Response to Referee 2 (Discussion Forum)

Limiting amplitudes of fully nonlinear interfacial tides and solitons (npg-2016-1)

Borja Aguiar-González[1,2*] and Theo Gerkema[2]

[1]Dpto. de Física, Facultad de Ciencias del Mar, ULPGC, E-35017 Las Palmas, Spain

[2]NIOZ Royal Netherlands Institute for Sea Research, P.O. Box 59, 1790 AB Den Burg, Netherlands

Anonymous Referee #2

The work is devoted to a numerical analysis of the MCC-type equations describing strongly nonlinear waves in a two-layer fluid. Its novelties are in adding earth rotation (Coriolis force) and an oscillating forcing imitating a tidal current over a bottom feature. Tidal forcing is represented by an oscillating bottom hill which in most cases gives a reasonable approximation for the case of a fixed hill and periodic current. Ten variants are computed which differ in forcing velocity, layer thicknesses ratio, relative height of the hill, and the Coriolis force (latitude). Some interesting results regarding the parameters of limiting solitons, rate of their formation (in tidal periods). Some results, such as decreasing of soliton amplitude with the increase of forcing, and chande of amplitude and width of a strong limiting soliton, remain unexplained; I agree with authors that it may be due to interaction with current induced by the oscillating source.

In general, the paper deserves publication. However, some questions and notes should be taken into account. Among them are:

1. The MCC system which is the base of the model, allows strong nonlinearity but only weak (quasi-hydrostatic) dispersion. On the other hand, stationary waves, including a soliton, realize a balance between nonlinearity and dispersion. Thus, (unlike the weakly nonlinear case), applicability of such systems for solitons cannot be taken for granted and need to be verified. It is even possible that some numerical 'paradoxes' are due to this limitation (see, e.g., Ostrovsky & Grue, Phys. Fluids, 15, 2934, 2993). This circumstance should at least be mentioned.

We agree on this important remark, which in a revised manuscript would be discussed in section 5 (Discussion and conclusions), as follows:

'As previously noted by Ostrovsky and Grue (2003), MCC-type models entail a paradox to the effect that strongly nonlinear solitons appear from a set of equations that have strong nonlinearity but weak dispersion, while the very existence of solitons presume a

*aguiar@nioz.nl

balance between the two. In our case, the MCC-type model is used, involving only the lowest-order nonhydrostatic dispersive terms. Despite the small parameter featuring in the nonhydrostatic terms, they may actually become large in practice (i.e., in the numerical runs) if wave profiles are steepening, contradicting the original assumption. Indeed, there is no guarantee that the higher-order dispersive terms, which were dropped from these equations, would always remain small. A suggestion for future work is, therefore, to check our results against a numerical computation with a fully nonlinear nonhydrostatic set of equations.

2. The weakly nonlinear and 'quasi-nonlinear' case is not quite clear for me. It should be close to the eKdV case (where the limiting solitons also exist) but the results seem somewhat different. The physic of this case should be better explained.

In a revised version, we would make more clear the distinction between the different set of equations being used so that the physical interpretations are also more clear.

The quasi-nonlinear case involves neglecting the baroclinic interactions but retaining the nonlinear terms involving a combination of barotropic and baroclinic fields. The equations are then still linear with regard to the baroclinic fields, but the coefficients become time-dependent due to barotropic factors (which are prescribed), so that higher harmonics will be generated. Hence one should not expect the quasi-nonlinear case to be close to the eKdV on showing limiting interfacial waves.

In a future version we will re-name the quasi-nonlinear interfacial waves as quasi-linear interfacial waves because we understand that the former name has led to confusion when we discussed the physical interpretations and findings.

3. Paragraph 120. 'c0 is an approximate measure of the linear long wave phase speed.' - Why approximate, what is the approximation?

It was not our intention to mean further approximations. The only point is that this quantity is indicative of the phase speed of linear long interfacial waves but not exactly equal to it, for the precise theoretical value has a factor H_1H_2/H, whereas the present factor is D. For clarity, we have rephrased this paragraph as follows:

'Since we allow waves to have large amplitudes (i.e. being strongly nonlinear), we may take horizontal current velocities to scale with $c_0 = (g'D)^{1/2}$, where g' is reduced gravity, $g' = g\ (\rho_2 - \rho_1)/\bar{\rho}$; and, c_0 is close to the linear long-wave phase speed for interfacial waves (which would have H_1H_2/H instead of D). Thus, u and v will be scaled with c_0. For the interfacial displacement being allowed to be large, an appropriate scale of Z is D.'

4. The reasoning in paragraph 325 should be made simpler and more clear.

We agree. In a revised version lines 323-330 would read:

'We use the generation model of weakly nonlinear, weakly nonhydrostatic interfacial waves derived in Gerkema (1996), which works with tidal motion over a fixed topography, as a benchmark for testing the impact of our 'non-inertial' frame of reference. If we compare interfacial waves generated from the nonlinear version of both models, differences are expected to arise from the fact that forced-MCC equations are fully nonlinear. For this reason we restrict the comparison to the linear and quasi-nonlinear cases. If the results between the models turn out to be similar, it thus seems reasonable to conclude that within the framework of study we can compare our present setting to that in the ocean setting.

5. Figures 8 and 13. How comes that the numerical (color) circles for soliton velocity do not go to 1 at zero amplitude limit? Is this due to some negative period-averaged current?

For clarity, in a revised version Fig. 8 and Fig.13 would also show the corresponding dimensional values along secondary axes (see the new figures at the end of this document).

In all cases the scaled nonlinear phase speed of tide-generated solitons does go to 1 (left y-axis) when solitons approach their minimum amplitude, meaning that they are approaching the linear long-wave phase speed for (baroclinic) interfacial waves. Nevertheless, we note that the soliton amplitude of our numerical solutions does never really reach the zero amplitude limit, but it is always above zero. This is because, to get our 'tracking algorithm' working, we need the leading soliton to be large enough so its characteristic points Z_a, Z_b, Z_c and Z_d, as described in Fig. 6, can be identified and used for computation of its amplitude and width. When the solitons are in their very early stage of generation, i. e. amplitudes near zero, the former characteristic points are not well defined yet. As a result, we can't track solitons at the nearly zero amplitude limit.

To account for a more clear and comprehensive discussion of these results, lines 533-543 would read differently in a revised version:

'Lastly, Fig. 8 compares the wave properties of tide-generated solitons from experiment A1 with solitary wave solutions of the KdV and eKdV theories (Kakutani and Yamasaki (1978), Ostrovsky and Stepanyants (1989), Helfrich and Melville (2006), Gerkema and Zimmerman (2008)).

For a fair comparison, we compute the soliton width for KdV and eKdV theories following the same procedure as for the forced-MCC solitons, i. e. we use points Z_c and Z_d (see Fig. 6c-e). The soliton amplitude and width are scaled, respectively, to the thickness of the upper layer and total water depth. The nonlinear phase speed is scaled to the linear long-wave phase speed for (baroclinic) interfacial waves, $c'_o = \sqrt{g' \ (h_1 h_2)/(h_1 + h_2)}$.

In Fig. 8a, small tide-generated solitons approach the linear long-wave phase speed for (baroclinic) interfacial waves, as expected, while larger solitons have a phase speed following a similar curve to that predicted from the eKdV theory. Nevertheless, tide-generated solitons propagate in all cases slower than their eKdV counterparts. Tide-generated solitons ride on interfacial tides and, hence, their wave properties are not simply the response to a settled two-layer fluid system as it occurs for eKdV solitons, but involve also the forcing of the system.

With respect to Fig. 13, former lines 603-621 would read now simpler:

'When compared to classical solitary wave theories, tide-generated solitons in experiments B1 and C1 resemble wave properties of strongly and weakly nonlinear solitons, respectively (Fig. 13). As it occurred for experiment A1, tide-generated solitons propagate in all cases slower than their eKdV (experiment B1) and KdV (experiment C1) counterparts of similar amplitude.

In experiment B1, the largest solitons start to slightly broaden and present amplitudes larger than those predicted by eKdV theory, although eventually they do not attain the 'table-top' form. In both experiments the largest solitons start to experience a decrease of their amplitude and width when the tidal forcing increases above a certain value, as previously noted from Figs. 11 and 12. These results highlight our finding that tide-generated solitons may be subjected to limiting conditions beyond classical KdV theories. We suggest that these limiting conditions are driven by the appearance of higher harmonics with increasing forcing, and which saturate the underlying quasi-nonlinear internal tide prior to its nonlinear disintegration (see Fig. 3).'

6. Arguments about the role of higher harmonics are unclear. First, there are no spectra shown in the paper. Second, it remains unclear how the Coriolis dispersion can enhance the table-top soliton form (paragraph 635).

We agree with the reviewer that spectra can be a useful tool to explore higher harmonics, as was done, e.g., in Mercier et al (2012), their Fig. 9. However, we think that our analyses presented in Fig. 3 and Fig. 4 already show convincingly that quasi-nonlinear interfacial tides present limiting amplitudes when the tidal forcing increases, in contrast to the linear regime where higher harmonics are neglected with the increase of the forcing. For clarity, we include in this document a figure where the spectra of power density is shown for several cases. Nevertheless, we prefer not to include these results in the final paper unless this is further recommended. The reason is we think these results may not be sheding more light beyond results from former analyses.

In a revised version, and regarding the mechanism by which the Coriolis dispersion can enhance the 'table-top' soliton form, lines 628-645 would read:

'Importantly, an unexpected feature must be noted for experiments A1, B1 and C1. At

relatively low latitudes off the equator (c. f., numerical solutions for the rotationless case and θ=15°), rotational effects appear to favour the development of the leading solitons up to reaching a 'table-top' form. This is especially noticeable in experiments B1 and C1, panels (d) and (f), respectively, for which the rotationless cases do not lead to 'table-top' solitons.

In previous sections we have shown that higher harmonics generated by strong tidal flows cause a saturation of the quasi-nonlinear tide by which solitons emerge, and that this factor limits the development of the leading soliton. We suggest that at relatively low latitudes, the saturation of the quasi-nonlinear interfacial tide weakens as the higher harmonics weaken too due to Coriolis dispersion. Accordingly, underlying quasi-nonlinear interfacial tides reach deeper troughs and favour, in the nonlinear regime, leading solitons attaining a 'table-top' form under forcing conditions which would not generate such solitary waves in the rotationless cases. As expected, at higher latitudes (θ=45°) the dispersive effect of the Coriolis force becomes stronger, thus not only dispersing the higher harmonics but also preventing the nonlinear interfacial tide from disintegrating into strongly nonlinear solitons.'

The above phenomenon, occurring at low latitudes, is also illustrated in Fig. 14, where a set of power density spectra (panel b) is shown for the linear, quasi-nonlinear and fully nonlinear interfacial waves of experiment B1 (panel a), without and with rotational effects included (θ=15°). Adding rotational effects at low latitudes causes a weakening of the power density of the higher harmonics (c. f., blue and red thick lines in panel b), favouring the rise of quasi-nonlinear interfacial tides with a deeper trough (c. f., blue and red thick lines in panel a). In the nonlinear regime, the addition of rotational effects follows the former pattern (c. f., blue and red thin lines in panels a and b), causing deeper troughs of the underlying quasi-nonlinear interfacial tides which eventually allow the leading solitons to develop a 'table-top' form that does not emerge in the rotationless case. Note here that the 'table-top' form for the rotational case is more evident in Fig. 13c,d, where interfacial waves are 'zoomed in' along the spatial domain.

In general: the work is interesting but it is overloaded with details at the expense of clear physical interpretations. If the authors agree to take the above into account, I do not insist on sending the revised paper back to me.

We agree with Referee #2 on that the discussion of the numerical results could be shortened in order to highlight more the physical interpretations we present. In a revised version we would shorten section 4 following this suggestion.

References

*Mercier, M. J., M.Mathur, L.Gostiaux, T.Gerkema, J. M.Magalhaes, J. C. B.DaSilva, and T.Dauxois (2012), Soliton generation by internal tidal beams impinging on a pycnocline: Laboratory experiments, J. Fluid Mech., 704, 37-60.

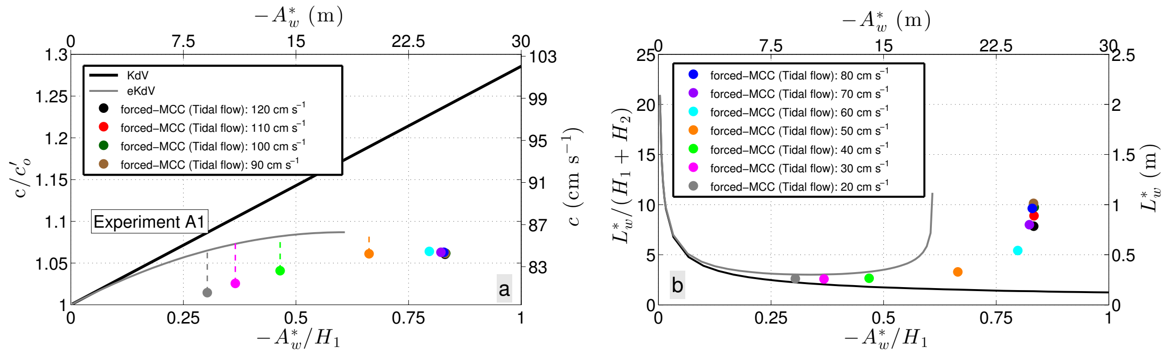

Figure 8: Solitary wave solutions of the KdV (black line) and eKdV (grey line) theories compared to tide-generated solitary waves derived from the forced-MCC equations (colored dots) for experiments A1. The tide-generated solutions correspond to the mean mature leading soliton propagating in Fig. 7 (experiment A1) along its 4^{th} tidal period of age. The different colors indicate the strength of the tidal flow (see legend). (a) Nonlinear phase speed scaled to the linear long-wave phase speed for (baroclinic) interfacial waves vs. soliton amplitude scaled to thickness of the upper layer. (b) Soliton width scaled to total water depth vs. soliton amplitude scaled to thickness of the upper layer. All panels are also shown for the corresponding dimensional form (top and right axes).

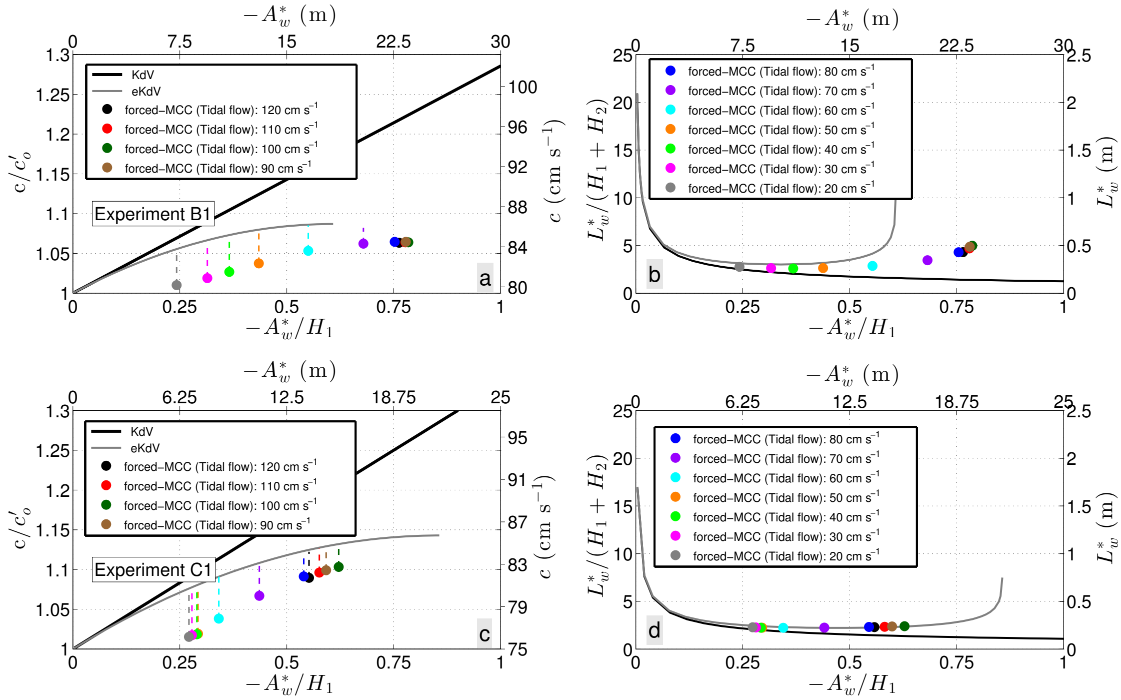

Figure 13: Solitary wave solutions of the KdV (black line) and eKdV (grey line) theories compared to tide-generated solitary waves derived from the forced-MCC equations (colored dots) for experiments B1 (top row) and C1 (bottom row). The tide-generated solutions correspond to the mean mature leading soliton propagating in Figs. 11 (experiment B1) and 12 (experiment C1) along its 4^{th} tidal period of age. The different colors indicate the strength of the tidal flow (see legend). (a,c) Nonlinear phase speed scaled to linear long-wave phase speed for (baroclinic) interfacial waves vs. soliton amplitude scaled to thickness of the upper layer. (b,d) Soliton width scaled to total water depth vs. soliton amplitude scaled to thickness of the upper layer. The dashed lines in (a) and (c) highlight the varying departure between soliton solutions from eKdV and forced-MCC equations.

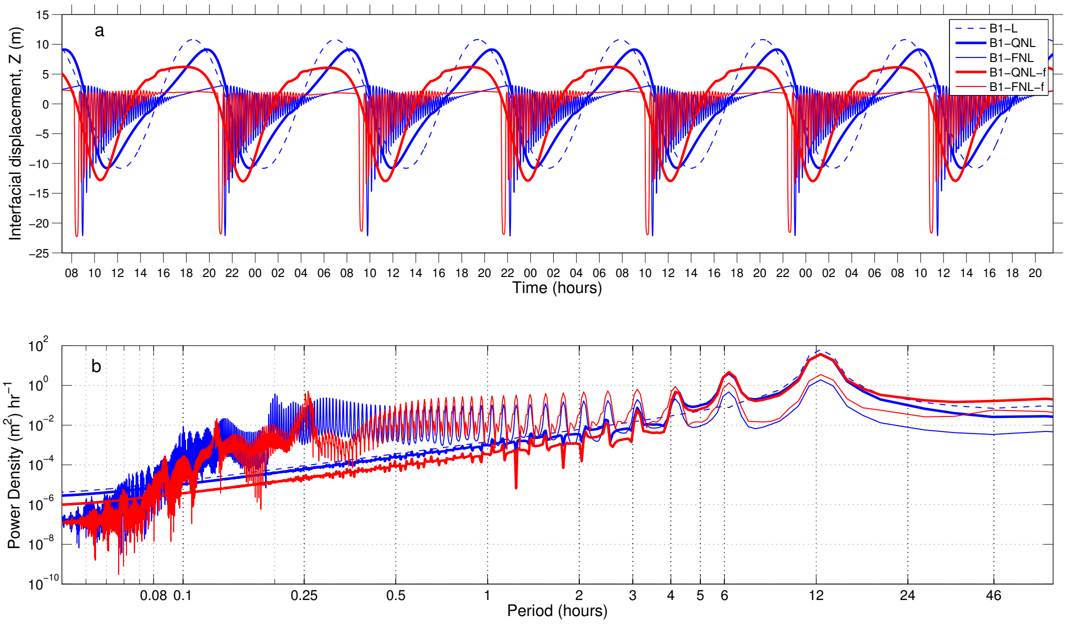

Figure 14: (a) Time evolution of linear (L), quasi-nonlinear (QNL) and fully nonlinear (FNL) tide-generated interfacial waves in experiment B1 computed from the forced-MCC-f equations, without and with rotational effects included (blue and red lines, respectively). Spectra of power density for interfacial waves in (a). This time-series corresponds to a 'mooring' located at 115 km leftward from the sill.