Response to Reviewer #2

We would like to thank the reviewer for his encouragement and positive assessment of our manuscript and the suggestions to improve the manuscript, which we have taken into account. Here we respond to all the comments made by the reviewer and indicate the changes in the manuscript made accordingly.

Several typographical errors to correct: 1) Overall, many missing commas. When starting a sentence with a prepositional phrase, separate it from the sentence with a comma. Page/Line 1/13 - For this reason, 7/11 - For the Lagrangian descriptor M_v, 7/16 - ... in a flow, 9/26 - For M_v, 14/4 - However, nowadays, 14/12 - However,

In some cases, there are commas that are unnecessary, page 5 line 8: "...dynamical evolution yield" (no comma).

Do a re-read of the paper and look for these prepositional phrases and clearly separate them grammatically.

We have done our best in re-reading the manuscript and improving grammar. Since we are not native English speakers, we admit having some problems with that.

At times, the papers language is too conversational. In general, the tone of the paper is scientific, and it should remain so throughout the paper. From above, "nowadays" is an example. Page/Line 3/15 - "Anyhow" can simply be removed. 7/6 - remove "Again".

Do a re-read and it helps to read it out loud so that you can catch the conversational tone when it comes up. That said, the language is outstanding for a non-English native speaker!

Thank you very much for pointing out the conversational language. We have removed it, whenever we noticed it ourselves.

Several words can be removed as they are unnecessary. In some cases, words need to be added or changed. Page/Line 1/14 - change "e.g. marine biology." to "marine biology for instance." 2/21 - "...but have been recently..." 4/31 - change on to of 5/10 - Change: "Manifold trajectories on both sides of the manifold have different behaviors compared..." 8/6 - change "distinction" to "distinguishing between ... and the identification " 9/2 - change "We use its" to "We use the feature" 14/3 - Change "non" to "none"

We have changed the text accordingly.

Check your formulae: Page/Line 5/3 - equation #3 - make sure the velocity is squared and the dt is not under the sqrt

We have corrected the formula.

My main problem with this paper is that it asserts things that it does not support directly in the *text. At times, there are conflicting statements about what the newly proposed Euler-Lagrangian descriptor can and cannot do. These discrepancies need to resolved in the text so that the reader is not confused or led astray. Also, in the beginning of the paper, the use of oceanographic data is discussed, but the paper is essentially about toy-models. I understand the need to verify a new metric by using toy-models, however, if you suggest that this metric is useful for geo-physical flows, then you need to demonstrate that in this paper, or put a disclaimer early within the text, that you intend to follow-up this paper with another paper demonstrating the metric on actual physical flows obtained from either satellite data or well-understood simulated oceanographic models such as CCSM4 or a variant of ROMS that is well-accepted as a good representation of historical data (flows).*

We agree completely with the reviewer that a demonstration of the method with a real oceanographic velocity field is much more convincing. To apply the Lagrangian descriptor M_v based on the modulus of vorticity and the eddy tracking employing it to an oceanographic velocity field, was already planned when we submitted the manuscript. Now, we have included an example for the western Baltic Sea in the revised version and replaced the Section about the seeded eddy model. Furthermore, we discuss on the basis of this example advantages and disadvantages of the method if it is applied to an oceanographic data set. The velocity field for the western Baltic Sea is from the ocean model described in Gräwe et al. (2015a). Further research aims at an eddy statistics for lifetime, size and track of eddies for the central Baltic Sea with the eddy tracking based on M_v. But this work is beyond the scope of this current manuscript.

Finally, when you compare your new metric to existing metrics, then state they your metric is better, you need to clearly state the differences and exactly HOW your metric out performs another. That is simply now done well in the text as it currently stands.

We have rewritten and complemented the parts of the text where we explain what the new metric searches for and what are the differences to existing methods. We hope it is now easier to understand and more precise. We now discuss in more detail the problems arising when applying this metric to a real oceanographic field. This sheds more light on the difficulties of an automated eddy detection. The comparison of the results obtained with M_v and the eddy tracking toolbox by Nencioli et al. (2010) reveals that for both methods false positives and false negatives exist. To improve those results is a future challenge.

From my understanding of M_v, you state its superiority over the M-value mainly because it maximizes when a fluid packet is part of gyre. In this case, for the duration of its stay within the gyre, the vorticity is high so the M_v will be maximal. For the M-value, the center of the gyre will be a minimum, such that the M_v can distinguish an elliptical point as well as a hyperbolic point, whereas, the M-value shows both types as minima. That is the main difference you quote in their behavior.

First, you state that your metric has excellent time resolution when seeking the beginning of a *gyres formation as well as its lifetime, because you can measure when the gyre dies off. In both of these measurements, you depend on the value of tau. You make a cases in figure 6 that the best value for tau is 0.15 times the lifetime of the gyre. This is a circular definition. You need to know the lifetime in order to determine tau if it is to be based on a percentage of that lifetime. Furthermore, you can only find a gyre once you vorticity values are maximized, meaning that you need a particle to have already been inside of a gyre long enough for the M_v to become maximal. This means that there is a lead-in time where you do not know whether you are in a gyre or not as the trajectory has not had enough time to sample to gyre. The problem is that in order to find the gyre in the first place, you need an initial value for tau simply to compute the M_v. So, do you propose to constantly be computing M_v for a range of tau values until you find a gyre - THEN you can adjust tau to be 0.15 the lifetime of the gyre? But wait, you need to know the end of the gyre as well to know the lifetime, so you cannot determine an optimal tau to find a gyre until it has formed and gone away. This suggests that an oceanographer will need to be computing M_v over a range of tau values constantly simply to see when/if a gyre has formed. Of course, this is also true for the M-value.*

The proper choice of tau is indeed the main problem with any Lagrangian descriptor including M_v and M. Hence, for a real oceanographic problem one has to vary tau to find all the eddies. This necessary choice is a practical limitation of the method. We point to that fact now better in the manuscript and provide an improved figure to show the dependence on tau (Fig. 6).

Figure 3 needs to be larger and with a better color contrast in order to show the manifold structure.

We have changed the colorcode to improve the contrast, because a larger tau does not lead to a clearer structure. Unfortunately, the colorcode does not take into account colour-blindness, but we did not find any colorcode with enough color-dimensions that is also valid for color-blindness.

Page 5, lines 6-15. At the beginning of the paragraph, you state that the M-value can distinguish between stable and unstable manifolds as well as hyperbolic and elliptic regions. On line 14, you state that the M-value cannot distinguish between elliptic and hyperbolic points.

Manifolds as well as hyperbolic and elliptic fixed points (more general distinguished hyperbolic trajectories and distinguished trajectories surrounded by an elliptic region in the sense of Mancho et al. (2013) correspond to singular features in the plot of M (singular lines and local minima). In this sense M can identify them. We apologize for the misleading use of the term “distinguish” on page 5 line 6 it was meant in the sense of “identify” (We are not native speakers.). We have rewritten the section on the Lagrangian descriptor M to make clear that M can of course identify distinguished hyperbolic trajectories and distinguished trajectory surrounded by an elliptic region but they are both displayed as a local minimum of M from which one cannot decide if it has elliptic or hyperbolic properties. Therefore, we constructed a vorticity based Lagrangian descriptor M_v that yields singular lines and local minima and maxima where the local maxima correspond to eddy cores (moving elliptic points) and the local minima to the distinguished hyperbolic trajectories ("moving saddle point").

Page 7, figure 2. It is implied in the previous literature as well as your own figures, that the M-value is good at finding the radius of the elliptic regions BECAUSE it has a minimum as the center, so that the contour of M-value maximizes as it moves away from the center and then decays as it moves far away - such that the maxima of M- values could be used to estimate the radii of elliptic regions. This is not explained in your paper, yet, you regularly refer to needing to use both the M_v and M-value to extract useful gyre information. Page 9, lines 3 and 4 - refer to using the M_v in combination with the M-value. Pages 15-17 also make it unclear in all of the figures which M function is used to extract the gyre location AND SIZE. In the figures, is it stated M and M_v. Why both?

As explained above M and M_v yield singular features (singular lines and local minima and maxima).

The eddy core in case of M corresponds to a local minimum and in case of M_v to a local maximum. Because the Lagrangian descriptor M would display a minimum in case of a DHT too a second criterion is needed to distinguish them properly. Therefore, we suggest M_v to simplify the automated eddy detection because one has only to search for a local maximum that corresponds to the eddy core.

The local maxima and the singular lines of M_v will be used to construct an eddy tracking tool based on the following concept of an eddy: We denote an eddy as being bounded by pieces of stable and unstable manifolds of DHTs (according to Branicki et al. (2011) and Mendoza and Mancho (2012)) surrounding an area in which the flow is rotating. The manifolds correspond to singular lines in M_v which are used to describe the eddy boundaries. The eddy core is considered as a local maximum of M_v within this bounded region, which can be interpreted as one point of a distinguished trajectory surrounded by an elliptic region.

For the detection of the eddy shape we have previously used a combination of M and M_v because M shows in our test case a clear line of minimum M values that was easier to detect automated than the line in M_v. In general, manifolds correspond to singular lines (Mancho et. al. 2013). To construct an eddy shape detection that is more general and only based on M_v, we have improved the shape detection algorithm. The improved shape detection is based on the assumption that the eddy boundary is the largest closed contour line of M_v where M_v is an extremum (large gradient of M_v).

Furthermore, we have rewritten the Sections 2 and 3 to clarify the idea of M and M_v and its correspondence to our understanding of an eddy.

Page 8, lines 6-10. This paragraph asserts that M_v is the best of four metric because it can discern between stable and unstable manifold lines - which can be used to get more insight into the size of the eddies. HOW exactly? I feel like a paragraph ex- plaining this statement is *missing. Perhaps it would precede this paragraph.*

We have rewritten this paragraph and parts of Sect. 2 to make clear what the idea of the description of an eddy boundary based on manifolds is, namely to describe a region that is separated from the rest of the flow (as explained above and in Branicki et al. (2011) and Mendoza and Mancho (2012)*).*

Can M_v distinguish between stable and unstable manifold lines? If so, how?

Singular lines in the plot of M or M_v correspond to manifolds. But one cannot distinguish based on the plot of M or M_v if it is a stable or unstable manifold. For the understanding of an eddy as a region bounded by pieces of stable and unstable manifolds of the distinguished hyperbolic trajectory ("moving saddle point") with an eddy core inside, it is only necessary to identify the manifolds and not the type of the manifold. Furthermore, if one is interested in the type of the manifold one can put tracers on the manifold close to the hyperbolic trajectory and track them forward and backward in time.

For that matter, in Figure 2, you show the four convective cell case, where M_v maximizes at the center. As tau increases, the maxima form a flatter and flatter plane centered on the gyre. Doesn't this make you less sensitive to the size of the gyre, not more sensitive? How does M_v determine the radius of a gyre. I'd like to know based on the text provided.

The maximum of M_v does not form a flatter and flatter plane in the centre, instead the centre becomes sharper and sharper as minimum of M in figure 2 f). This cannot be seen so clear in the colorcode used because the maximum is a light yellow point in a yellowish region. As mentioned above we have changed the colorcode to improve the contrast.

*Page 11, figure 6. The resolution shown for this figure does not convince me that 0.15*lifetime is the optimal tau value. It could be any value from 0.06 up to 0.21*lifetime. There should be many more points to determine the best value.*

We have improved the figure and calculated more values. The chosen value of tau=0.15*lifetime is in our case the beginning of a 0.02 small region of the optimal tau values. We have chosen the lower bound of this region to minimize the computational effort for calculating M_v.

Finally, in the beginning, I thought I was going to see this metric applied to an oceano- graphic data set. By the end, I did not find it. Please show me something geo-physical or tell me that it is coming in a later publication.

We have applied the method to an example of the western Baltic Sea to give an outlook on the application to oceanographic data sets.