Dear Referee,

Thanks for your constructive comments. We have tried our best to rebuttal your comments line by line as following:

(1)Referee: why 3-state variable systems have not been studied in the literature, laboratory friction experiments simply do not seem to suggest the need for this additional complexity. While some large rate-step experiments seem to point towards a second state variable (Marone, 1998; Ruina 1983) even this, in itself, is by no means a robust feature of experimental rate-stepping data. I find it difficult to understand the need for adding yet another, by all accounts unnecessary, degree of freedom to the system.

Authors: we agree with above opinion of the reviewer. But our Motivation to study the stability of 3sRSF emanates from the numerical simulations concerning the effect of temperature, viscosity as well as normal stress on chaotic behaviour of the 2sRSF model with the slip law. Our study establishes that either of these parameters (temperature, inertia, viscosity etc.) always results in diminishing the chaotic behaviour of the sliding system. However if an additional state variable "θ_3" is added in the 2sRSF model, unlike the effect of temperature, viscosity or normal stress on chaos, the present dynamic system that is,3sRSF model, in fact, becomes more chaotic. For instance, Fig.1 presents the results concerning the effect of temperature parameter "q" on chaos. As q varies from $q=1$ (corresponds to the RSF model) to $q=1.01$, period doubling oscillations reduces to periodic oscillations. It is important to mention that the Ruina-Rice slip law/slip law shows the chaos with 2sRSF model but the chaotic behaviour is not seen with the Dieterich-Ruina aging law/aging law. This is despite the fact that the both friction laws for stable variable "θ" were derived from the same expression for friction of rock surfaces (Ruina, 1983). The reason for this contradiction is not known in literature. So we believe that the present study will be useful to unravel the reason behind aforementioned “chaos contradiction”. <u>Although we have not included the results in Fig.1 in the present manuscript, the results (Fig.1) can be included in the manuscript if editor/reviewers suggest us to do so.</u>

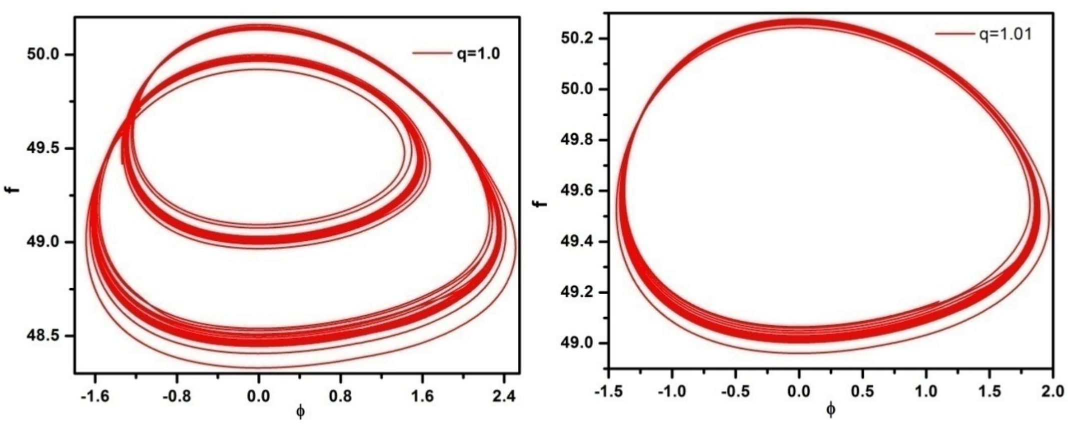

Fig.1 Effect of temperature parameter q on chaotic behaviour of the 2sRSF model with slip law in the form of phase diagram f vs. ϕ for $q = 1.0$ and $q = 1.01$, $\beta_1 = 1.0$, $\beta_2 = 0.84$, $\beta_3 = 0.38$, $\rho = 0.048$ and $\rho_1 = 0.034$ for initial condition [0,0,0,0]. It is obvious from two plots that period doubling reduces to single period orbits just upon slight modification in q .

(2)Referee: Besides this clear motivational short-coming, the paper suffers from a lack of proper discussion of the background of the RSF formalism. For example, in Eq. 1 (which misses the summation sign on θi), the authors neglect to mention that they are using the Slip law for their simulations. Further, there is no discussion of why they choose the Slip law for their simulations.

Authors: We have further modified the background of the RSF formalism. It is also clarified that the slip law of the RSF model is presently used in the present numerical simulations. We have also corrected the missing summation sign ($\sum$) in state variable "θ". It is already reported in literature that the 2sRSF with the slip law shows the chaos but the same is not seen with the aging law (Liu, 2007).

<u>We have also added in the manuscript at page no 2 and line no 18-25 as following:</u>

“Motivated from the numerical simulations concerning the effect of temperature, viscosity as well as normal stress on chaotic behaviour of the 2sRSF model with the slip law. We find that either of these parameters (temperature, viscosity etc.) results in diminishing the chaotic behaviour of the sliding system. However if an additional state variable "θ_3" is added in the 2sRSF model, unlike effect of temperature, viscosity or normal stress on chaos, the present dynamic system becomes more chaotic. Ruina(1983) has predicted that the RSF model with more than two state variables should also show the complex dynamical behaviour”.

(3)Referee: There is also no discussion of how these existing formulations of RSF fail to explain observed experimental data.

Author: We would like to clarify that the present study is not at all motivated from the fact whether the established RSF models in literature fails to explain observed experimental data. As mentioned in the beginning that present study motivated to study the effect of "third state variable" that is, "θ_3" after concluding that temperature, pore pressure, normal stress, inertia of the sliding as well viscous damping eliminates the chaotic behaviour of the 2sRSF model with slip law. Another motivation for the present study why does not chaos is seen with aging law? In fact, our recent studies have shown that the RSF model with the aging law does not show the chaos at all irrespective of number of state variables added in the friction model. Ruina (1983) has also stated that the sliding system may become more complex upon addition of two or more number of state variables in the slip law. That is why we have chosen the slip law with three state variables.

(4)Referee: There needs to be some discussion of how sensitive their results are to the choice of the state evolution law i.e. 3 Slip law state variables versus 3 Aging law state variables. One would expect the critical stiffness estimate to be insensitive to this choice (given Aging and Slip laws are asymptotically identical near steady state) but behaviour under large perturbations from steady state is likely to be sensitive to this choice.
Authors: The RSF model with the aging law does not the chaotic behaviour at all irrespective of number of state variables considered in the dynamical sliding system. Further we do not see chaos with the aging law even for large perturbations. This is the reason why the present study focuses on slip law which shows the chaos even after adding one more state variable. We are agree with the reviewer that critical stiffness determined using either with the slip law or with aging law, the final expression for critical stiffness is the same (Singh and Singh, 2016 published Geophysical Journal International).

(5)Referee: In a related point, the authors make the following claim in the abstract: "Linear stability analysis shows that critical stiffness, at which dynamical behaviour of the sliding system changes, increases with number of state variables"-it is likely that this conclusion is not generally true. This paper would definitely benefit from a section which systematically studies how the critical stiffness varies as a function of A, B1, B2, B3, L1, L2 and L3.

Authors: We have checked the effect of state variables on critical stiffness and find critical stiffness increases in the present dynamical system. Fig.2 presents the effect of the friction parameter on the critical stiffness of the sliding system. It is obvious from the plots that critical stiffness of the sliding system varies linearly with each of the friction parameters β_2, β_3, ρ and ρ_1. Since the trend is linear in every case as obvious in Fig.2, as a result we have presented only one result that is, k_{cr} vs. β_1. <u>However the editor/reviewers suggest us to do so, we can add these results in the manuscript as well.</u>

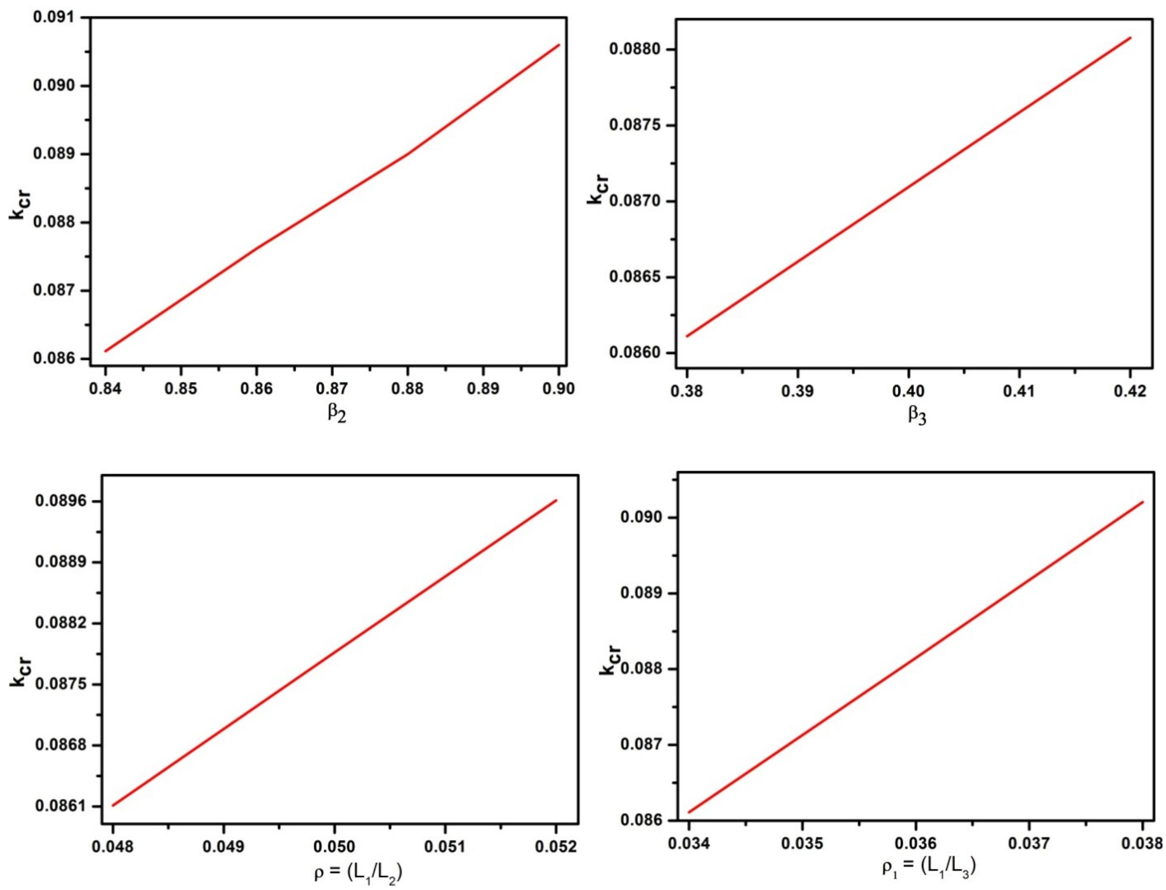

Fig.2. Effect of friction parameter on critical stiffness k_{cr} for β_2, β_3, ρ and ρ_1 for keeping remaining are to be fixed as $\beta_1 = 1.2$, $\beta_2 = 0.84$, $\beta_3 = 0.38$, $\rho = 0.048$, $\rho_1 = 0.034$

(6)Referee: Pg. 2, L6: It is incorrect that a second state variable was required to explain chaos in frictional slip. The second state variable is used to explain experimental observations of slip-weakening in response to rate steps.

Authors: We are fully agreed with the reviewer that the second state variable was added in the RSF model to explain the step velocity experiments (Ruina, 1980, 1983; Tullis, 1986). But this is only friction law of the RSF model which shows the chaos. Now present study establishes that even adding one more state variable that is, third state variable, also shows the chaos. This is most significant result of the present paper.

(7)Referee: Pg. 3, L4, L10: The conditions on the Lyapunov exponents for hyper-chaos as stated are confusing. Does the sum of all LE's require to be +ve or -ve? The authors seem to suggest both at some point.

Authors: We clarify that the present system is not hyper chaotic as we initially claimed in the paper. This is despite the fact that all the Lyapunov exponents are positive. In order to be a hyper chaotic system, the sum of all Lyapunov exponents must be negative. This condition not being met by the present dynamical system as far as the sum of Lyapunov exponents are concerned. We wish to modify at Page no 14 and line no 5-11 as following:

"The present analysis of the 3sRSF model shows in Fig.8 that the magnitude of LEs are LE_1=1.6358, LE_2=0.0525, LE_3=0.0662, LE_4=0.0294. Since sum all LEs is not negative, as a result the present dynamical system is not hyper chaotic."

5. Conclusions

We have established numerically that the three state variables based RSF model with slip law is also chaotic. The route of chaos is established to be period doubling bifurcation. Moreover, linear stability shows that critical stiffness of spring-mass model increases with number of state variables in the RSF model. Moreover, all Lyapunov exponents are positive thus the 3sRSF is more chaotic than the 2sRSF model. Nonetheless the present system is not hyper chaotic as sum of all Lyapunov exponents are not negative.

(8)Referee: Pg. 12, L9: How is the fractal dimension 5.7 for a 4D (3 states, 1 slip rate) system?

Authors: We have already clarified and corrected this error in the uploaded rebuttal to the first reviewer. Moreover, calculation of the fractal dimension of the present dynamical system, which is equal to 5.7, is not correct at all. Since all the Lyapunov exponents are positive thus the formula for calculating fractal dimension may not be valid. As an error, we shall remove this estimation from the present article. <u>We have also removed the following from the paper the following paragraph at Page no.12 and line no 3-9</u> "The relationship between the Lyapunov exponents and fractal dimensions is established by Kaplan and Yorke (1979). They have proposed the Lyapunov or Kaplan-Yorke dimension D_{KY} which is given by the formula: $D_{KY} = D + \frac{1}{|h_D + 1|}\sum_{i=1}^{D} h_i$ Where D is the largest integer for which $\sum_{i=1}^{D} h_i > 0$. As a result, D_{KY} is a convenient geometrical measure of objects in phase space if Lyapunov exponents are known. The fractal dimension of the present dynamical system is calculated to be as 5.70."

(9) Referee: The writing style, grammar needs to substantially improve throughout for this paper to be publishable in an international journal. Copy-editing by a native English speaker might be necessary.

Author: We have revised the whole manuscript with the help of a native English speaker. We believe the present manuscript has improved greatly. We have revised the ABSTRACT at page no 1 and line no 8-15 as following:

Abstract

In this article, we study linear and non-linear stability of the three state variables based rate and state friction (3sRSF) model with spring-mass sliding system. The motivation emanates from the observation in the numerical simulations, unlike "state" variable, temperature, normal stress etc. diminishes the chaotic behaviour of the sliding system. Linear stability analysis shows that critical stiffness, at which dynamical behaviour of the sliding system changes, increases with number of state variables. The bifurcation diagram reveals that route of chaos is period doubling and this has also been confirmed with the Poincaré maps. Moreover, since all Lyapunov exponents of the present dynamical system are positive there by the present 3sRSF is not hyper chaotic. Finally, we believe that this study may be useful to establish the reason why slip law results in chaos but not the aging law of the rate and state friction model.

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We have also checked the results/plots in the present manuscript; we do not find any error except slight improvement in Figs.6 & 8. Otherwise the results are alright. The modified plots are as following:

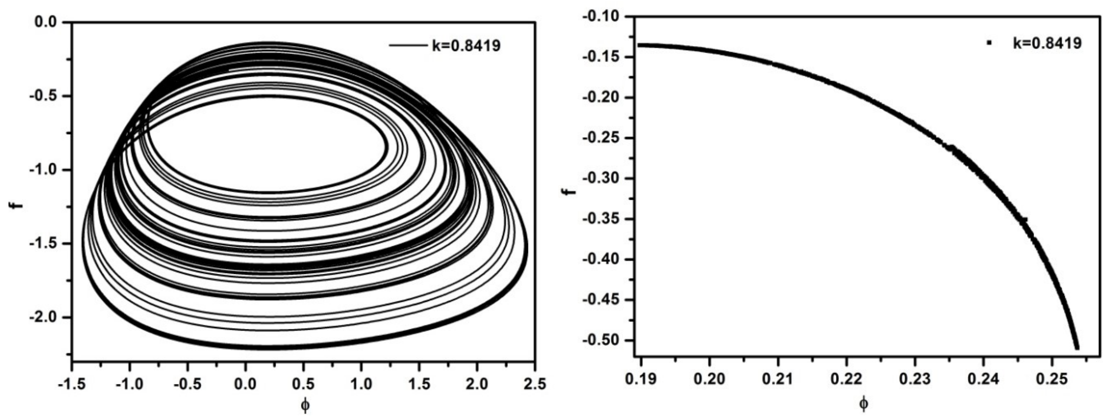

Fig.6. Phase diagram f vs. ϕ and corresponding Poincaré section for $k = 0.08419$, $\beta_1 = 1.0$, $\beta_2 = 0.84$, $\beta_3 = 0.38$, $\rho = 0.048$ and $\rho_1 = 0.034$ for initial condition [0,0,0,0].

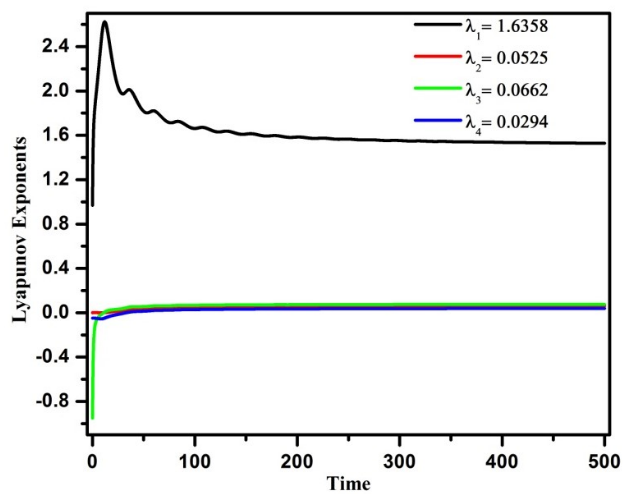

Fig.8. All four Lyapunov exponents vs. time for parameters: k=0.08421, $\beta_1 = 1.0$, $\beta_2 = 0.84$, $\beta_3 = 0.38$, $\rho = 0.048$ and $\rho_1 = 0.034$ and initial condition [0,0,0,0]

We have also removed the twisted phase plots in Fig.9 as we believe that these plots are not significant for the present study. The removed paragraph and plots are as following:

"we have also investigated whether the present friction model fulfils the other condition of hyperchaos. The results in Fig.9 shows the phase diagrams and corresponding. This is also a feature of hyper chaos.

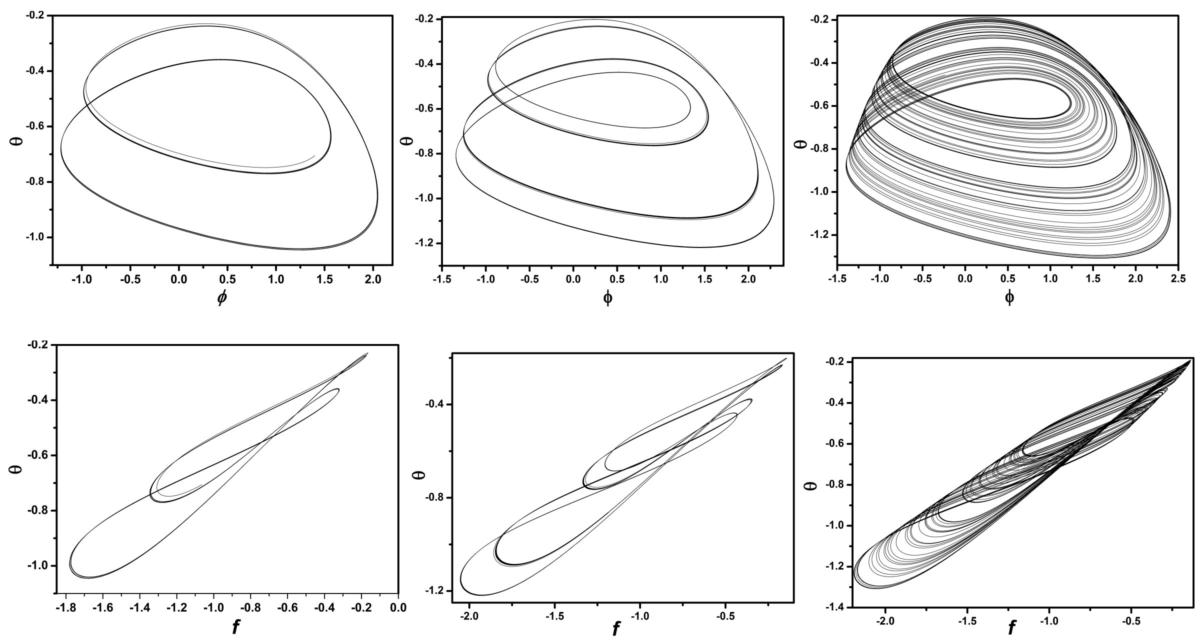

Fig.9. Phase diagrams and corresponding twisted phase diagram for stiffness value (a) $k = 0.085$,(b) $k = 0.08437$,(c) $k = 0.08421$, $\beta_1 = 1.0$, $\beta_2 = 0.84$, $\beta_3 = 0.38$, $\rho = 0.048$ and $\rho_1 = 0.034$ for initial condition [0,0,0,0]."