quantity Eulerian-Lagrangian descriptor". This is completely misleading and contrary to the methodology introduced in Mancho et al 2013. Indeed, all of the quantities in Mancho et al. 2013 would then be `'Eulerian-Lagrangian descriptors" in the terminology of the authors of the paper under review as all of the quantities of Table 1 in Mancho et al. 2013
proposed for the construction of Lagrangian descriptors are Eulerian quantities. In this sense all Lagrangian descriptors are constructed from Eulerian and Lagrangian quantities, with the purpose of providing Lagrangian transport information. So there is nothing new methodologically" in this paper.

We completely agree with the reviewer and also reviewer #3 that already Mancho et al. 2013 pointed out, that any fluid property can be used to construct a Lagrangian descriptor. This has been pointed out by us explicitly already in the first version of the manuscript (cf. the sentence “As already pointed out by Mancho et al. (2013) any intrinsic physical ...” in the beginning of the paragraph before formula (4)). Our motivation to introduce the name Euler-Lagrangian descriptor was a more practical one. Because we found it difficult to read talking about one and another Lagrangian descriptor, we introduced a distinction by the names Euler-Lagrangian for one of them and Lagrangian for the other. Since this has been found misleading by two reviewers (#1 and #3) because it would look like the definition of a new descriptor, which indeed is not the case as we of course know, we have changed this in the revised version. To emphasize this we have now avoided the name Euler-Lagrangian descriptor in the title and throughout the whole manuscript. Furthermore, we have even more than before emphasized that the original idea had been already formulated in Mancho et al. 2013. We now write “We would like to emphasize, that it has been already pointed out by Mancho et al. (2013)”.

The authors claim that Lagrangian descriptors are not objective, and justify this claim by referring to reference [15]. However, we have looked at reference [15], and I cannot find any proof of non-objectivity for Lagrangian descriptors in that reference. If the authors are going to make such a strong claim, then they must provide a reference to a proof of their claim.

We have removed the word heuristic as well as the whole discussion about objectivity from the text to avoid any further discussion of this issue since it is not the aim of this manuscript to clarify a question, which is debated in the literature. Our aim is much more practical and does not claim to develop a new mathematical method. Since the reviewer does not find Ref. [15] appropriate, we are thankful to George Haller to provide his comment #2 containing the arguments requested. Ana Mancho has answered the question of objectivity of Lagrangian descriptors in an editor's comment. The main focus of this manuscript is completely different and does not concentrate on the question whether the method is objective or not. It is just an application of an existing method to tackle the question of providing a robust method to detect and count eddies in an oceanographic flow. We regret that the problem of objectivity distracted the reviewer from the main focus of the manuscript. However, since this problem has been addressed in several comments, we cannot ignore it and refer the reader now to this discussion in the discussion section of the journal.

The authors are proposing what they refer to as a new characterization of eddies based on an elliptic region bounded by segments of stable and unstable manifolds of a hyperbolic trajectory. This allows the lobe dynamics mechanism to control transport in and out of the elliptic region. However, a careful development of eddies from this point of view has already been given in the following references:
M. Branicki and S. Wiggins, Finite-time Lagrangian transport analysis: stable and unstable manifolds of hyperbolic trajectories and finite-time Lyapunov exponents, Nonlin. Processes Geophys., 17, 136 (2010).
M. Branicki, A. M. Mancho, and S. Wiggins, A Lagrangian description of transport associated with a front-eddy interaction: Application to data from the North-Western Mediterranean Sea, Physica D, 240(3), 282-304, (2011).

We would like to thank the reviewer for pointing out those papers to us, which we now cite in the text.

The authors claim that Lagrangian descriptors cannot detect eddies. I find this to be a very surprising statement based on several papers in the literature that characterize eddies in the Gulf stream and Gulf of Mexico, for example, in terms of Lagrangian descriptors.

We did not claim in the manuscript that Lagrangian descriptors cannot detect eddies. We have only written that the Lagrangian descriptor using the path length identifies both the eddy core and the DHTs with a minimum of M. This is a bit cumbersome when designing an algorithm for which it is impossible to check for each whether it corresponds to a hyperbolic point or elliptic point (eddy core). Therefore, we were looking for another Lagrangian descriptor not posing this difficulty. Using the Lagrangian descriptor M_V has the advantage that no distinction is needed between eddy cores and DHTs, since they are displayed as maxima and minima of M_V respectively. By contrast, the Lagrangian descriptor M needs an additional criterion since eddy cores and DHTs are both displayed as minima. Since the reviewer misunderstood our statements we have now reformulated it in a more precise way.

The word "heuristic" and the phrase "to identify Lagrangian coherent structures in a flow" are used very bizarrely, and incorrectly, here. First, there is nothing "heuristic" about this approach. A hyperbolic trajectory is a trajectory of the fluid flow having stable and unstable manifolds. The stable and unstable manifolds are made up of trajectories, this is why trajectories cannot cross them. They ARE ow barriers by construction (they do not have to be "identfied"). In other words, they are the direct construction of Lagrangian coherent

structures. FTLEs and Lagrangian descriptors are methods to detect these structures (not to construct them).

According to our previous answer to the same criticism above we have deleted the word heuristic.
The word “identified” was used in the meaning that the manifolds are visible in the plot of M. We apologize the misleading use of the word. We are not native speakers.

In several places the authors use the word "ridges" to refer to some property of Lagrangian descriptors. It is not clear what they mean by this since, to my knowledge, it has not been used in the literature to refer to any property of Lagrangian descriptors. It was used in the original Shadden and Marsden paper to refer to a feature of FTLE fields, and in that paper it was given a precise mathematical meaning. However, that meaning appears to have largely been "lost" as people now tend to throw around the term rather cavalierly (as these authors have done) without providing an understanding of its context and mathematical definition in the situation in which they are writing.

Indeed, as the reviewer pointed out, the word „ridges“ has been used in the context of FTLE fields, not only in the paper by Shadden and Marsden, but also in the papers by the group of Hernandez-Garcia/Lopez and coworkers from Spain. It just describes the local maxima of a certain quantity under investigation. We have deleted the word ridges in the context of Lagrangian descriptors.

In Section 4.2 the authors add noise to their velocity field. The details of this are not clear, especially if, and how, Lagrangian descriptors would fit into this framework. Lagrangian descriptors are integrations of positive quantities over trajectories. Noisy velocity fields give rise to stochastic ODEs, whose solutions are stochastic processes, not trajectories.

The reviewer is correct in saying that one hast o study stochastic differential equations when dealing with noisy velocity fields. This would apply if one would be interested in a stochastic view of the problem. However, in the manuscript we look at this problem from a very different point of view. Since we are interested to design an algorithm searching and detecting eddies in a real velocity field from oceanography, we wanted to test the algorithm against noisy velocity field in the way, that the noise is observational noise or in other words measurement errors. For many problems, the velocity field will not be given as solutions of a numerical integration but from observational data, which are corrupted by measurement errors. Therefore, we just added noise to the computed velocity field. This approach enables us to “mimic” in a simple way a noise, which comes from errors in field measurements. This is at the same time a test, how a Lagrangian descriptor would respond to velocity fields which do not fulfill the mathematical criteria of a two-dimensional divergence free velocity field. We have now explained this approach in more detail in the text and have rewritten most of the noise section accordingly.