

## Comments from Referees: J. M. Miras Avalos (Editor)

The manuscript entitled “Fractal behavior of soil water storage at multiple depths” (Reference number NPG-2015-81) authored by W. Ji, M. Lin, A. Biswas, B.C. Si, H.W. Chau, and P. Cresswell presents results from a five-year study on the soil water storage from a transect in a hummocky landscape of central Canada. The authors applied multifractal and joint multifractal theories to this huge dataset in order to describe the fractal behavior of this variable at different depths along the transect.

I agree with the comments posted by reviewer1 and consider this manuscript very-well written and acceptable for publication after several modifications. Unfortunately, I have not received the comments from the second reviewer yet. Anyway, I carefully read the submitted manuscript and performed some comments and suggestions in order to improve its quality.

The reported work is interesting and fits perfectly well within the scope of the Special Issue “Multifractal analysis in soil systems” to be published in *Nonlinear Processes in Geophysics*. However, the manuscript is rather long and information can be condensed as well as reduced since it seems repetitive in some portions. Tables can be improved and, from my point of view, figure 9 is not needed and can be deleted. Finally, a few English mistakes must be corrected.

In the attached file (supplement), I provide the authors with some suggestions in order to improve their manuscript. Therefore, the authors must address these issues prior to the acceptance of their manuscript. They must correct them in order that this manuscript achieves the standard quality for being published in *Nonlinear Processes in Geophysics*.

Therefore, I recommend a moderate revision prior to its publication in this journal.

Please also note the supplement to this comment: <http://www.nonlin-processes-geophys-discuss.net/npg-2015-81/npg-2015-81-EC1-supplement.pdf>

*-- Response: Thank you very much for your detailed comments. Your comments and suggestions have greatly help improve our manuscript. We have tried to condense the context by deleting some repetitive parts and combining the information by figures. The tables and figures have been revised and supplemented according to all the review comments. New figures are also added and figure sequence have been changed to improve the structure of manuscript. We have also worked carefully on the English.*

*We have also documented all the changes we made in the revised version and responded to each comment individually.*

### Author’s changes

Lines 28-30: Only for the dry period? This is somewhat unclear.

*-- Response: Not only dry period, here it is just to discuss the dry period specifically as a comparison to wet period mentioned above. We changed the “in contrast” to “on the other hand”, which may shows the logic better. (L26)*

Lines 44-46: You used the word “scale” too many times in this sentence.

-- Response: "Scale" has been changed to "extent" in some sentences to increase the variability according your suggestions. (L40, L47, etc)

Lines 68-69: I am not sure that "indicating the superficial properties" is needed.

-- Response: We think it is required, for it acknowledges the achievement of previous studies. (L71)

### **Materials and Methods:**

Line 87: Please, indicate the elevation above sea level of the study site.

-- Response: The elevation data included. (L89)

Line 96: "at every 20 cm depth", you should indicate down to what depth.

-- Response: Has added "down to 140cm" (L104)

Lines 135-138: This last sentence is not clear, please, re-phrase it

-- Response: The sentence has been rewritten (L150-152)

Lines 226-229: These values are not included in Table 1. Why did you mention this table here?

-- Response: Yes, that was a mistake. We have deleted this.

Lines 230-231: "The highest average SWS..." this is not true for all depths.

-- Response: The clause of "for the surface layer" has been added. (L257)

Line 231: "large amount of spring rainfall", data on rainfall are not shown.

-- Response: Data is added. (L258)

Line 245: The minimum is 6.71 cm according to table 1 and not 6.72 cm as you said in the text

-- Response: The text has been changed to 6.71cm as it was a typo. (L273)

Line 146: Define UM when first used, please.

-- Response: The UM is expanded during first use. (L160)

Lines 321-329: This is messy and unclear, even somewhat repetitive. Please, re-phrase.

-- Response: We have modified the paragraph and tried to make it clearer. (L347-359)

Line 371: "A very similar trend was observed in other years". These data are not shown. Indicate this and also briefly specify the similarity.

-- Response: We have added a new figure (Fig. 11) to explain the pattern.

Line 396: "average SWS in a year", only in one year?

-- Response: No, it was for other years too. This is a general observation. We have deleted the word 'in a year'. (L445)

Lines 271-273: This looks like materials and methods and not results.

-- *Response: It reads like that as the result was generalized. This is necessary to main the flow of the results and discussion. So, we kept this section. However, we did modify a bit. (L484-496)*

Discussion

This section can be reduced since it seems repetitive and information can be condensed because some paragraphs look like materials and methods.

-- *Response: We have reorganized the paragraphs and reinterpreted the figures to provide further information after the analysis in the result section.*

Lines 451-457: Is this paragraph really needed? It repeats the former paragraphs.

-- *Response: We think this summary is required to increase the readability. Therefore, we kept this paragraph.*

Line 498: Is figure 9 really needed?

-- *Response: We think the figure is a summary of the main conclusion which is needed for the ease of understanding. This also shows the general trend in the data and the analysis result. Therefore, we kept the figure in the revised manuscript.*

Lines 504-506: The idea mentioned in this last sentence is not well developed throughout the text, 7 especially it is not discussed at all in the discussion section. However, it appeared in the abstract.

-- *Response: Yes, it seemed little weak and we have deleted this sentence.*

Lines 520-522: "Therefore, the observation completed...", I am not sure about this conclusion.

-- *Response: Through this sentence, we mean to say the general pattern of the scaling indices. The relationship can be directly transferred to other field situations given the similar kind of landscape and climate condition.*

Table 1: Please, include the five-year averages, since you refer to them in the text.

-- *Response: Added.*

Table 2: Please, indicate the number of data used for each correlation. Was it the same for all dates and depths?

-- *Response: The number of data is same for all and is mentioned in the title.*

Figure 1: Why not showing the Y-scale in all left graphs?

-- *Response: We have added y-axes for all the plots.*

Figure 2 and 3: It is very difficult to distinguish the points from each depth. Besides, the UM model is missing from the graphs.

-- *Response: Yes, there were too many graphs. We have reduced to only three. All the graphs have the UM model but it is not visible due to the condition of the plots.*

Figures 6 and 7: Some values are overlapped in the Y-axis.

-- *Response: We have modified the Fig. 6 and Fig 7. These are assigned to new numbers.*

Figure 9: Is this figure really needed?

-- *Response: We think the figure is a summary of the main conclusion which is needed for the ease of understanding. This also shows the general trend in the data and the analysis result. Therefore, we kept the figure in the revised manuscript.*

### **Comments from Referees: Anonymous Referee #1**

**General Comments:** This manuscript repeatedly evaluates soil water storage (SWS) across an transect consisting of 128 individual measurement points by multifractal and joint multifractal analysis. In each point SWS was determined at seven depth increments, and this several times per year during a five years period. Therefore the relationship between several multifractal parameters obtained from SWS transects as a function of depth and time were investigated. This study follows previous studies carried out with in the same site, with the same experimental design and using similar methods of analysis. The rationale and the objectives exposed in the Introduction are worthwhile and in general the work appears well justified and appealing for the international reader of this journal. The main findings, such as the usefulness of either multifractal or monofractal parameters to assess patters of heterogeneity and evenness of SWS transects with increasing soil depth and for different seasons of the year. In general, the paper is well written and organized, and represents an original contribution, even if it follows previous similar work. The results are based in robust data analysis. This study also is compatible with the aims Nonlinear Processes in Geophysics and may fit well into the scope of the current special section titled “Multifractal analysis in soil systems”. In my opinion, it should be acceptable for publication following minor to moderate revisions. In my opinion the manuscript could be ameliorated by plotting selected multifractal parameters (for example, the amplitude of the singularity spectrum,  $\tau$ ,  $A_{\max}$ ,  $A_{\min}$ , or the information dimension  $D1$ ) as a function of depth and time, or both, depth and time. Next I'm indicating two places where plots are recommended, but this is not exhaustive.

*Response: Thank you very much for your detailed comments. It really helped modify the manuscript. We have addressed the comments individually and documented the responses below.*

**Specific Comments** 1. Page 4, Lines 95-100. I recommend to briefly describe the methods used to measure soil water content and to evaluate soil water storage (SWS), even if they have been already detailed described before.

*Response: We have included a brief description of the data collection in the materials an methods (L102-111).*

2.- Page 9, Lines 226-243. I suggest to draw a graphic with these statistical information; then check if including this graph increases readability.

*Response: We have included a new figure (Fig. 11) showing the joint multifractal spectra between two spatial series of soil water storage measure on 22 October 2008.*

3.- Page 12 and 13, Lines 342-352. Again, I suggest to draw a graphic plotting ( $\tau(A_{\max}) - \tau(A_{\min})$ ) as a function of depth for several measurement periods.

*Response: Thanks for the suggestion. We have included two figures; Fig. 5 showing the  $\alpha_{\max} - \alpha_{\min}$  values for all the measurements at all depths and Fig. 8 showing the D1 values for all the measurements at all depths.*

4.- Page 23, Table 2. I suggest to include figures showing some multifractal spectra either in the main manuscript or as supplementary content.

*Response: We have included a new figure (Fig. 11) as an example showing the joint multifractal spectra between two spatial series of soil water storage measure on 22 October 2008.*

5.- Page 25, Figures 2 and 3. I recommend to show only two or four selected plots of mass exponent functions to increase visibility. (because of the small size of the Figures, differences are hardly to view).

*Response: We have modified the figures. Now we have only included only 3 measurement dates for both the figures. The figure numbers have been changed. The new numbers are Fig. 3 and Fig. 4.*

6.- Page 26, Figures 4 and 5. I suggest to take into account the shape of the singularity spectra and not only the amplitude in this and the Results and in the Discussion sections; also these shapes should provide valuable information, I guess.

*Response: Thank you very much for the comments. We discussed about the shape of the spectra specifically the non-uniformity and the tails of the spectra and their meaning in terms of the distribution of scaling indices (L360-371).*

7.- Page 28, Figure 8. I suggest to move Figure 8 (scheme of the vegetation growth patterns) either to the Material and Method section, or to the section 3.1 (Spatial pattern of soil water storage at different depths). Indeed this Figure is related to the Discussion section, but it is also pertinent to previous section.

*Response: We have moved the figure into materials and methods section and first introduced in L 95. Now the new figure is marked as Fig. 1.*

## **Comments from Referees: Anonymous Referee #2**

This work makes use of multifractal analysis and joint multifractal analysis to study the spatio-temporal behavior of soil water storage (SWW) at multiple depths. Several interesting implications about the scaling nature of SWW that are relevant to transfer information from one scale to another, are shown. The manuscript is well structured and conclusions are drawn from sound mathematical theories applied to a rich enough database consistent with the algorithms

used to estimate theoretical parameters. Therefore, I recommend acceptance following minor revisions.

My specific comments are itemized below: page 6, line 163 and page 6, line 167: I suggest to use Chhabra and Jensen (1989) as a reference instead of Everest and Mandelbrot (1992).

*Response: The reference has been revised from Everest and Mandelbrot (1992) to Chhabra and Jensen (1989) (L187 and L223)*

page 7, line 197 and page 17, line 478: This parameter was first introduced in Caniego, Martín and San José (2003) page 14, line 384: There are two points instead of one to the end of the line. Reference: Caniego, F.J., Martín, M.A. and San José, F., 2003. Rényi dimensions of soil pore size distribution. Geoderma 112 (2003) 205– 216.

*Response: Thanks. We have included the reference (L505).*

# 1 Fractal behavior of soil water storage at multiple depths

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10 7990)

11

12 **Abstract** Spatio-temporal behavior of soil water is essential to understand the science of  
13 hydrodynamics. Data intensive measurement of surface soil water using remote sensing has  
14 established that the spatial variability of soil water can be described using the principle of self-  
15 similarity (scaling properties) or fractal theory. This information can be used in determining  
16 land management practices provided the surface scaling properties ~~held are kept~~ at deep layer.  
17 Current study examined the scaling properties of sub-surface soil water and ~~its~~ their  
18 relationship to surface soil water, thereby serving as ~~the~~ supporting information for ~~the~~ plant  
19 root and vadose zone models. Soil water storage (SWS) down to 1.4 m depth at seven equal  
20 intervals was measured along a transect of 576 m for 5 years in Saskatchewan. The surface  
21 SWS showed multifractal nature only during the wet period (from snowmelt until mid to late  
22 June with large SWS) indicating the need ~~of~~ for multiple scaling indices in transferring soil  
23 water variability information over multiple scales. However, with increasing depth, the SWS  
24 became monofractal in nature indicating the need ~~of~~ for a single scaling index to  
25 upscale/downscale soil water variability information. The dynamic nature ~~made of~~ the surface  
26 layer soil water in the wet period ~~is~~ highly variable compared to the deep layers. In contrast,  
27 all soil layers during the dry period (from late June to the end of the growing season in early  
28 November ~~with low SWS~~) were monofractal in nature, probably resulting from the high  
29 evapotranspirative demand of the growing vegetation that surpassed other effects. This strong  
30 similarity between the scaling properties at the surface layer and deep layers provides the  
31 possibility of inferring about the whole profile soil water dynamics using the scaling properties  
32 of the easy-to-measure surface SWS data.

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Commented [r411]: Only dry period?

Commented [r412R1]: Lines28-30: Not only dry period, here it is just to discuss the dry period specifically as a comparison to wet period mentioned above. We changed the "in contrast" to "on the other hand", which may shows the logic better.

33 **Keywords** ~~Scaling, s~~Scale invariance, monofractal, multifractal, root zone, remote sensing

## 34 1 Introduction

35 Knowledge on the spatial distribution of soil water over a range of spatial ~~scales~~ and time has  
36 important hydrologic applications including assessment of land-atmosphere interactions  
37 (Sivapalan, 1992), performance of various engineered covers, monitoring soil water balance  
38 and validating various climatic and hydrological models (Rodriguez-Iturbe et al., 1995;Koster  
39 et al., 2004). However, high variability in soil is a major challenge in hydrology (Quinn, 2004)  
40 as the distribution of soil water in the landscape is controlled by various factors and processes  
41 operating at different intensities over a variety of ~~extent~~scales (Entin et al., 2000). The  
42 individual and/or combined influence of these physical factors (e.g. topography, soil  
43 properties) and environmental processes (e.g. runoff, evapotranspiration, and snowmelt) gives  
44 rise to complex and nested effects, which in turn evolve a signature in the spatial organization  
45 (Western et al., 1999) or patterns in soil water as a function of spatial scale (Kachanoski and  
46 Dejong, 1988;Kim and Barros, 2002;Biswas and Si, 2011a). This complexity makes the  
47 management decision difficult at a scale other than the scale of measurement. Therefore, it is  
48 necessary to transfer variability information from one ~~extent~~scale (e.g. pedon scale) to another  
49 (e.g. large catchment scale), which is called scaling.

50 The scaling of soil water is possible if the distribution of some statistical parameters (e.g.,  
51 variance) remain similar at all studied ~~scope~~scales. This feature, known as scale-invariance,  
52 means that the spatial feature in the distribution of soil water will not change if the length scales  
53 are multiplied by a common factor (Hu et al., 1997). Generally, the soil water will have a typical  
54 size or scale, a value around which individual measurements are centered. So the probability  
55 of measuring a particular value will vary inversely as a power of that value, which is known as  
56 the power law decay, a typical ~~principle~~of scaling process. Now, as the spatial distribution of  
57 soil water follows the power law decay (Hu et al., 1997;Kim and Barros, 2002;Mascaro et al.,  
58 2010), the spatial variability can be investigated and characterized quantitatively over a large  
59 range of measurement ~~seales-extents~~ using ~~the~~ fractal theory (Mandelbrot, 1982). When the  
60 spatial distribution of soil water is the response of some linear processes, the scaling can be  
61 done using a single ~~scaling-coefficient~~ over multiple scales and the distribution shows  
62 monofractal ~~scaling-behaviour~~behavior. However, the spatial distribution of soil water is the  
63 nonlinear response of multiple factors and processes acting over a variety of scales and  
64 therefore needs multiple scaling indices (multifractal ~~scalings~~) ~~in-for~~ quantifying spatial  
65 variability (Hu et al., 1997;Kim and Barros, 2002;Mascaro et al., 2010).

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~~The multifractal scaling behaviour of soil water has been used in developing models to downscale soil water estimate from remotely sensed measurements with a large foot print area.~~

**Commented [r414]:** Has been implied in the next sentences

The multifractal ~~behaviour~~behavior in the surface soil water as a result of temporal evolution of wetting and drying cycles ~~haves~~ have been reported from a sub-humid environment of Oklahoma by Kim and Barros (2002). Mascaro et al. (2010) reported the multifractal ~~behaviour~~behavior of soil water, which was ascribed as a signature of the rainfall spatial variability. Though these measurements can provide a ~~quick~~ estimate of soil water over a large area ~~quickly~~, they are limited to very few centimeters of the soil profile. These studies reported the multifractal ~~behaviour~~behavior of only the surface soil water ~~indicating the superficial scaling properties.~~ Surface soil layer is exposed to direct environmental ~~forcing forces~~ and ~~are is the~~ most dynamic in nature. The scaling properties of surface soil water can be used for land management practices provided the observed scaling properties ~~holds~~remain the same characteristics for the deep layers such as vadose zone or the whole soil profile. Understanding overall hydrological dynamics in soil profile needs information on the scaling properties and the nature of the spatial variability of soil water over a range of scales at deep layers as well (Biswas et al., 2012b). The information on the similarity in the nature of the spatial variability of soil water between the surface layer and deep layers may also help inferring about the soil profile hydrological dynamics. Therefore, the objectives of this study were to examine ~~over time~~ the scaling properties of sub-surface layers and their relationship with surface layers at different initial soil water conditions ~~over time~~. We have examined the scaling properties of soil water storage at ~~multiple depth layers each layer~~ and ~~their trend at soil layers~~ with increasing depth from the surface (cumulative depth) over a 5-year period from a hummocky landscape from central Canada using the multifractal analysis. The relationship between the scaling properties of the surface layer and the subsurface layers was also examined using the joint multifractal analysis.

**Commented [r415]:** Not sure that this clause is needed

**Commented [r416R5]:** We think it is required, for it acknowledges the achievement of previous studies.

## 2 Materials and Methods

### 2.1 Study site and data collection

A field experiment was carried out at St. Denis National Wildlife Area (52°12'N ~~latitude-~~ ~~And~~ ~~-106°50'W longitude and ~549 m above sea level~~), which is located 40 km east of Saskatoon, Saskatchewan, Canada. The landscape of the study area is hummocky with a complex sequence of slopes (10 to 15%) extending from different ~~ly~~ sized rounded depressions to irregular complex knolls and knobs, a characteristic landscape of the North American Prairie pothole region encompassing approximately 780,000 km<sup>2</sup> from north-central United States to south-central Canada (National Wetlands Working Group, 1997). ~~Some of these potholes are~~

**Commented [r417]:** Please indicate the elevation above sea level

99 seasonal in nature meaning to store water in the spring (wet period) and drying out during later  
100 summer and in fall season (dry period) (Fig. 1). Variables water distribution within the  
101 landscape and in different landform elements such as side slopes, knolls, and depressions  
102 support vegetation differently. For example, the large amount of stored water in depressions  
103 provide a luxurious supply of water to growing plants compared to knolls (Fig. 1). A transect  
104 of 128 points (576 m long) extending in the north-south direction covering multiple knoll-  
105 depression cycles was established in 2004 at the study site to examine the soil water variation  
106 at field scale. The sample points were selected at 4.5 m regular intervals along the transect to  
107 catch the systematic variability of soil water. Soil water measurements were carried out at every  
108 20 cm depth down to 140cm along the transect over the period of 2007 to 2011, among which,  
109 the surface soil water (0 to 20 cm) was measured using vertically installed time domain  
110 reflectometry (TDR) probes and a metallic cable tester (Model 1502B, Tektronix, Beaverton,  
111 OR), while the rest deeper soil, down to 140 cm depth was measured using a neutron probe  
112 (Model CPN 501 DR Depthprobe, CPN International Inc., Martinez, CA) (Biswas et al.,  
113 2012a), and were used in this study. These measured data of soil water content from either the  
114 neutron probe or TDR were then multiplied with depth and added together to obtain the overall  
115 soil profile water storage so as -to examine the fractal behavior of SWS at different depths-  
116 over time-  
117 A detailed description of the study site, development of the transect, measurement  
118 of soil water and the calibration of measurement instruments can be found in earlier  
119 publications from this project (e.g. Biswas et al. (2012a)).

## 119 2.2 Data analysis

120 Various methods including geostatistics, spectral analysis, and wavelet analysis have been used  
121 to examine the scale-dependent spatial patterns of SWS. These methods generally deal with  
122 how the second moment of SWS changes with scales or frequencies. When the statistical  
123 distribution of SWS is normal, the second moment plus the average provide a complete  
124 description of the spatial series. However, for other distributions (e.g. left skewed distribution),  
125 higher-order moments are necessary for a complete description of the spatial series. For  
126 example, let's define the  $q^{\text{th}}$  moment of a spatial series  $z$  as  $z^q$ . In this situation, for a positive  
127 value of  $q$ , the  $q^{\text{th}}$  moment magnify the effect of larger numbers and diminish the effect of  
128 smaller numbers in  $z$ . While, on the other hand, for a negative value of  $q$ , the  $q^{\text{th}}$  moment  
129 magnify the effect of small numbers and diminish the effect of large numbers in the spatial  
130 series  $z$ . In this way, using variable moments, we can look at the effect of the magnitude of the  
131 data in a series and better characterize its spatial variability better. There is a pressing need to

**Commented [r418]:** Anonymous #1: recommend to briefly describe the methods used to measure soil water content and to evaluate soil water storage (SWS), even if they have been already detailed described before.

**Commented [r419R8]:** Has been added with the contents from Biswas et al., 2012a)

**Commented [r4110]:** Indicate down to what depth

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132 ~~summarize how these moments change with scales so that we can compare and simulate~~  
133 ~~spatially variable SWS.~~

### 134 2.2.1 Statistical self-similarity or scale invariance

135 Soil water is highly variable in space and time. If the variability in the spatial/temporal  
136 distribution remains statistically similar at all studied scales, the SWS is assumed to be self-  
137 similar (Evertsz and Mandelbrot, 1992). Self-similarity, also called scale invariance, is closely  
138 associated with the transfer of information from one scale to another (~~scaling~~). We used the  
139 multifractal analysis to explore self-similarity or inherent differences in scaling properties of  
140 SWS in this study.

### 141 2.2.2 Multifractal analysis

142 On the spatial domain of the studied field, multifractal analysis was used to characterize the  
143 scaling property of SWS by statistically measuring the mass distribution (Zelege and Si, 2004).  
144 The spatial domain or the data along the transect was successively divided into self-similar  
145 segments following the rule of the binomial multiplicative cascade (Evertsz and Mandelbrot,  
146 1992). This method required that the two segments divided from a unit interval to be of equal  
147 length. With regards to a unit mass  $M$  (a normalized probability distribution of a variable or  
148 measured in a generalized case) relating to the unit interval, the weight was also partitioned  
149 into  $[h \times M]$  and  $[(1-h) \times M]$ , where  $h$  was a random variable ( $0 \leq h \leq 1$ ) governed by a  
150 probability density function. Sequentially, the new subsets with ~~their~~ associated mass were  
151 equally divided into smaller parts. In this way, multifractal analysis was able to describe the  
152 scaling properties for the higher-order moments compared to semivariogram which can only  
153 measure the scaling properties of the second moment. In a special case, if the scaling properties  
154 do not change with  $q$ , the spatial series can be identified as monofractal, when one scaling  
155 coefficient is enough to characterize scaling property of SWS. Generally, the multifractal  
156 analysis is good at measuring the highly fluctuated mass (box size) within a scale interval. This  
157 ~~also well as providing provides~~ physical insights at all scales regardless of any ad hoc  
158 parameterization or homogeneity assumptions in the analysis (Schertzer and Lovejoy, 1987).

159 For SWS spatial series, the scale-invariant mass exponent, was termed as  $\tau(q)$  (Liu and  
160 Molz (1997):

$$161 \langle [\Delta z(x)]^q \rangle \propto x^{-\tau(q)} \quad [1]$$

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Commented [r4111]: Need to be rephrased

162 where  $z$  was the SWS spatial series,  $x$  was the lag distance and the symbol  $\propto$  indicated  
163 proportionality. The  $\tau(q)$  is widely used in multifractal analysis. If the plot of  $\tau(q)$  vs.  $q$  [or  $\tau(q)$   
164 curve] has a single slope (i.e. a linear line), then the series is a simple scaling (monofractal)  
165 type. If  $\tau(q)$  curve is nonlinear and convex (facing downward), then the series is a multiscaling  
166 (multifractal) type. In this study, we used the [universal multifractal \(UM\)](#) model of Schertzer  
167 and Lovejoy (1987) to create a ~~linear~~ reference line ~~that~~~~which~~ represented the perfect  
168 monofractal type of scaling. Assuming the conservation in mean value of SWS, this model  
169 simulated a cascade process with a scaling function in an empirical moment. It is thus used  
170 here to compare and characterize the observed scaling properties with a reference to the  
171 monofractal behavior. The goodness-of-fit between the  $\tau(q)$  curves and the UM model was  
172 tested using the chi-square test. The sum of squared residuals (SSRs) between the  $\tau(q)$  curve  
173 and the UM model was also calculated to test the deviation. The  $\tau(q)$  curves over the range of  
174  $q$  values (in this study -15 to 15 at 0.5 intervals) were fitted with a linear regression line  
175 (referred to as a single fit). The linear fitting of the  $\tau(q)$  curves with  $q < 0$  and  $q > 0$  (referred to  
176 as segmented fit) ~~were~~~~was~~ also completed. The difference between the mean of slopes and  
177 segmented fits (for positive and negative  $q$  values) was ~~tested~~~~checked~~ using the Student's  $t$   
178 test.

179 With similar manner to Eq. [1], the  $q^{\text{th}}$  order normalized probability measure of SWS,  
180  $\mu(q, \varepsilon)$  (also known as the partition function), is ~~proved~~ to vary with the scale size, as below

$$181 \mu_i(q, \varepsilon) = \frac{[p_i(\varepsilon)]^q}{\sum_i [p_i(\varepsilon)]^q} \propto (\varepsilon/L)^{\tau(q)} \quad [2]$$

182 where  $\varepsilon$  is scale size in the  $i^{\text{th}}$  segment and  $p_i(\varepsilon)$  is the probability of a measure.  $p_i(\varepsilon)$  and  
183 measures the concentration of a variable of interest (e.g. SWS) by dividing the value of the  
184 variable in the segment to the whole support length (e.g. to the whole transect of length  $L$  units)  
185 (Meneveau et al., 1990; [Chhabra and Jensen, 1989](#); [Evertsz and Mandelbrot, 1992](#)). The mass  
186 exponent  $\tau(q)$  was related to the probability of mass distribution of SWS.

187 Moreover, the fractal dimension of the subsets of segments in scale size  $\varepsilon$  was measured by  
188 the multifractal spectrum  $f(q)$ . When a coarse Hölder exponent (local scaling indices) of  $\alpha$  was  
189 in the limit as  $\varepsilon \rightarrow 0$ ,  $f(q)$  was calculated as below ([Chhabra and Jensen, 1989](#); [Evertsz and](#)  
190 [Mandelbrot, 1992](#)):

191 
$$f(q) = \lim_{\varepsilon \rightarrow 0} \left( \log \left( \frac{\varepsilon}{L} \right) \right)^{-1} \sum_i \mu_i(q, \varepsilon) \log \mu_i(q, \varepsilon) \quad [3]$$

192 and the local scaling indices,  $\alpha$ , were given by

193 
$$\alpha(q) = \lim_{\varepsilon \rightarrow 0} \left( \log \left( \frac{\varepsilon}{L} \right) \right)^{-1} \sum_i \mu_i(q, \varepsilon) \log p_i(\varepsilon) \quad [4]$$

194 Noting that  $f(\alpha)$  was determined through the Legendre transform of the  $\tau(q)$  curve:

195 
$$f(\alpha) = q\alpha(q) - \tau(q) \text{ (Chhabra and Jensen, 1989).}$$

196 The multifractal spectrum is a powerful tool in portraying the similarity and/or differences  
 197 between the scaling properties of the measures (e.g. SWS). ~~This spectrum also enabled us to~~  
 198 ~~examine the local scaling property.~~ The width of the spectrum ( $\alpha_{\max} - \alpha_{\min}$ ) was used to examine  
 199 the heterogeneity in the local scaling indices. The wider the spectrum, the higher was the  
 200 heterogeneity in the distribution of SWS and vice versa. Similarly, the height of the spectrum  
 201 corresponded to the dimension of the scaling indices. The small  $f(q)$  values indicated rare  
 202 events (extreme values in the distribution), whereas the largest value was the capacity  
 203 dimension ( $D_0$ ) obtained at  $q = 0$ .

204 In addition to the multifractal spectrum, [ $f(q)$  vs.  $\alpha(q)$ ], for many practical applications, we  
 205 used models to incorporate a few selected indicators to describe the scaling property and  
 206 variability of a process. One of the widely used models for multifractal measure were the  
 207 generalized dimensions, which was calculated as below:

208 
$$D_q = \frac{1}{q-1} \lim_{\varepsilon \rightarrow 0} \frac{\log \sum_i p_i(\varepsilon)}{\log(\varepsilon)} \quad [5]$$

209 when  $q = 1$ ,  $D_1$  was referred to as the information dimension (also known as entropy dimension)  
 210 which provided information about the degree of heterogeneity in the measure distribution in  
 211 analogy to the entropy of an open system in thermodynamics (Voss, 1988). If the value of  $D_1$   
 212 is close to unity, it indicated the evenness of measures over the sets of cell size, while the value  
 213 approaching 0 indicated a subset of scale in which the irregularities were concentrated. The  $D_2$ ,  
 214 known as the correlation dimension, was associated with the correlation function and measured  
 215 the average distribution density of the SWS (Grassberger and Procaccia, 1983). For a  
 216 monofractal distribution, the  $D_1$  and  $D_2$  tend to be equal to the  $D_0$ . The same value of  $D_0$ ,  $D_1$   
 217 and  $D_2$  indicates that the distribution exhibits perfect self-similarity and is homogeneous in  
 218 nature. Contrarily, in multifractal type scaling, the  $D_1$  and  $D_2$  tend to be smaller than  $D_0$ ,

219 showing  $D_0 > D_1 > D_2$ . Accordingly, the  $D_1/D_0$  value can be used to describe the heterogeneity  
 220 in the distribution (Montero, 2005; Caniego, Martin and San José, 2003). The ~~When this value~~  
 221 equals to 1, it indicated exact monoscaling of the distribution.

### 222 2.2.3 Joint multifractal analysis

223 While the multifractal analysis characterized the distribution of a SWS spatial series along its  
 224 geometric support, the joint multifractal analysis was used to characterize the joint distribution  
 225 of two SWS spatial series along a common geometric support. As an extension of the  
 226 multifractal analysis, the length of the datasets was also divided into several segments of size  
 227  $\varepsilon$ . Two variables ( $P_i(\varepsilon)$  and  $R_i(\varepsilon)$  representing two spatial series of SWS) were used here to  
 228 measure the probability of the measure in the  $i^{\text{th}}$  segment, when  $P_i(\varepsilon) \propto (\varepsilon/L)^\alpha$  and  $R_i(\varepsilon) \propto (\varepsilon/L)^\beta$   
 229 . Among them,  $\alpha$  and  $\beta$  were the local singularity strength which respectively represented the  
 230 mean local exponents of  $P_i(\varepsilon)$  and  $R_i(\varepsilon)$  in the corresponding expressions above. The  
 231 partition function for the joint distribution of  $P_i(\varepsilon)$  and  $R_i(\varepsilon)$ , was calculated as below  
 232 (Chhabra and Jensen, 1989; Meneveau et al., 1990; Zeleke and Si, 2004):

$$233 \mu_i(q, t, \varepsilon) = \frac{p_i(\varepsilon)^q \cdot r_i(\varepsilon)^t}{\sum_{j=1}^{N(\varepsilon)} [p_j(\varepsilon)^q \cdot r_j(\varepsilon)^t]} \quad [6]$$

234 where the normalized  $\mu$  is the partition function,  $q$  and  $t$  were the real numbers for weighting.  
 235 And the aforementioned local singularity strength (coarse Hölder exponents)  $\alpha$  and  $\beta$  were the  
 236 function to  $q$  and  $t$  as well:

$$237 \alpha(q, t) = -[\ln(N(\varepsilon))]^{-1} \sum_{i=1}^{N(\varepsilon)} [\mu_i(q, t, \varepsilon) \cdot \ln(p_i(\varepsilon))] \quad [7]$$

$$238 \beta(q, t) = -[\ln(N(\varepsilon))]^{-1} \sum_{i=1}^{N(\varepsilon)} [\mu_i(q, t, \varepsilon) \cdot \ln(r_i(\varepsilon))] \quad [8]$$

239 To indicate the dimension of the joint distribution, the multifractal spectra  $f(\alpha, \beta)$ , was given  
 240 by

$$241 f(\alpha, \beta) = -[\ln(N(\varepsilon))]^{-1} \sum_{i=1}^{N(\varepsilon)} [\mu_i(q, t, \varepsilon) \cdot \ln(\mu_i(q, t, \varepsilon))] \quad [9]$$

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242 In fact, the joint partition function in Eq. [6] can be simplified to Eq. [2] when  $q$  or  $t$  is equal  
243 to 0. In this case, the joint multifractal spectrum was transformed to the multifractal spectrum  
244 with a single measure. When both ~~value of~~  $q$  and  $t$  were 0,  $f(\alpha, \beta)$  reached maximum and  
245 indicated box dimension of the geometric support of the measures. Pair value of  $\alpha$  and  $\beta$  were  
246 ~~fluctuates determined by with the change of~~ variable  $q$  and  $t$ . Therefore, it is possible to examine  
247 the distribution of high or low values (different intensity levels) of one variable with respect to  
248 another by varying the values of  $q$  or  $t$ . As the joint multifractal spectra  $f(\alpha, \beta)$  represents the  
249 frequency of the occurrence of certain values of  $\alpha$  and  $\beta$ , high values of  $f(\alpha, \beta)$  represents  
250 strong association between the values of  $\alpha$  and  $\beta$ . The Pearson correlation coefficient was used  
251 to quantitatively describe their relations across similar moment orders. In addition, correlation  
252 coefficients between the surface layer and subsurface layers were used as well to examine the  
253 similarity in the scaling properties. Additionally, a contour was used to represent the joint  
254 distribution of a pair of variables by permuting similar values (highs vs highs or lows vs lows)  
255 of  $q$  and  $t$ . The bottom left part of the contour graph presents the joint distribution of high data  
256 values of both variables while top right part represents the low data values of both variables.  
257 Therefore, a diagonal contour with low stretch indicate strong association between the variables  
258 in consideration (Biswas and Si, 2012b).

### 259 3 Results

#### 260 3.1 Spatial pattern of soil water storage at different depths

261 Average SWS for the surface 0-20 cm layer over five year period was 5.51 cm. A slight  
262 decrease in SWS was observed at the immediate deep layer (20-40 cm) and a gradual increase  
263 thereafter. Five-year average SWS was 5.45 cm, 5.48 cm, 5.56 cm, 5.61 cm, 5.69 cm and 5.77  
264 cm for the 20-40 cm, 40-60 cm, 60-80 cm, 80-100 cm, 100-120 cm and 120-140 cm layers,  
265 respectively (~~Table 1~~). Average SWS for a single measurement varied from 3.40 cm to 7.16  
266 cm. The highest average SWS for the surface layer was observed on 29 June 2011. The study  
267 area received large amount of spring snowmelt (2010 received 642 mm, double the annual  
268 average precipitation) and rainfall during 2011 leading to the high SWS in the surface layer  
269 (Weather Canada historical report). The lowest average SWS for surface was observed on 23  
270 August 2008, which was one of the driest summers within the five-year study period. The  
271 highest average SWS (on 29 June 2011) at the surface layer gradually decreased to 6.55 cm  
272 and the lowest average SWS (on 23 August 2008) at the surface layer gradually increased to  
273 5.28 cm at the 120-140 cm layer (Table 1). These top and bottom boundaries formed yielded

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Commented [r4112]: Not included in Table 1

Commented [r4113]: Not true for all depths

Commented [r4114R13]: So I add " for the surface layer"

Commented [r4115]: The data on rainfall should be shown here

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274 a bigger-wider range (3.76 cm) in-of the average SWS at the surface layer compared to that at  
275 the deepest layer (1.27 cm). A big range (2.00 cm) in the standard deviation (maximum=2.43  
276 cm and minimum=0.43 cm) of the measurement at the surface layer (0-20 cm) was also  
277 observed compared to that at the deepest layer (120-140 cm; maximum=1.28 and  
278 minimum=0.76). This indicated large variations in SWS at the surface layer and gradually  
279 decreased at deeper layers. The coefficients of variations (CVs) at the surface layer (0-20 cm)  
280 varied from 10% to 43% and the deepest layer (120-140 cm) varied from 13% to 23%  
281 (Supplementary Table S.1).

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282 The maximum SWS at the surface layer also varied widely (maximum=13.96 cm and  
283 minimum=4.64 cm) compared to the deepest layer (maximum=9.81 cm and minimum=6.71  
284 cm) (Table 1). There was a gradual decrease in the maximum value and increase in the  
285 minimum value from the surface to the deepest layer. A similar trend was also observed for the  
286 minimum SWS at different layers. The maximum SWS at different layers was much localized.  
287 For example, there was high SWS at different layers at the locations of 100 to 140 m and 225  
288 to 250 m from the origin of the transect. These locations had very high SWS compared to the  
289 field-average and-because they were situated in the depressions while low SWS was observed  
290 on the knolls.

Commented [r4117]: According to the table

291 The variations in SWS with time were evaluated within a year. There was little change in  
292 the average SWS over measurements within the years from 2007-2011 except 2008 (Table 1).  
293 For example, average SWS was 6.47 cm, 6.03 cm, 6.54 cm, and 6.33 cm on 6 April 2010, 19  
294 May 2010, 14 June 2010 and 28 September 2010, respectively. However, the average SWS in  
295 2008 drops from 6.28 cm on 2 May 2008 to 3.51 cm on 17 September 2008 in the surface 0-  
296 20 cm layer. This falling trend was even-observed at all soil layers. When compared between  
297 years, the trend over time and with depth was very similar in 2007 and 2009 while slightly  
298 different between 2010 and 2011 (Table 1). A decreasing trend of the variability was also  
299 observed with time. For example, the CV of the surface layer was around 28% on 2 May 2008,  
300 which gradually decreased to around 13% on 17 September 2008 (Supplementary Table S.1).

301 The average water storage for soil layers with increasing depth was also calculated by  
302 adding the individual layers together. The time-averaged values of SWS were 10.96 cm, 16.44  
303 cm, 22.00 cm, 27.61 cm, 33.30 cm and 39.07 cm for the 0-40 cm, 0-60 cm, 0-80 cm, 0-100 cm,  
304 0-120 cm and 0-140 cm, respectively (Supplementary Table S.2). The CV of the 0-20 cm layer  
305 was the highest during the wet period and gradually declined to the smallest during the dry



306 period (Supplementary Table S.3). The variability also gradually increased with depth. ~~This~~  
307 ~~trend with depth and time has also been verified by the standard deviation of measurement.~~

### 308 3.2 Statistical scale invariance

309 ~~The distribution of a statistical measure is considered as fractal (monofractal/multifractal)~~  
310 ~~provided the moments obey the power law (Evertsz and Mandelbrot, 1992).~~ The power law  
311 relationships and the statistical scale invariance were evaluated using a log-log plot of the  
312 aggregated variance of SWS spatial series at different depths of soil layers and the level of  
313 disaggregation (or scales) at different  $q$  values or statistical moments. The linear relationship  
314 of the logarithm of the variance with scale indicated the presence of statistical scale invariance  
315 (Fig. 42). The scale invariance was observed for all measurements and at all depths though  
316 only all depths of selected three ~~measurements-dates~~ were presented as example. The  
317 coefficient of determination ( $r^2$ ) for a linear fit ( $n=7$ ) was between 0.99 and 1.00 (significant at  
318  $P=0.001$ ) for any measurement days and depths. The **scale invariance was also observed for**  
319 **SWS trend at soil layers with cumulative-increasing depths.**

### 320 3.3 Multifractal analysis

321 The  $\tau(q)$  curves for the surface layer displayed deviation from the UM model during the wet  
322 period (Fig. 23). A high **SSR value** was observed between the  $\tau(q)$  curves and the UM model.  
323 Nonlinearity in the  $\tau(q)$  curve was observed and the slopes of the segmented fit of the  $\tau(q)$   
324 curves were significantly different from each other. For example, the SSR values between the  
325  $\tau(q)$  curve and the UM model were 27.74 and 50.49 for the surface layer (0-20 cm) on 2 May  
326 2008 and 31 May 2008, respectively. The slopes of the  $\tau(q)$  curve for (single fit) were 0.97 and  
327 0.96, respectively for the surface layer of 2 May 2008 and 31 May 2008 (Fig. 23). The slopes  
328 of the segmented fit for these measurements were 1.04 ( $q<0$ ) and 0.87 ( $q>0$ ) and, 1.06 ( $q<0$ )  
329 and 0.82 ( $q>0$ ), respectively (Fig. 23; Supplementary Table S.4).

330 With the maximum deviation at the surface layer, the  $\tau(q)$  curves gradually became very  
331 similar to the UM model with depth. The SSR value decreased considerably in ~~the~~ deep layers.  
332 The slopes of the  $\tau(q)$  curve (single fit) became almost unity with no significant difference with  
333 the UM model. There was no significant difference between the slopes of the segmented fit.  
334 For example, the SSR value was 6.17, 4.98, 8.80, 8.50, 8.86, and 6.16 respectively for the 20-  
335 40, 40-60, 60-80, 80-100, 100-120, and 120-140 cm layer of 2 May 2008 (Supplementary Table  
336 S.4). The slopes (single fit) for these layers were 0.99, 1.00, 1.01, 1.01, 1.00, and 0.99,

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337 respectively (Fig. 23). The slopes of the segmented fit were also very close to unity with no  
338 significant difference between them.

339 The SSR values gradually decreased and the slopes became almost unity with ~~the increase~~  
340 ~~of increasing~~ depth of soil layers (Fig. 34). For example, the SSR values were 14.11, 9.31, 7.71,  
341 6.86, 6.71 and 6.30 and the slopes (single fit) were 0.98, 0.99, 0.99, 1.00, 1.00, and 1.00,  
342 respectively for 0-40, 0-60, 0-80, 0-100, 0-120 and 0-140 cm layer (Supplementary Table S.5).  
343 The slopes of the segmented fit for the  $\tau(q)$  curve became almost the same as soil layers ~~going~~  
344 ~~went~~ deeper (Fig. 34). The linearity of the  $\tau(q)$  curves was gradually strengthened and the SSR  
345 value gradually fell with the depth increase of soil layers at any time. A ~~statically~~-significant  
346 difference was observed between the slopes of the  $\tau(q)$  curves in segmented fitting at the  
347 surface layer of ~~the~~ first three measurements in 2007 (Supplementary Fig. S.1), two  
348 measurements in 2008 (Fig. 34), three measurements in 2009 (~~Supplementary Fig. S.2~~), and  
349 all measurements in 2010 and 2011 (~~Supplementary Fig. S.2~~)-(Fig. 3).

350 A decreasing trend in the SSR value was also observed over time within a year. During the  
351 dry period, the slopes (single fit and segmented fit) became almost unity with no significant  
352 difference (Supplementary Table S.6). For example, the SSR value was 14.12, 8.25, 1.30, 1.46,  
353 and 0.52 and the slope was 0.99, 0.99, 1.00, 1.00, and 1.00, respectively for the surface layer  
354 (0-20 cm) of 21 June 2008, 16 July 2008, 23 August 2008, 17 September 2008 and 22 October  
355 2008 (Fig. 23). Similarly, a small SSR value and consistent slope were also observed at the  
356 deepest layer (120-140 cm). The SSR values of the 120-140 cm were 2.47, 2.47, 3.31, 3.44 and  
357 4.57, respectively for the measurements on 21 June 2008, 16 July 2008, 23 August 2008, 17  
358 September 2008 and 22 October 2008 (Supplementary Table S.6). The slope (single fit) for all  
359 these measurements was equal to 1.01 (Fig. 23). There was very little difference in the slopes  
360 of the segmented fits.

361 A significant difference in the slopes of the segmented fit was observed for the surface  
362 layer (0-20 cm) of three measurements in 2007 (17 July, 7 August, and 1 September;  
363 Supplementary Fig. S.1), and three measurements in 2009 (21 April, 7 May, and 27 May)  
364 (Supplementary Table S.4; Supplementary Fig. S.2). ~~The difference became non-significant~~  
365 ~~with depth and during other measurement times~~. The trend in deep layers over time was very  
366 similar to that of 2008. However, the trend in the SSR values and the slopes with time was  
367 ~~scarcely~~-different ~~between-in~~ 2010 and 2011 (Supplementary Table S6). There was very little  
368 difference in the SSR values at different times of the year. For example, the SSR value for the  
369 surface layer (0-20 cm) was 20.79, 27.18, 24.63 and 26.66 and the slope (single fit) was 0.97,

370 0.97, 0.97, and 0.97, respectively for the measurements on 6 April 2010, 19 May 2010, 14 June  
371 2010, and 28 September 2010 (Fig. 23). The slope of the segmented fit of the surface layer (0-  
372 20 cm) was statistically significant for all measurements in 2010 and 2011 (Fig. 2). However,  
373 the trend with depth was similar to other years (Supplementary Table S.7).

374 The height of the multifractal spectrum at different depths of measurement ~~over time~~ was  
375 very similar over time. The width of the spectrum ( $\alpha_{\max}-\alpha_{\min}$ ) varied with depth and time (Fig.  
376 5). Generally, a comparative large value of  $\alpha_{\max}-\alpha_{\min}$  was observed at the surface layer during  
377 the wet period and the value gradually became smaller ~~at~~ with depths. For example, the value  
378 of  $\alpha_{\max}-\alpha_{\min}$  for the surface soil layer (0-20 cm) was 0.23 and 0.31, respectively for the  
379 measurements of 2 May 2008 and 31 May 2008 (Fig. 5). Meanwhile, the value of  $\alpha_{\max}-\alpha_{\min}$  for  
380 the soil layers of 20-140 cm with 20 cm increment was 0.15, 0.14, 0.19, 0.20, 0.20, and 0.18  
381 for 2 May 2008 and 0.25, 0.19, 0.11, 0.14, 0.12, and 0.11 for 31 May 2008, respectively (Fig.  
382 64). In the later part of the year, the width of the spectrum gradually decreased (Supplementary  
383 Table S.8). For example, the  $\alpha_{\max}-\alpha_{\min}$  values were 0.19, 0.16, 0.07, 0.08, and 0.05, respectively  
384 for the surface layer ~~measurement on~~ 21 June 2008, 16 July 2008, 23 August 2008, 17  
385 September 2008 and 22 October 2008. Similar trend in values of  $\alpha_{\max}-\alpha_{\min}$  was also observed  
386 at deep layers (Fig. 64).

387 The trend of the  $\alpha_{\max}-\alpha_{\min}$  values in 2007 and 2009 was very similar to that of 2008  
388 (Supplementary Table S.8). A higher value of  $\alpha_{\max}-\alpha_{\min}$  was observed in the first three  
389 measurements of 2007 (Supplementary Fig. S.5) and three measurements of 2009  
390 (Supplementary Fig. S.6). However, the values in the surface layer (0-20 cm) of measurements  
391 in 2010 and 2011 were always higher compared to the deep layers (Fig. 46). There was no  
392 decreasing trend in values for the surface layer over time. For example, the  $\alpha_{\max}-\alpha_{\min}$  value was  
393 0.21, 0.24, 0.21, and 0.22, respectively for the measurements on 6 April 2010, 19 May 2010,  
394 14 June 2010, and 28 September 2010 (Fig. 46). However, the trend in the  $\alpha_{\max}-\alpha_{\min}$  value of  
395 deep layers was similar to that of other years. A similar trend was observed for cumulative  
396 SWS with increasing depth over the years (Fig. 57). Generally, the value of  $\alpha_{\max}-\alpha_{\min}$  was also  
397 small with the highest in the 0-20 soil layers and gradually decreased with depth (Fig. 57;  
398 Supplementary Table S.9).

399 A very similar height of the  $f(q)$  curve for all depths and all periods indicated a consistent  
400 frequency distribution of the scaling indices (Fig. 6 and 7). Additionally, the position and the  
401 symmetry of the curve revealed the distribution of scaling exponents. A symmetric  $f(q)$  curve  
402 indicated uniform distribution of the scaling exponents. The left side of the spectrum

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Commented [r4120R19]: I guess the table and figures have been revised after writing this text. There are not just three measurements now. I have difficulty in revising here.

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403 corresponded to the large SWS that were amplified by the positive values of  $q$  while the right  
404 side indicated smaller SWS that were amplified by negative  $q$  values. Symmetry leaning  
405 towards the left side during the early spring and in the surface layers in 2008 clearly showed  
406 the wider distribution of scaling indices and multifractal nature of the SWS (Fig. 6). While the  
407 shifting of the symmetry towards right side clearly indicated less variable scaling indices and  
408 thus reduction of multifractal behavior. During the wet year of 2010 and 2011, the symmetry  
409 towards left side indicated the variability in the scaling indices. This also persisted with depth.  
410 A similar trend was observed for different years at all layers of cumulative depths (Fig. 7).

411 Generally, the  $D_1$  and  $D_2$  values for different depths of different measurements were very  
412 close to 1 (only varied at 3 decimal points; Fig. 8 and Supplementary Table S.10). In general,  
413 the  $D_1$  value of the surface layers gradually increased with depth. Similarly, at any depth, the  
414  $D_1$  values gradually increased from spring to fall season through summer (Fig. 8). Highest  
415 variation in  $D$  values with  $q$  was observed in the surface layer and in the spring season and  
416 gradually decreased with depth and later part of the growing season. Specifically, the  $D$  values  
417 for the surface layer during the wet period increased at high  $q$  values. For example, the first  
418 three measurements in 2007 and 2009 ~~all~~ presented high  $D$  values at high  $q$  values  
419 (Supplementary Figs. S.9 and S.10). This high  $D$  value gradually decreased in the dry period  
420 of the year. For example, the  $D$  value with positive  $q$  was high in the surface layer of 2 May  
421 2008 and 31 May 2008 (Fig. 69), whereas it gradually decreased at the later part of the year  
422 (e.g. 17 September 2008). The trend with time and depth in 2007 and 2009 was very similar to  
423 that of 2008 (Supplementary Tables S.10 and S.11). A consistent high  $D$  value was observed  
424 in the surface layer for all 2010 and 2011 measurements (Fig. 69). The trend in  $D$  values with  
425 depth in 2010 and 2011 was also similar to other years. A high value of  $D_1$  and  $D_2$  were also  
426 observed at all layers of cumulative depths for all measurements (Fig. 710; Supplementary  
427 Table S.11).

### 428 **3.4 Joint multifractal analysis**

429 There were strong correlations between the scaling property of the joint distribution of the  
430 surface soil layer and the deep soil layers. The narrow width and the diagonally oriented  
431 contours between SWS measured on 22 October 2008 at 0-20 cm and 20-40 cm layers clearly  
432 demonstrates strong association between those two layers (Fig. 11). The correlation between  
433 the surface 0-20 cm and the deep layers on 2 May 2008 (wet period) was larger than 0.9  
434 (significant at  $P=0.001$ ; Table 2). The highest correlation was observed between the layers  
435 closest to each other. The correlations gradually increased over time and showed high

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436 consistency between different layers on 17 September 2008 (Table 2). A very similar trend was  
437 observed in other years.

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#### 438 4 Discussion

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439 The amount of water stored in ~~soil layer~~the soil is the result of the dominant underlying  
440 hydrological processes. Located in semi-arid climate, the study area receives about 30% of the  
441 long term annual average precipitation as snowfall during winter months (Pomeroy et al.,  
442 2007). Generally, the depressions receive snow from surrounding uplands or knolls as  
443 redistributed by strong prairie wind (Pomeroy and Gray, 1995;Fang and Pomeroy, 2009). The  
444 snow melts within a short period of time during the early spring and contributes a large  
445 amount of water. The frozen ground restricts infiltration and redistributes excess water within  
446 the landscape with greater accumulation in depressions (Fig. 81) (Gray et al., 1985). Apart  
447 from the snowmelt, the spring rainfall also contributes to the water inflow in the landscape  
448 (Fig. 81). This created a spatial pattern of SWS that was almost a mirror image of the spatial  
449 distribution of relative elevation (Biswas and Si, 2011a, b;Biswas et al., 2012a).

450 In the spring, the sources of water loss were the deep drainage and the evaporation-. As the  
451 loss of water through deep drainage in the study area was as low as 2 to 40 mm per year,  
452 occurring mainly through the fractures and preferential flow paths (Hayashi et al., 1998;van  
453 der Kamp et al., 2003), the major loss occurred mainly through evaporation from the surface  
454 of the bare ground and standing water in depressions. These processes lose a very small amount  
455 of water compared to the input of the water in spring and early summer leaving the soil wet.  
456 Moreover, the surface soil with high organic matter content and low bulk density stored a larger  
457 amount of water than the deep layers where the organic matter gradually decreased and the  
458 bulk density increased. Reflecting the long-term history of vegetation growth in the landscape,  
459 the variability of organic matter content (CV=41%) may be one of the main factor of the high  
460 variability in surface layer SWS (Biswas and Si, 2011c).

461 As the vegetation developed in summer, strong evapotranspiration resulted in the lowest  
462 average SWS in a year. High amount of water in the depressions allowed grasses to grow faster  
463 and transpire more water comparing to the knolls (Fig. 81). For example, the aquatic  
464 vegetation growth within the depressions was as high as 2 m, while the grasses on the knolls  
465 grew to a maximum up to a meter tall. The uneven growth of vegetation and the high  
466 evapotranspirative demand in summer narrowed the range of SWS. In the soil where water is  
467 more available, evapotranspiration will be stronger while the less evapotranspirative demand

Commented [r4123]: Only in a year?

468 will be shown in the relatively dry soil. As a result, the excessive water in the relatively wet  
469 soil will be offset by evapotranspiration, ~~Stronger demand extracted more water from the soil~~  
470 ~~where available and comparative less water from the soil where the availability was restricted,~~  
471 ~~thus reducing the disparities between maximum and minimum values.~~ This variable water  
472 uptake was visible in the growth of vegetation in the later part of the growing season as well  
473 (Fig. 81). The reduction in the range of SWS was the largest in the surface layer and gradually  
474 decreased at deeper layers. This is because the surface layer was exposed to various  
475 environmental forcing ~~and was very dynamic in nature.~~ For example, plants can take up  
476 more than 70% of the water they need from the top 50% of the root zone (Feddes et al., 1978).  
477 This dynamic behavior of the surface layer exhausted readily available water and finally  
478 reduced the range in water storage. This decrease in range also happened in the later part of the  
479 growing season.

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480 The multifractal and joint multifractal analyses explained the scaling behavior of SWS at  
481 different depths over time. The linearity in the log-log plot between the aggregated variance in  
482 SWS and the scale at all soil layers over time indicated ~~the presence of that SWS behaved under~~  
483 ~~scaling laws (Fig. 12). The mass exponent,  $\tau$  calculated over a range of moment orders ( $q$ ) was~~  
484 ~~used to examine the scaling behavior (monofractal and multifractal). The shape of the curve~~  
485 ~~described the type of scaling involved. The curve with a single slope implied a monofractal~~  
486 ~~scaling, while a convex downward curve with different slopes for negative and positive~~  
487 ~~moment orders implied a multiple scaling (multifractal) (Evertsz and Mandelbrot, 1992). The~~  
488 ~~deviation in the scaling property of SWS from the monofractal was also examined by~~  
489 ~~comparing the  $\tau(q)$  curve with the theoretical UM model and the SSR between them (Fig. 2).~~  
490 The near unity slope of the  $\tau(q)$  curves and the insignificant difference from the UM model  
491 indicated a monofractal type scaling at all layers except the surface layer during the wet period  
492 (until mid to late June) where a multifractal behavior led to a slight convex downward curve  
493 (Fig. 23). This was also supported by a significant difference between the slope of single and  
494 segmented fit in the surface layer during the wet period.

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495 Generally during the wet period, excess water fills and drains macropores quickly and  
496 creates variations in SWS. Variations in the evaporation due to uneven solar incidence over  
497 micro-topography also triggered SWS variability in the surface layer. Additionally, the snow  
498 melt and the release of water controlled by local (e.g. soil texture) and non-local (e.g.  
499 topography) factors also affected the spatial distribution of SWS, making it more heterogeneous  
500 in the wet period (Grayson et al., 1997; Biswas and Si, 2012). Contrarily, as depth increased,

501 less impact of environmental ~~forcing-factors~~ tended to create less variability in SWS and  
502 exhibited a monofractal behavior which was consistent with the uniform slope shown in Figure  
503 23. During the dry period or later part of the growing season, the SWS storage variability at all  
504 depths was small and exhibited monofractal behavior (Fig. 23). Accordingly, the deeper layers  
505 in the wet period and all layers in the dry period can be accurately represented by only one  
506 scaling exponent while the surface layer in the wet period may require a hierarchy of exponents  
507 ~~to describe scaling property~~. A similar trend was observed in SWS of cumulative depth layers  
508 (Fig. 34). Resulting from increasingly buffering capacity of the deeper soil layers, the  
509 variability of cumulative SWS overlaid the multifractal nature of the surface layer, and finally  
510 exhibited monofractal behavior in general.

511 [The scaling patterns of SWS at different depths and different periods were further examined  
512 using multifractal spectrum [ $f(q)$  vs.  $\alpha(q)$ ] (Fig. 46 & Fig. 57). The degree of convexity was  
513 used to characterize the heterogeneity of scaling exponents or the degree of multifractality.  
514 Large value of  $\alpha_{\max}-\alpha_{\min}$  indicated stronger heterogeneity in the local scaling indices of SWS  
515 or cumulative SWS and vice versa. The largest value for the surface layer(s) in the wet period  
516 indicated the most multifractal behavior of SWS. However, the value decreased with depth and  
517 gradually converged in deep layers (Fig. 46). This decline manifested a conformity in the  
518 scaling behavior of SWS at deeper layers. Over time, the  $\alpha_{\max}-\alpha_{\min}$  value of the surface soil  
519 layer decreased and became very similar to that of deep layers. This indicated a reduction in  
520 the degree of multifractality for surface soil layers from the wet period to the dry period. A  
521 consistent  $\alpha_{\max}-\alpha_{\min}$  value for all depths during the dry period suggested the homogeneity and  
522 least multifractal nature of SWS. A similar behavior was observed in the cumulative SWS (Fig.  
523 57).

524 To sum up, both the unity slope of the  $\tau(q)$  curves (Fig. 23 and Fig. 34) and the degree of  
525 convexity of the  $f(q)$  spectrum (Fig. 46 & Fig. 57) jointly demonstrated that dynamic behavior  
526 of surface soil layers in the wet period made SWS highly variable and exhibited multifractal  
527 nature, while less environmental forcing and increased buffering capacity of deep layers led  
528 to monofractal nature. As a result, multiple scaling exponents were required to characterize the  
529 variability of SWS in the surface layer during the wet period, while less number of exponents  
530 was necessary for deeper layers during wet period or all layers during dry period.

531 The height of the spectrum,  $f(q)$  revealed the dimension or frequency distribution of the scaling  
532 indices (Caniego, Martín, San José, 2003). A low height of  $f(q)$  curve indicated rare events or  
533 extreme values in the distribution, while a high value represented uniform distribution in all

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534 segments. A very similar height of the  $f(q)$  curve for all depths and all periods indicated a  
535 consistent frequency distribution of the scaling indices. ~~Additionally, the position and the~~  
536 ~~symmetry of the curve revealed the distribution of scaling exponents. A symmetric  $f(q)$  curve~~  
537 ~~indicated uniform distribution of the scaling exponents. The left side of the spectrum~~  
538 ~~corresponded to the large SWS that were amplified by the positive values of  $q$  while the right~~  
539 ~~side indicated smaller SWS that were amplified by negative  $q$  values.~~

540 ~~Surface one or two layers~~The two upper soil layers during the wet period tended to exhibit  
541 longer tail of the curve on the left, showing more heterogeneity in the distribution of large  
542 values. However, when stepping into the dry period, the spectrum tended to display a longer  
543 tail on the right compared to the left side, suggesting more heterogeneity in the distribution of  
544 smaller values. ~~A few locations had with standing water leads to the spatial differences, thus~~  
545 ~~large SWS during the wet period while a compared to few points with very small SWS due to~~  
546 ~~high evapotranspiration by growing vegetation during the dry period results in the heterogenic~~  
547 ~~distribution in smaller values, owing to stronger demand by growing vegetation.~~

548 The generalized dimension,  $D_q$  was subsequently used to characterize the scaling property  
549 and variability in SWS (Fig. 6-9 and Fig. 7-10). The largest value of  $f(q)$ , referred to as the  
550 capacity dimension ( $D_0$ ) obtained at  $q = 0$ , was close to unity for all layers at different times  
551 (Fig. 6-9). ~~The information dimension ( $D_1$ ) obtained at  $q = 1$  was different from correlation~~  
552 ~~dimension ( $D_2$ ), which is denoted as~~ the average distribution density of the measurement for  
553 the surface layers in the wet period (Grassberger and Procaccia, 1983). In this case, the different  
554 values of  $D_0$ ,  $D_1$  and  $D_2$  indicated multifractal nature of the distribution of SWS. Similarly, a  
555 non-unity value of  $D_1/D_0$  (Montero, 2005) also indicated ~~the~~ multifractal nature of SWS at the  
556 surface layer(s) during the wet period. However, over the growing season, the  $D_1$  and  $D_2$  value  
557 approached ~~closer~~ to  $D_0$  and indicated ~~a~~ monofractal type behavior. Similar values of  $D_0$ ,  $D_1$   
558 and  $D_2$  during the dry period also indicated homogeneous distributions.

559 Joint multifractal distribution between the surface to various subsurface layers indicated  
560 the similarity in the scaling patterns (Table 2). Basically, the hydrological processes of  
561 shallower layers ~~were as more~~ similar to ~~those of~~ the top layer, while deeper layers showed  
562 more ~~observable~~ disparities from the surface. The nearest subsurface (20-40 cm) layer showed  
563 generally the highest similarity with the surface (0-20 cm) layer. However, in the wet period,  
564 the subsurface layers displayed the smallest similarity to the surface layer, suggesting ~~a~~ higher  
565 dynamic nature of hydrological processes. In the dry period, ~~a~~ stronger effect of vegetation  
566 overwhelmed the effect of small variations ~~of water distribution~~, thus creating a more uniform

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567 distribution of SWS at all soil layers and showed stronger similarity to the surface layers (Table  
568 2).

569 Overall, our result revealed a multifractal behavior of surface soil layers during the wet  
570 period due to its dynamic nature of hydrological processes. This behavior gradually changed  
571 with depth and time (Fig. 9.12). In the deeper layers during the wet period, the behavior became  
572 less multifractal or nearly monofractal. Similarly, in the dry period, the vegetation development  
573 and its high evapotranspirative demand in the semi-arid climate of the study area increasingly  
574 buffered the variation of SWS, as a result, all the soil layers with less effect from environment  
575 factors ~~ereing~~ showed uniform distribution or monofractal behavior (Fig. 9.12).

## 576 5 Summary and Conclusions

577 The transformation of information on soil water variability from one scale to another requires  
578 knowledge on the scaling ~~behaviour~~ behavior and the quantification of scaling indices. Surface  
579 soil water can be easily measured (e.g. remote sensing) and presents multi-scaling  
580 ~~behaviour~~ behavior (requiring multiple scaling indices). However, land-management practices  
581 requires the understanding of the hydrological dynamics in the root zone and/or the whole soil  
582 profile. ~~The scaling properties of the surface soil layer can be used in the decision making  
583 provided the similar behavior holds at the deep soil layer.~~

584 In this manuscript, the scaling properties of soil water storage at different soil layers  
585 measured over a five-year period were examined using multifractal and joint multifractal  
586 analysis. The scaling properties of soil water storage mainly suggested a monofractal scaling  
587 behavior. However, the surface layer in the wet period or with high soil water storage tended  
588 to be multifractal ~~in nature~~, which gradually became monofractal with depths. With the  
589 decrease in soil water storage, the scaling behavior became monofractal ~~in nature at the later  
590 part of the year or during the~~ growing season. ~~In~~ The year with high annual precipitation, the  
591 soil stored more water in the surface layer throughout the growing period and displayed nearly  
592 multifractal scaling behavior. This multifractal nature indicated that the transformation of  
593 information from one scale to another at the surface layer during the wet period requires  
594 multiple scaling indices. On the contrary, the transformation requires a single scaling index  
595 during the dry period for the whole soil profile. The scaling properties of the surface layer were  
596 highly correlated with that of the deep layers, which indicated a highly similar scaling  
597 ~~behaviour~~ behavior in the soil profile. The study was conducted in an undulating landscape from  
598 a semi-arid climate and the results were very ~~persistence~~ consistent over the years. Therefore,

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599 the observation completed at the field scale in this type of landscape and climate may be  
600 generalized in similar landscapes and climatic situations, otherwise may need to be examined  
601 thoroughly. The method used here can be transferred to examine the scaling properties in other  
602 experimental situations.

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## 603 6 Acknowledgements

604 The project was funded by the Natural Science and Engineering Research Council of Canada.  
605 The help from the graduate student and the summer students of the Department of Soil Science  
606 at the University of Saskatchewan in collecting field data is highly appreciated.

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721

## 722 Figure captions

723 Fig. 81: Conceptual schematics showing the vegetation growth patterns in the different section  
724 of landscapes at different times of the year. The figure is developed based on field observations  
725 and the scale is arbitrary.

726 Fig. 42. Log-log plot between the aggregated variance of the SWS spatial series and the scale.  
727 A linear relationship indicated the presence of scale invariance and scaling laws for three  
728 selected dates.

729 Fig. 23. Mass exponents for soil water storage spatial series measured at each-selected 20 cm  
730 soil layer down to 140 cm in 2008, ~~and 2010 and 2011~~ for a range of  $q$  (-15 to 15 at 0.5  
731 increments). The solid line is a linear reference created following the UM model of Schertzer  
732 and Lovejoy (1987) passing through ( $q = 0$ ).

733 Fig. 34. Mass exponents for selected soil water storage spatial series from surface to different  
734 soil layers (cumulative storage) at 20 cm increment down to 140 cm in ~~2008 and 2010~~2008,  
735 ~~2010 and 2011~~ for a range of  $q$  (-15 to 15 at 0.5 increments). The solid line is a linear reference  
736 created following the UM model of Schertzer and Lovejoy (1987) passing through ( $q = 0$ ).

737 Fig. 5. The width of the multifractal spectrum ( $\alpha_{\max} - \alpha_{\min}$  value) for soil water storage at different depths  
738 (20 cm increment) for all measurements completed during the study period.

739 Fig. 46. Multifractal spectra of soil water storage spatial series measured at each 20 cm soil  
740 layer down to 140 cm in ~~2008 and 2010~~2008, 2010 and 2011 for a range of  $q$  (-15 to 15 at 0.5  
741 increments).

742 Fig. 57. Multifractal spectra of soil water storage spatial series from surface to different soil  
743 layers (cumulative storage) at 20 cm increment down to 140 cm in ~~2008 and 2010~~2008, 2010  
744 ~~and 2011~~ for a range of  $q$  (-15 to 15 at 0.5 increments).

745 Fig. 8. The information dimension (D1) for soil water storage at different depths (20 cm  
746 increment) over the whole measurement period.

Commented [r4133]: Would be preferable to show the differential results rather than cumulative plots, which are very smooth and give no indication of multifractality

747 Fig. 69. Generalized dimension spectra of soil water storage spatial series measured at each 20  
748 cm soil layer down to 140 cm in ~~2008 and 2010~~2008, 2010 and 2011 for a range of q (-15 to  
749 15 at 0.5 increments).

750 Fig. 710. Generalized dimension spectra of soil water storage spatial series from surface to  
751 different soil layers (cumulative storage) at 20 cm increment down to 140 cm in ~~2008 and~~  
752 ~~2010~~2008, 2010 and 2011 for a range of q (-15 to 15 at 0.5 increments).

753 ~~Fig. 8. Conceptual schematics showing the vegetation growth patterns in the different section~~  
754 ~~of landscapes at different times of the year. The figure is developed based on field observations~~  
755 ~~and the scale is arbitrary.~~

756 Fig. 11. Join multifractal spectra between surface (0-20 cm) and immediate (20-40 cm)  
757 subsurface soil layer soil water storage measured on 22 October 2008.

758 Fig. 911: Conceptual schematics showing vegetation development over time, dominant water  
759 loss processes and the scaling behavior of soil water storage at different depths. The figure is  
760 developed based on field observations and scaling analysis. The scale of the figure is arbitrary.

## 761 Tables

762 Table 1

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Table 1. Maximum, minimum, and average soil water storage at different depths (20 cm increment) over the whole measurement period.

	0-20 cm			20-40 cm			40-60 cm			60-80 cm			80-100 cm			100-120 cm			120-140 cm		
	Maximum (cm)	Minimum (cm)	Average (cm)	Maximum (cm)	Minimum (cm)	Average (cm)	Maximum (cm)	Minimum (cm)	Average (cm)	Maximum (cm)	Minimum (cm)	Average (cm)	Maximum (cm)	Minimum (cm)	Average (cm)	Maximum (cm)	Minimum (cm)	Average (cm)	Maximum (cm)	Minimum (cm)	Average (cm)
Jul 17 2007	13.96	3.25	5.65	11.55	3.09	5.63	9.43	2.59	5.73	9.06	3.34	5.90	9.51	3.22	5.89	9.81	3.55	6.05	9.81	3.54	6.14
Aug 7 2007	13.96	3.05	4.90	9.28	2.73	5.04	8.30	2.40	5.21	9.36	2.75	5.48	8.23	2.96	5.57	7.52	3.17	5.62	9.11	3.17	5.67
Sept 1 2007	13.96	2.26	5.29	9.28	3.00	5.08	8.08	2.42	5.23	6.98	2.75	5.38	7.17	2.92	5.52	8.08	3.20	5.64	9.07	3.23	5.73
Oct 12 2007	8.30	3.40	5.04	6.92	3.07	5.03	6.74	2.43	5.19	7.60	2.81	5.36	8.39	2.93	5.48	7.92	3.25	5.60	8.55	3.25	5.67
May 2 2008	13.96	4.49	6.28	9.96	4.09	6.03	9.43	3.69	5.80	8.83	3.16	5.74	9.51	2.90	5.66	9.81	3.26	5.70	9.81	3.30	5.75
May 31 2008	13.96	3.30	5.21	9.28	1.54	5.51	8.08	1.58	5.55	6.85	3.00	5.58	7.08	3.08	5.64	8.08	3.22	5.70	8.39	3.25	5.79
Jun 21 2008	8.77	3.06	4.70	7.84	3.43	5.25	6.86	2.80	5.38	6.78	2.77	5.52	7.08	3.04	5.61	7.73	3.28	5.69	8.48	3.23	5.77
July 16 2008	7.07	2.78	4.03	6.78	3.06	4.77	6.71	2.60	5.10	6.75	2.56	5.30	6.84	2.91	5.43	6.98	3.17	5.56	7.01	3.16	5.64
Aug 23 2008	4.96	2.44	3.40	5.66	2.73	4.11	6.02	2.37	4.59	6.44	2.36	4.90	6.56	2.63	5.12	6.85	3.04	5.30	6.81	2.99	5.42
Sept 17 2008	4.64	2.66	3.51	5.63	2.79	4.07	5.91	2.49	4.55	6.28	2.45	4.85	6.59	2.63	5.05	6.68	3.05	5.25	6.91	2.96	5.37
Oct 22 2008	6.11	3.83	4.96	6.03	3.10	4.37	5.92	2.52	4.53	6.13	2.46	4.79	6.55	2.63	5.00	6.61	3.00	5.18	6.73	1.22	5.28
April 20 2009	13.96	4.73	6.67	11.55	3.62	5.84	10.49	3.23	5.62	8.83	2.97	5.48	9.51	2.67	5.38	9.81	3.08	5.49	9.81	2.85	5.66
May 7 2009	13.96	4.45	5.97	9.51	3.68	5.70	8.08	3.26	5.49	8.30	3.00	5.36	7.85	2.73	5.35	9.81	3.01	5.43	8.91	2.84	5.51
May 27 2009	12.60	3.67	5.43	8.15	3.55	5.52	8.08	3.43	5.39	6.78	3.13	5.37	7.16	2.64	5.39	8.08	2.96	5.51	8.45	2.80	5.53
July 21 2009	6.92	3.16	4.56	7.24	3.16	4.83	6.55	2.91	5.00	6.72	2.95	5.23	6.77	2.58	5.24	6.91	3.02	5.34	6.89	3.24	5.43
Aug 27 2009	6.64	3.42	5.01	6.67	3.57	5.07	6.32	2.84	4.92	6.50	2.85	5.03	6.76	2.57	5.16	6.79	3.00	5.25	6.90	3.02	5.34
Oct 27 2009	6.65	3.89	5.30	6.44	3.44	4.90	6.04	2.74	4.80	6.36	2.68	4.91	6.55	2.60	5.05	6.71	3.05	5.17	6.71	2.79	5.29
April 6 2010	13.96	4.67	6.47	9.51	3.53	5.52	9.43	3.19	5.31	8.83	2.91	5.35	9.51	2.61	5.23	9.81	3.01	5.34	9.81	2.83	5.41
May 19 2010	13.96	4.08	6.04	11.32	4.28	5.94	10.49	4.46	5.94	8.75	4.08	5.93	8.60	3.55	5.90	9.81	4.03	5.91	9.81	3.96	5.85
June 14 2010	13.96	4.38	6.54	11.55	4.48	6.32	10.49	4.58	6.31	8.83	4.27	6.29	9.51	3.86	6.22	9.81	4.37	6.24	9.81	4.50	6.20
Sept 28, 2010	13.96	4.51	6.33	11.55	4.48	6.16	9.43	3.77	6.08	8.83	3.91	6.13	9.51	3.83	6.12	9.81	4.11	6.16	9.79	4.18	6.20
May 13, 2011	13.96	4.82	7.12	11.55	4.87	6.61	10.49	4.75	6.50	9.21	4.54	6.40	9.51	4.16	6.34	9.96	3.17	6.32	9.79	4.30	6.45
Jun 6, 2011	13.96	4.31	7.05	11.55	4.56	6.59	10.49	3.85	6.52	9.06	4.75	6.44	9.51	4.21	6.40	9.96	3.17	6.39	9.79	4.77	6.52
Jun 29, 2011	13.96	4.93	7.16	11.55	4.96	6.73	10.49	4.29	6.64	9.74	4.42	6.57	9.51	4.28	6.49	9.96	3.17	6.46	9.79	4.30	6.55
Sept 29, 2011	12.60	3.11	5.25	8.15	3.46	5.50	8.08	2.88	5.68	7.58	4.03	5.82	9.19	3.77	5.89	9.51	3.81	6.02	9.36	4.14	6.04
<u>5 year average</u>			<u>5.51</u>			<u>5.45</u>			<u>5.48</u>			<u>5.56</u>			<u>5.61</u>			<u>5.69</u>			<u>5.77</u>

Commented [r4135]: Please include 5-year averages since the text is mentioned.

Formatted Table

765 Table 2: ~~Correlation between joint multifractal coefficients of surface to different subsurface~~  
 766 ~~layers measured at 20 cm interval in 2008~~Correlation coefficients between joint multifractal  
 767 ~~indices ( $\alpha$  and  $\beta$ ) of the surface layer with those from subsurface layers at 20cm intervals in~~  
 768 ~~2008~~. The number of data points are same for all the analysis.

	2 May 2008	31 May 2008	21 Jun. 2008	16 Jul. 2008	23 Aug. 2008	17 Sep. 2008	22 Oct. 2008
0-20 cm vs. 20-40 cm	0.96	0.98	0.99	0.99	0.99	1.00	1.00
0-20 cm vs. 40-60 cm	0.93	0.96	0.96	0.97	0.97	1.00	1.00
0-20 cm vs. 60-80 cm	0.93	0.94	0.95	0.95	0.96	0.99	0.99
0-20 cm vs. 80-100 cm	0.92	0.92	0.93	0.94	0.94	0.98	0.99
0-20 cm vs. 100-120 cm	0.92	0.92	0.93	0.93	0.93	0.97	0.99
0-20 cm vs. 120-140 cm	0.93	0.94	0.95	0.94	0.94	1.00	1.00

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**Commented [r4136]:** Please indicate the number of data used for each correlation, was it the same for all the dates and depths?

**Commented [r4137]:** Anonymous Referee #1: I suggest to include figures showing some multifractal spectra either in the main manuscript or as supplementary content.

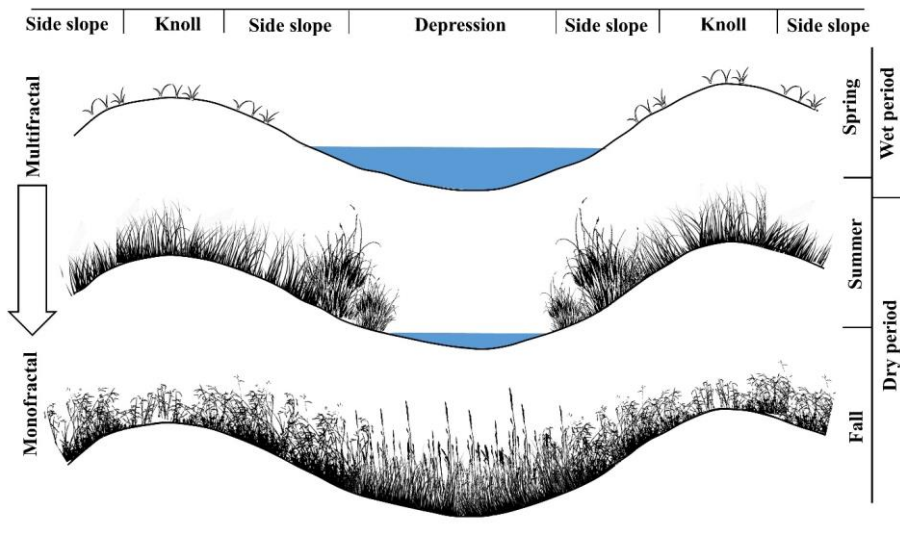
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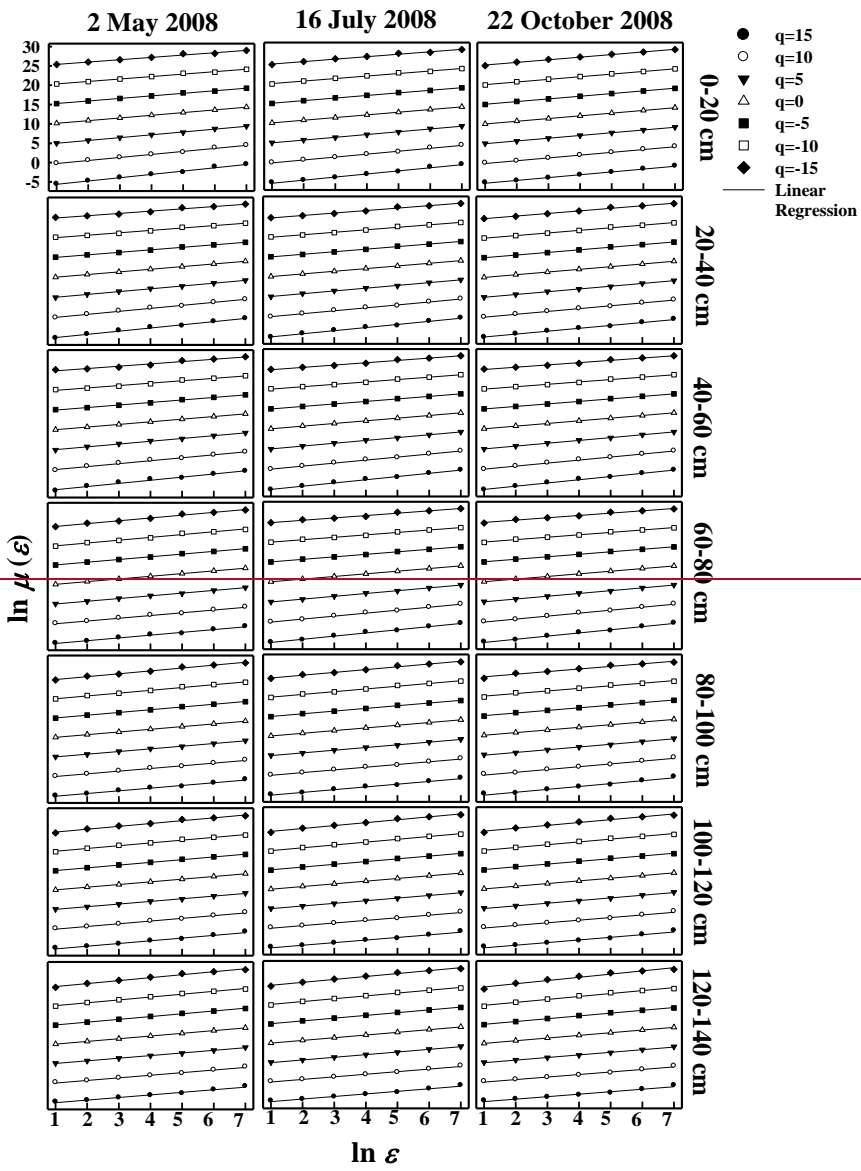
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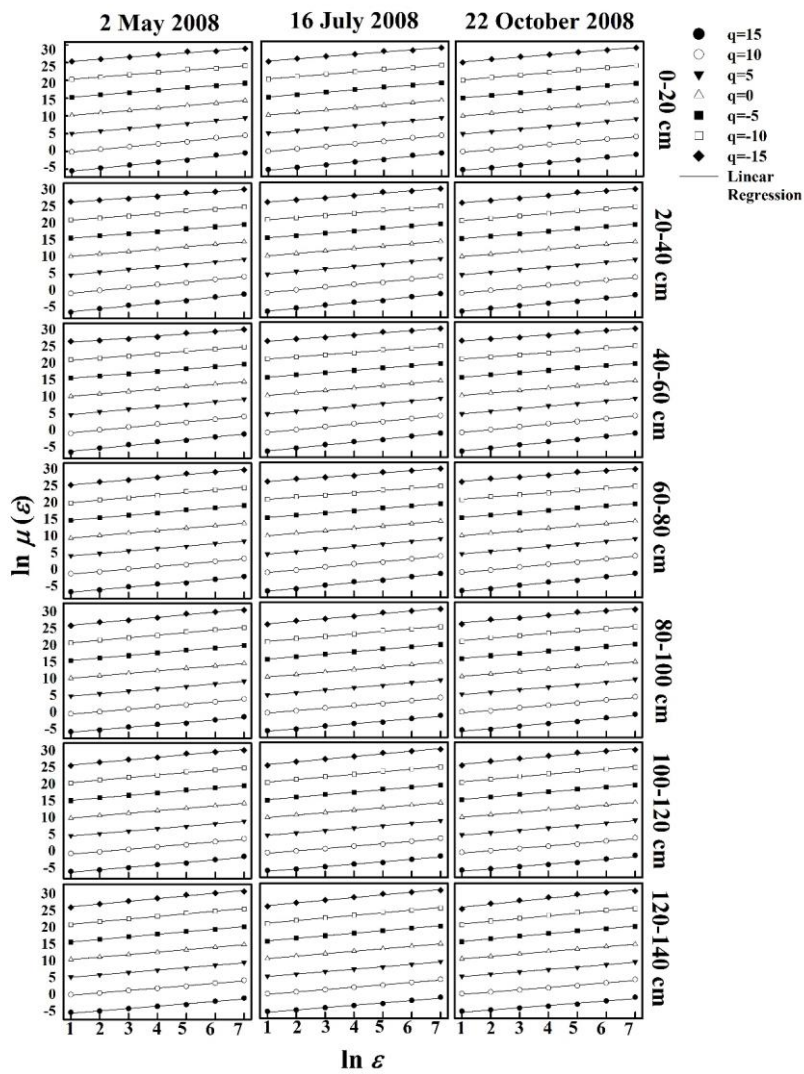
784 **Figures**



785

786 **Figure 1**



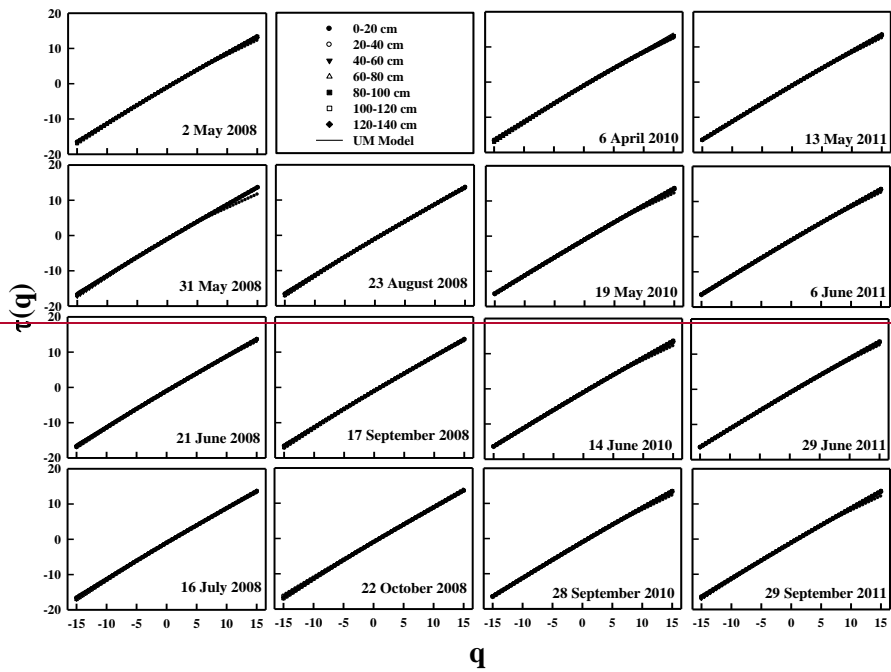


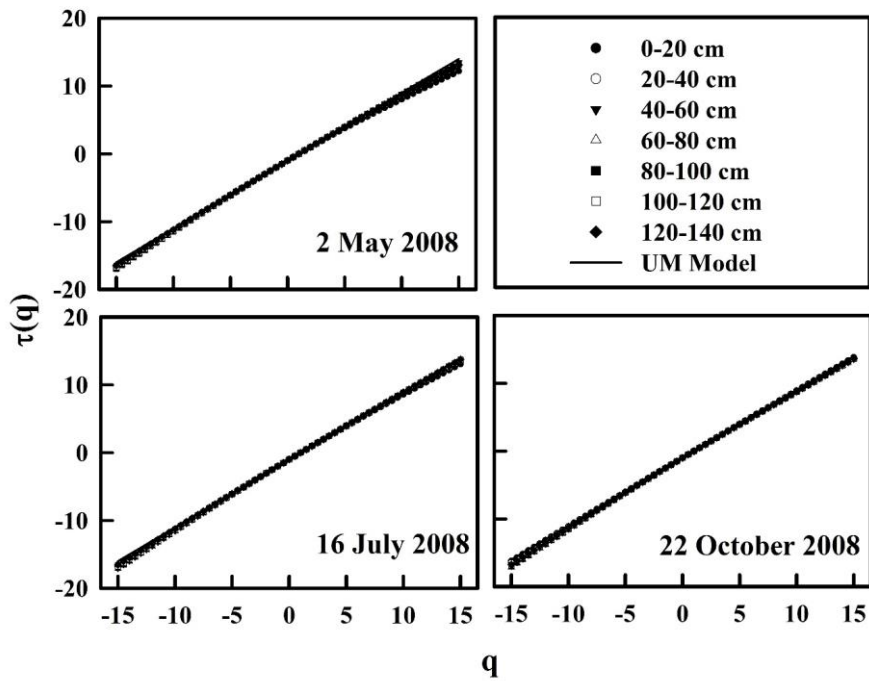
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789 Figure 42

Commented [r4138]: Please show Y-scale in all left graphs

Commented [r4139]: Anonymous Referee #1: I recommend to show only two or four selected plots of mass exponent functions to increase visibility. (because of these small size of the Figures, differences are hardly to view).



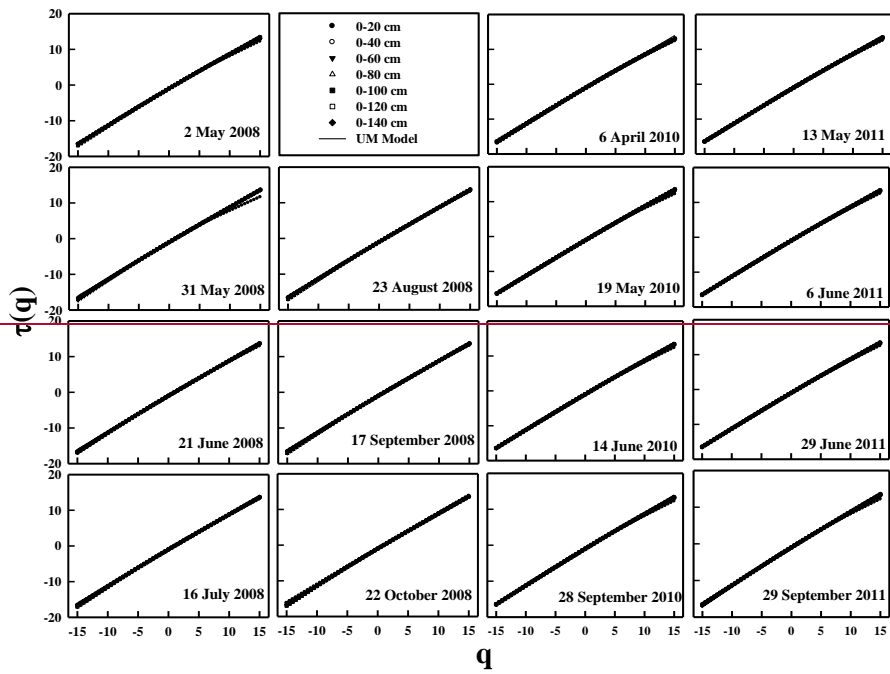


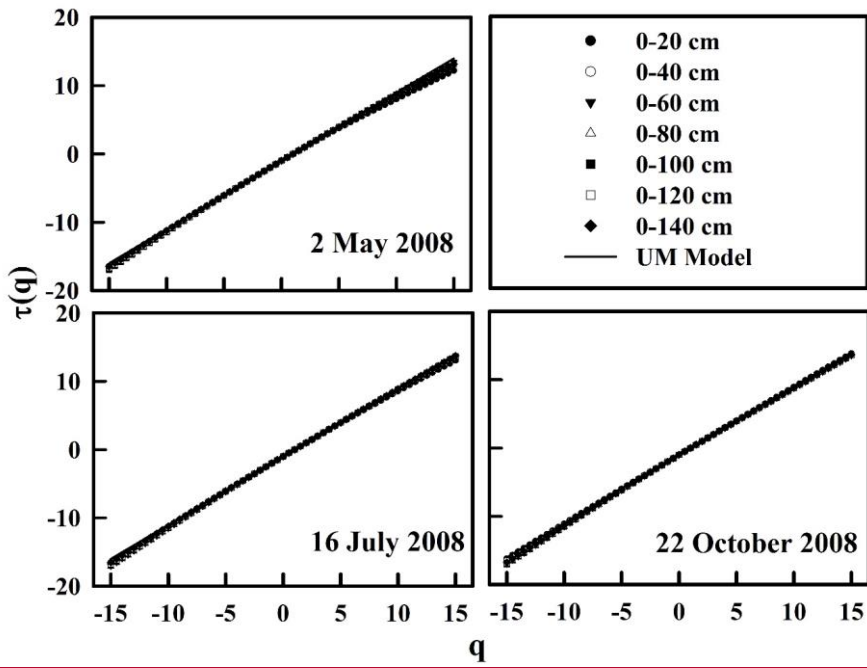
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792 Figure 23

Commented [r4140]: Very difficult to distinguish the points from each depth

Commented [r4141R40]:



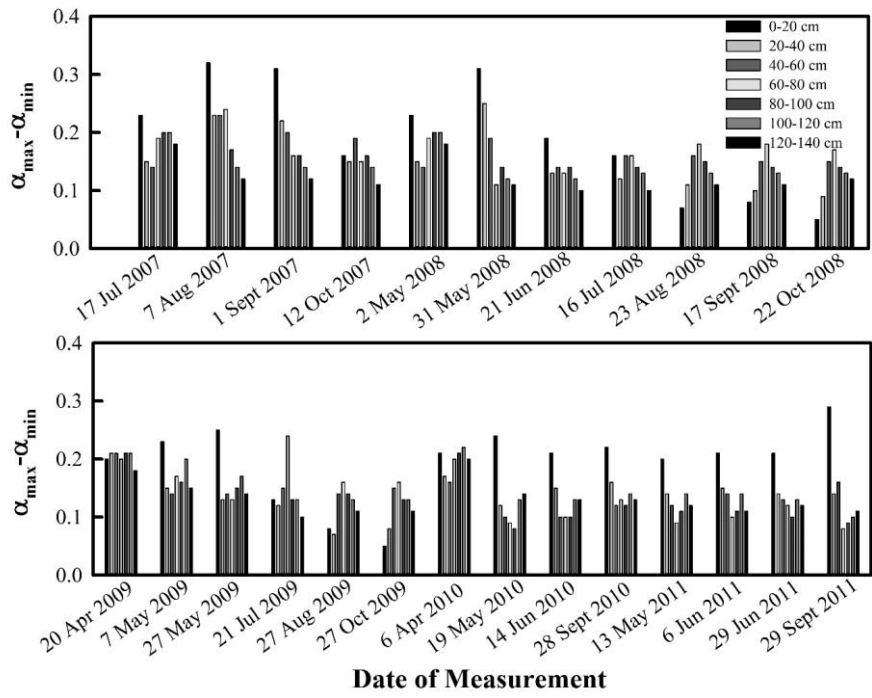


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Figure 34

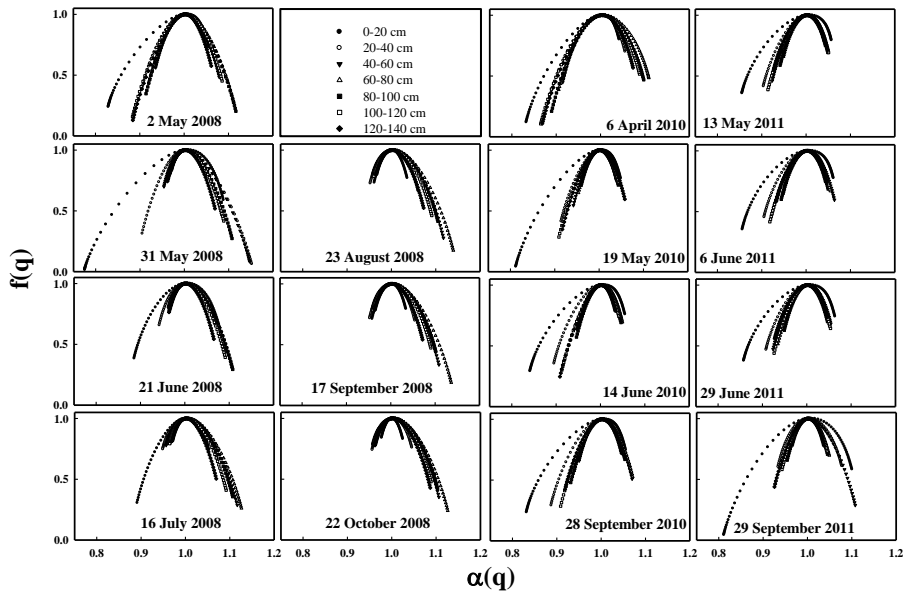
Commented [r4142]: Very difficult to distinguish the points from each depth, UM models is missing from the graphs



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797 [Figure 5](#)

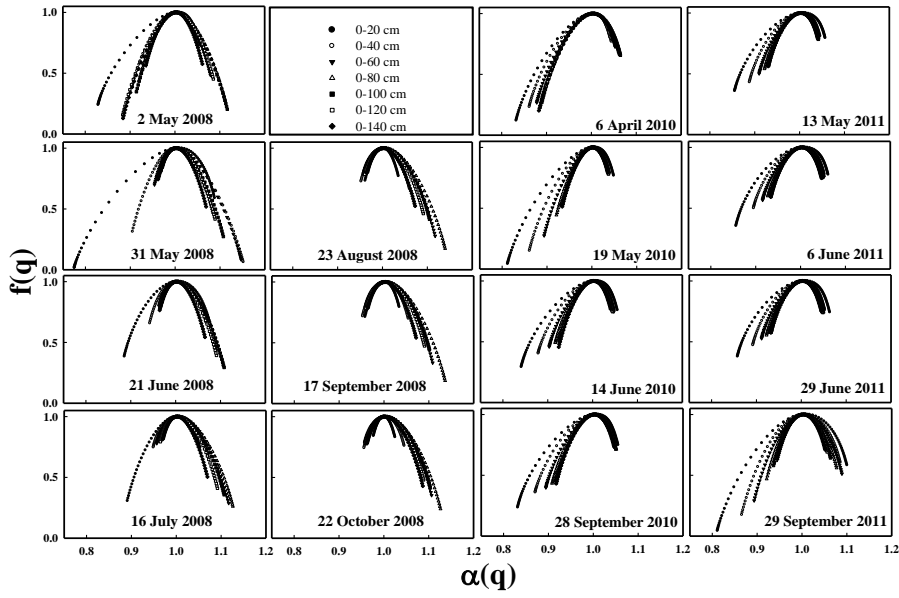




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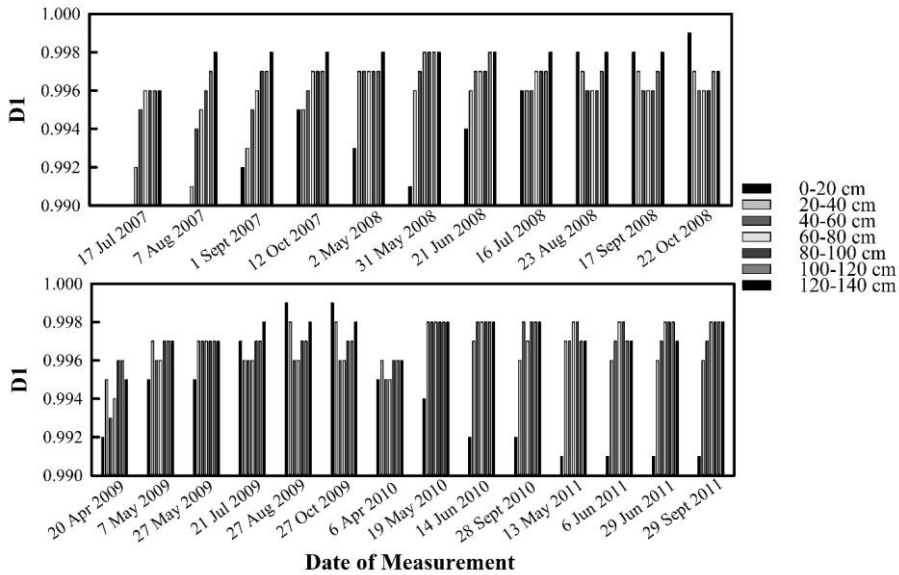
800 Figure 46

**Commented [r4143]:** Anonymous Referee #1: I suggest to take into account the shape of the singularity spectra and not only the amplitude in this and the Results and in the Discussion sections



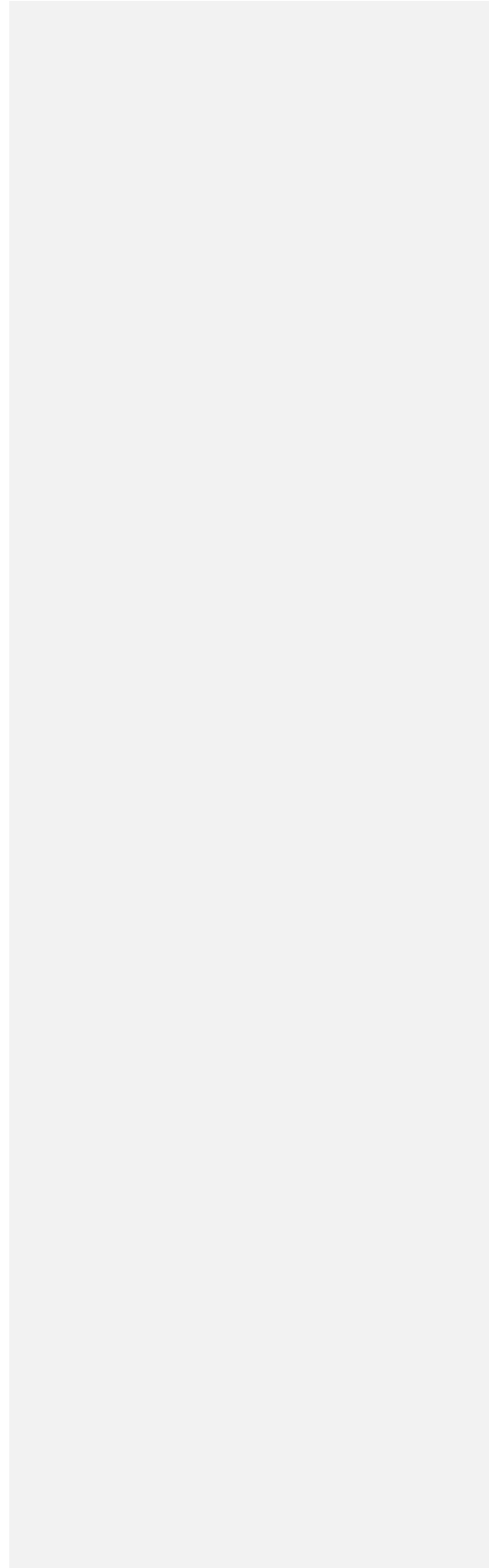
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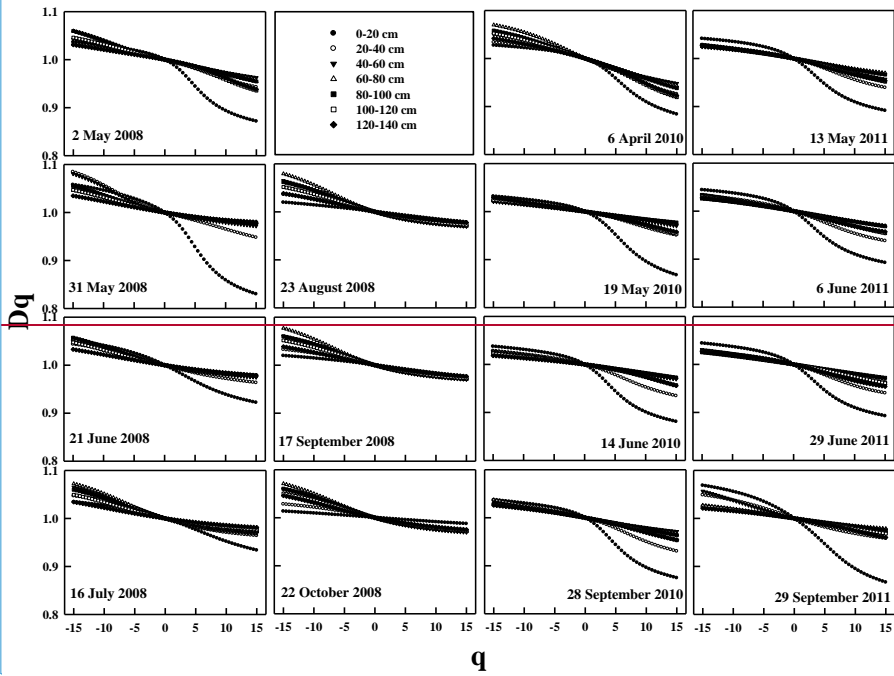
803 Figure 75-

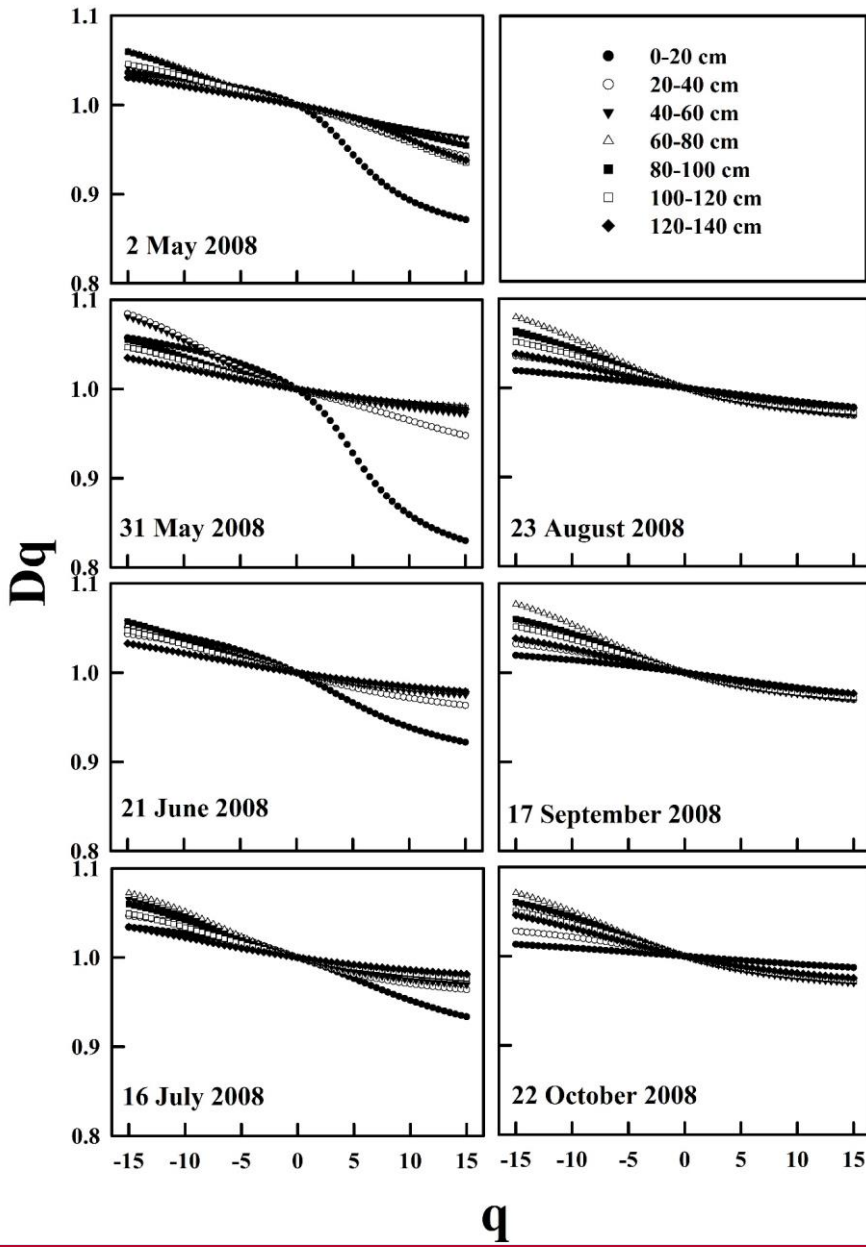


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**Commented [r4144]:** Anonymous Referee #1: I suggest to take into account the shape of the singularity spectra and not only the amplitude in this and the Results and in the Discussion sections, also these shapes should provide valuable information, I guess.





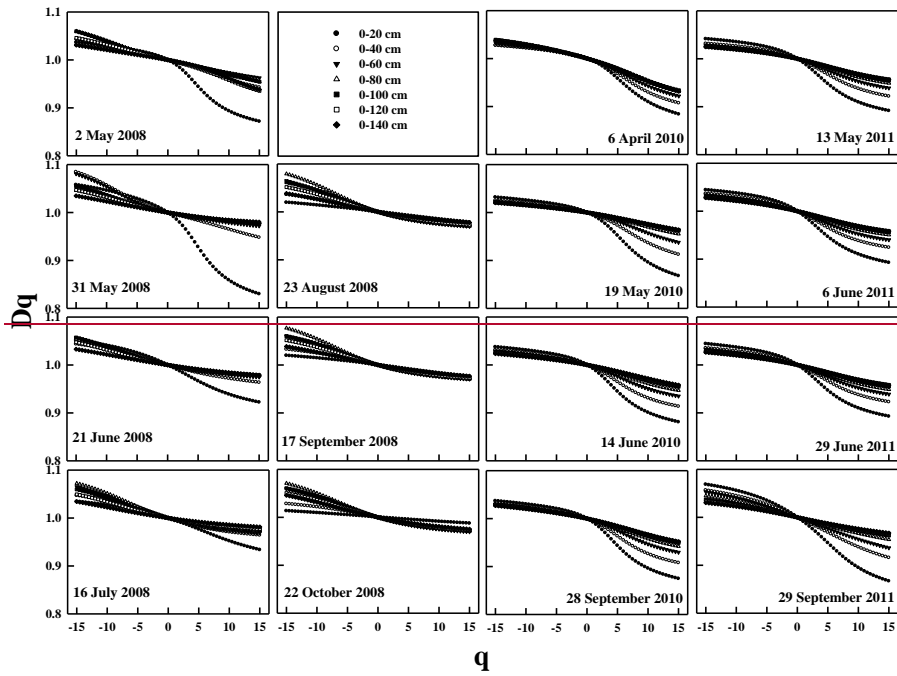


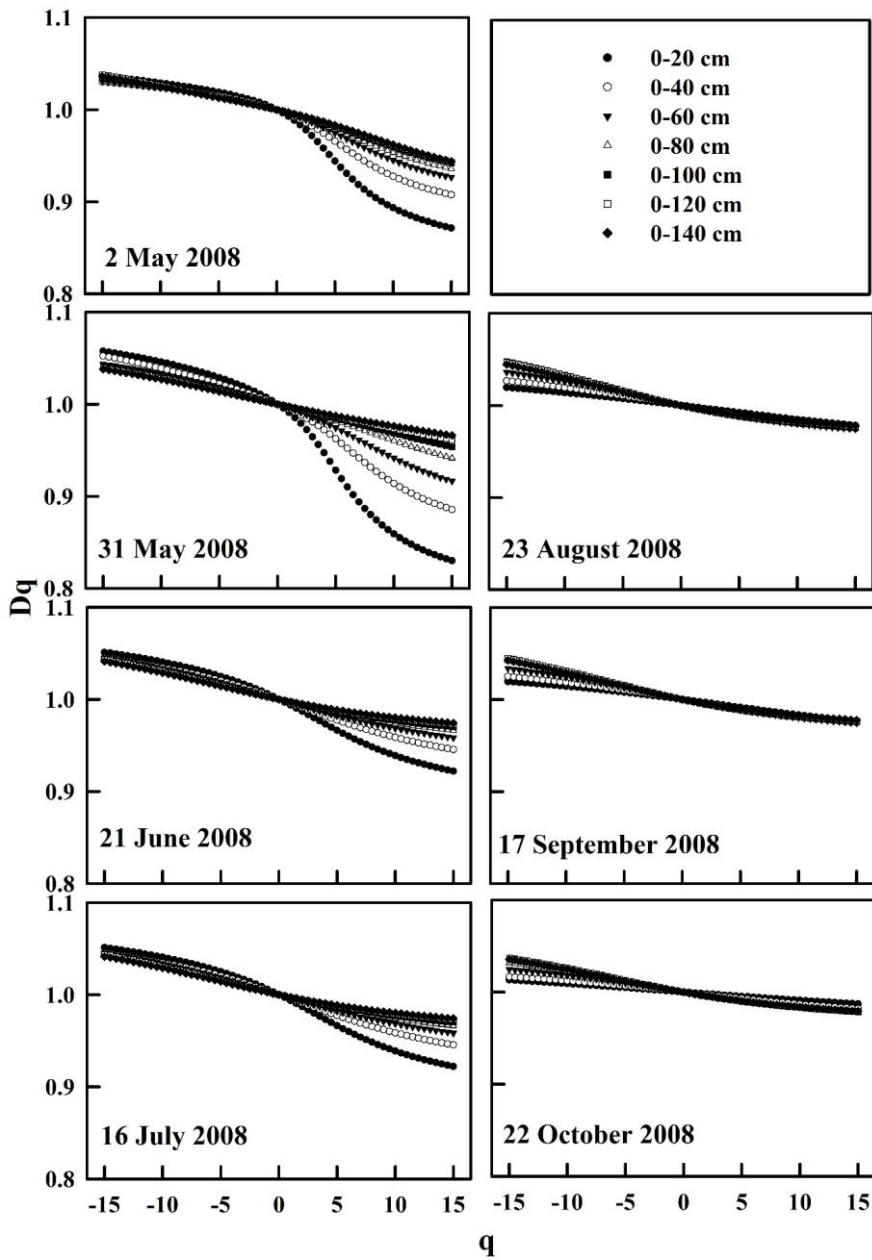
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808 Figure 69

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Commented [r4145]: Some values are overlapped in the Y-axis

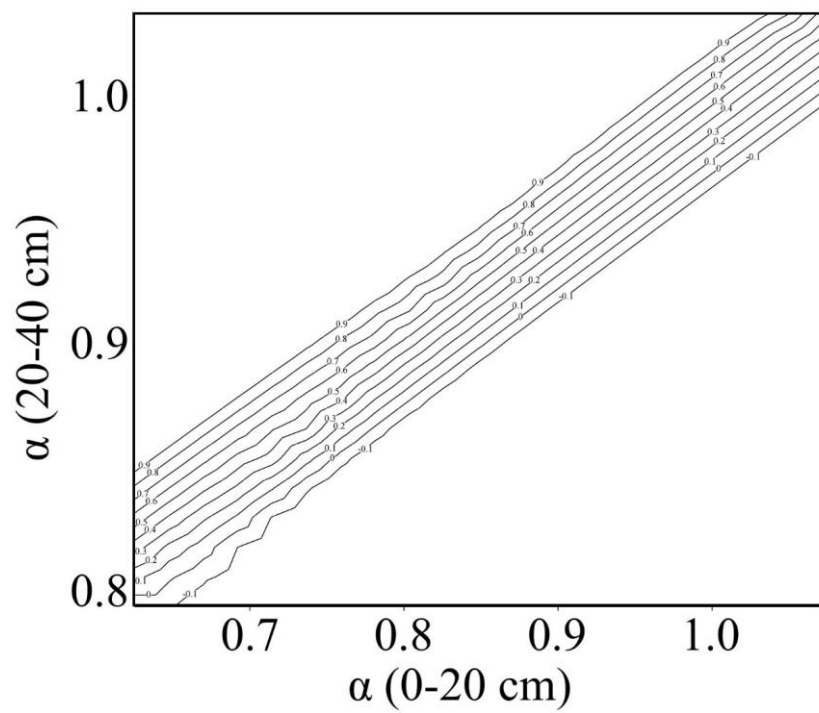




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812 Figure 7.10

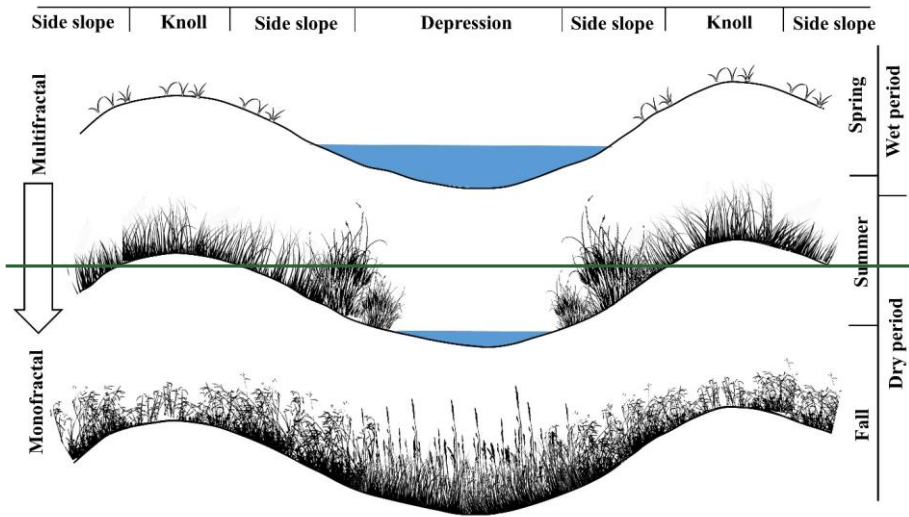
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814 [Figure 11](#)



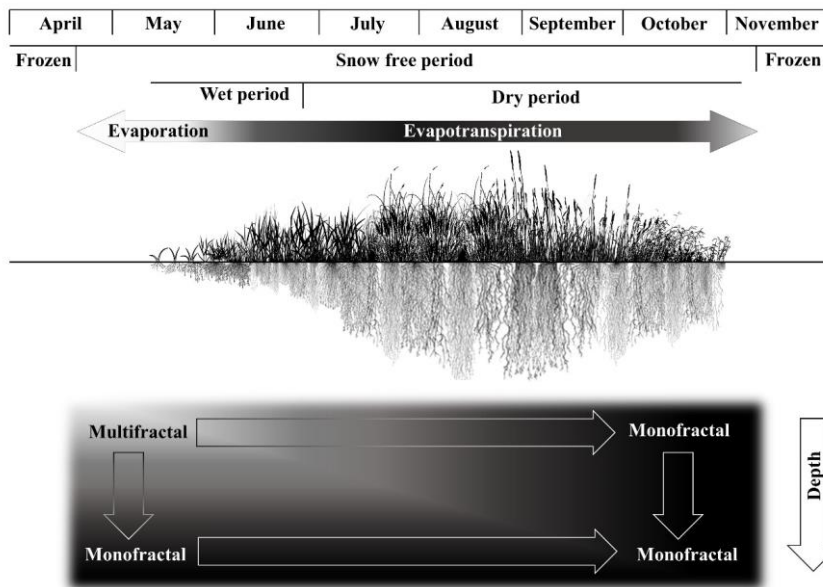


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816 **Figure 8**

**Commented [r4147]:** Anonymous Referee #1: I suggest to move Figure 8 (scheme of the vegetation growth patterns) either to the Material and Method section, or to the section 3.1 (Spatial pattern of soil water storage at different depths). Indeed this Figure is related to the Discussion section, but it is also pertinent to previous section.

**Commented [r4148R47]:** Once confirmed, I can do the revision.



817

818 **Figure 9**<sup>12</sup>

**Commented [r4149]:** The reviewer has asked for three times whether this figure is needed. I have answered in the text, however, maybe need to clarify again in the mail.