



1 **Influence of Atmospheric Stratification on the Integral**
2 **Scale and Fractal Dimension of Turbulent Flows.**

3

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12

13 **Abstract**

14 In this work the relation between integral scale and fractal dimension and the type of
15 stratification in fully developed turbulence is analyzed. Integral scale corresponds to
16 that in which energy from larger scales is incoming into turbulent regime. One of the
17 aims of this study is the understanding of the relation between the integral scale and the
18 Bulk Richardson number, which is one the most widely used indicators of stability close
19 to the ground in atmospheric studies. This parameter will allow us to verify the
20 influence of the degree of stratification over the integral scale of the turbulent flows in
21 the Atmospheric Boundary Layer (ABL).The influence of the diurnal and night cycle in
22 the relationship between the fractal dimension and integral scale is also analyzed.
23 Fractal dimension of wind components is a turbulent flow characteristic as it has been
24 shown in previous works, where its relation to stability was highlighted. Fractal
25 dimension and integral scale of the horizontal (u') and vertical (w') velocity fluctuations
26 have been calculated using the mean wind direction as framework. The scales are
27 obtained using sonic anemometer data from three elevations 5.8 m, 13 m and 32 m
28 above the ground measured during the SABLES- 98 field campaign. In order to
29 estimate the integral scales a method that combines the normalized autocorrelation
30 function and the best gaussian fit ($R^2 \geq 0.70$) has been developed. Finally, by comparing,



1 at the same height, the scales of u' and w' velocity components it is found that almost
2 always the turbulent flows are anisotropic.

3

4 **1 Introduction**

5 The aim of this paper is to bridge the considerable gap that exists between the fractal
6 dimension and the integral scale. The size of the integral scale of the horizontal and
7 vertical components and fractal dimension of wind velocity near the earth's surface in
8 boundary layer are determined. Also, these magnitudes are compared between them and
9 versus other parameters such as the Bulk Richardson number. It is assumed that the
10 turbulence is the primary agent that causes changes in the boundary layer. In turbulent
11 flows it is observed that time series of meteorological variables as wind velocity,
12 temperature, pressure and other atmospheric mechanical magnitudes fluctuate in a
13 disordered way with peaks extremely sharp and irregular space and time variations. The
14 complicated nature of these series indicates that the motion of the air is turbulent. If we
15 take a good look at the variety of fluctuations of different periods and amplitudes
16 observed in them we could explain the complicated structure of turbulence. The
17 irregularity of the time series obey to the existence of different size and time scales and
18 also to the nonlinear transfer of energy that exists between them in the turbulent flows
19 (Monin and Yaglom, 1971).

20 The irregular behavior of these flows is also due to waves and turbulence that are often
21 superimposed on a mean wind (Stull, 1998). If we filter the mean wind and waves in the
22 appropriate range we will only have turbulence. Some previous works present results
23 about this procedure (Tijera et al., 2008). In this paper we have carried out the necessary
24 transformation to get the mean wind series in short intervals, namely 5 minutes. We
25 filter horizontal and vertical mean wind velocity obtaining the time series of
26 fluctuations of the velocity in both directions ($u' = u - \bar{u}$, $w' = w - \bar{w}$).

27 When we observe these time series such as wind velocity, they vary in an irregular
28 shape and in spite of their complexity presents a self-similarity structure (Frisch, 1995).
29 This is a common property of the fractals, so that wind velocity could be considered as a
30 fractal magnitude. The modern physical notion of fractals is largely known due to
31 Mandelbrot (1977, 1985), but the mathematical notion of curves lines or sets having
32 noninteger dimensions is much older (Hausdorff, 1919, Besicovitch, 1929). An analysis



1 that compare the Hausdorff dimension and Kolmogorov capacities of self-similar
2 structure with non integer fractal dimensions (Kolmogorov capacity or box counting
3 dimension) was presented by Vassilicos (Vassilicos and Hunt, 1991). The wind
4 velocity versus time are irregular curves of this type, with noninteger dimensions. These
5 values correspond to the fractal dimension. A way of measuring the complexity of these
6 series is by means of fractal dimension. The Fractal Dimension of wind components is a
7 characteristic of turbulent flow as it has been shown in previous works where its
8 relation to stability was highlighted (Tijera et al., 2012)

9 In this paper the integral scale of u' and w' component are compared. The scales are
10 calculated using sonic anemometer data from three elevations 5.8 (~ 6), 13 and 32 m
11 above the ground at the main tower site of the Sables 98 field campaign. Turbulent
12 motion of the atmospheric fluxes that occurs from the smallest scales are usually
13 defined as the scale at which the motion dissipates into heat due to the viscosity of the
14 fluid until the larger scales that correspond the integral scale. The integral scale can be
15 defined in several ways: the larger scale of the flow, the scale above which the Fourier
16 transform has a slope inferior a $-5/3$ slope, as which the turbulent kinetic energy (TKE)
17 is maximum. Micrometeorological studies have found integral scale varying in a huge
18 range, from around a hundred to a thousand meters (Teunissen, 1980, Kaimal and
19 Finningan, 1994).

20 We study the anisotropy of the turbulent atmospheric flows in these scales comparing
21 integral scale of fluctuations of the velocity component along of the mean wind
22 direction and the vertical component at three different levels above the ground (5.8 m,
23 13.5 m, 32 m).

24

25 **2 Theoretical background**

26 The irregular behavior of the atmospheric turbulent fluxes in the boundary layer at large
27 Reynolds number leads us to be interested in calculating their fractal dimension. Fractal
28 dimension could help us to classify the irregularity of these flows. The more irregular
29 the flow the greater its fractal dimension. Turbulent flows are characterized by the
30 formation of many eddies of different length scales. These irregularities are due to the
31 superimposition of eddies of different sizes and it is related with a broad range of scales
32 which exist in turbulence. These scales vary from the smallest scales as dissipative scale
33 to larger scale as integral scale. This paper is concerned with the analysis of the



1 relationship between the integral scale and fractal dimension. As well as the relationship
2 between the integral scale with the Bulk Richardson number, a turbulent and stability
3 parameter which is used to characterize the degree of stratification in the atmosphere.

4 In this section we describe the methodology applied to calculate the fractal dimension
5 and the integral scale. The estimation of the fractal dimension of time series has been
6 the most commonly used criteria to measure their chaotic structure, there exist different
7 works in that direction (Grassberger and Procaccia, 1982, Shirer et al, 1997). One of the
8 methods most commonly used to estimate to fractal dimension of atmospheric flows has
9 been the mean slope method through box-counting dimension using mean slopes of the
10 graph of $\ln N(L)$ versus $\ln(L)$ for small ranges of L , where $N(L)$ is the number of the
11 boxes of side L necessary to cover the different points that have been registered in the
12 physical space (velocity-time) (Falconer, 2000, Peitgen et al., 2004). As $L \rightarrow 0$ then
13 $N(L)$ increases, N meets the following relation:

$$14 \quad N(L) \cong kL^{-d} \quad (1)$$

15 The value d is the box-counting dimension that is an approximation of the Hausdorff
16 dimension and is calculated approximately by means of least-square-fitting of the
17 representation of $\log N(L)$ versus $\log L$ obtaining the straight line regression given by
18 the following equation:

$$19 \quad \log N(L) = \log k - d \log L \quad (2)$$

20 The fractal dimension d will be given by the slope of this equation as is shown in the
21 Fig.1.

22 In this paper we focus on calculating the integral scales for the mean wind direction u'
23 as horizontal component and vertical velocity as component w' and we studied their
24 variations with respect to the fractal dimension and with the Bulk Richardson number, a
25 turbulent parameter of stability.

26 These integral scales have been estimated using the normalized autocorrelation function
27 and a Gaussian fit. The velocity autocorrelation function as a function of τ (lags
28 number) for u' component is:

$$29 \quad R(\tau) = \frac{\overline{u'(t)u'(t+\tau)}}{u'^2(t)} \quad (3)$$

30 Integral time scale is:



$$1 \quad T_L = \int_0^{\infty} R(\tau) d\tau \approx \int_0^{\tau'} R(\tau) d\tau \quad (4)$$

2 The integral time scale provides a measure of the scales of eddies in the x direction of
3 a flow field. In the Eq. (2) we observed that τ' denotes the last lag in the data series. In
4 boundary layer observations this time scale can be related to a length by multiplying the
5 mean wind velocity by time scale. This requires the assumption of frozen turbulence
6 known as Taylor's hypothesis (Panofsky and Dutton, 1984). The integral length scale
7 can be defined as:

$$8 \quad \lambda = \bar{v} T_L \quad (5)$$

9 The used method is based on Gaussian fit of the normalized autocorrelation function
10 $R(\tau)$ and we calculated the value of τ that verifies the following equation:

$$11 \quad \tau - \int_0^{\tau} R(\tau) d\tau = \int_{\tau}^{\tau'} R(\tau) d\tau \quad (6)$$

12 The Fig. 2 shows the Gaussian fit for an example of a data series of wind velocities with
13 τ that verifies Eq. 6. This value allows us to calculate the integral time scale
14 multiplying it by the time interval between each lag.

15

16 **3 Description of Data**

17 The data set was recorded in the Research Centre for the lower Atmosphere (CIBA in
18 the Spanish acronymus), located in Valladolid province (Spain) and were measured in
19 the experimental campaign Sables-98. This research centre was set up primarily to study
20 the atmospheric boundary layer. The campaign took place from 10th to 27th September
21 1998 (Cuxart et al., 2000). This experimental site is a quite flat and homogeneous
22 which forms a high plain of nearly 200 Km², surrounded by crop fields and some small
23 bushes strewn over ground. Duero river flows along the SE border of the high plain.
24 The synoptic conditions during the period of study of eight consecutive days (from 14
25 to 21 September) were controlled by a high pressure terrain system which produces
26 thermal convection during the diurnal hours and from moderate to strong stable
27 stratification during the nights.

28 Here we analyze sonic anemometer data installed at 5.6 (~ 6), 13 and 32 m, these data
29 set are five minute series. These series have been obtained once we have carried out the



1 necessary transformation to get the mean wind velocity series in short periods of 5
2 minutes. At a rate of 20 data points per second, sonic anemometers can resolve integral
3 scales between about 10 m to 2000 m of u' horizontal component and 1 m to 1000 m of
4 the w' vertical component, depending on the height in which the sonic anemometer is
5 positioned and at the wind speeds typically measured in the Sables-98 experiment.

6

7 **4 Results**

8

9 **4.1 Fractal Dimension, integral scale and stability of stratification.**

10 In this paper we analyze the influence of stability of stratification on fractal dimension
11 and integral scale. Different atmospheric surface-layers data are separated into thermal
12 and dynamics stability classes based on a dimensionless parameter such as the Bulk
13 Richardson number Ri_B . This parameter represents the ratio of the production or
14 destruction of turbulence by buoyancy and by wind shear strain that is caused by
15 mechanical forces in the atmosphere:

$$16 \quad Ri_B = \frac{g}{\bar{\theta}} \frac{\Delta\bar{\theta}\Delta z}{(\Delta\bar{u})^2} \quad (7)$$

17 where g is the gravity acceleration and $\bar{\theta}$ the average potential temperature at the
18 reference level, the term $\frac{g}{\bar{\theta}}$ is known as the buoyancy parameter. Ri_B is positive for
19 stable stratification, negative for unstable stratification and approximate zero for neutral
20 stratification (Arya, 2001). The way to calculate this number is described next:

- 21 1. Calculation of the mean potential temperatures at height $z = 32$ m, and close to the
22 surface $z = 5.8$ m, namely $\bar{\theta}_{32}$ and $\bar{\theta}_{5.8}$ respectively. Being $\Delta\bar{\theta} = \bar{\theta}_{32} - \bar{\theta}_{5.8}$.
- 23 2. Obtaining of \bar{u}_z the mean wind velocity module at the height $z = 32$ m and $z = 5.8$
24 m, denoted by \bar{u}_{32} and $\bar{u}_{5.8}$ respectively, where $\Delta\bar{u} = \bar{u}_{32} - \bar{u}_{5.8}$.

25 Once the values of $\Delta\bar{\theta}$, $\Delta\bar{u}$ and Δz have been obtained by means of Eq. (7) we
26 calculate the Bulk Richardson number in the layer between 32m and 5.8m.

27 In Fig. 3 we present the variation of the fractal dimension of the u' horizontal
28 component of the velocity fluctuations along time at the three considered heights: 5.8 m,



1 13 m and 32 m. The behaviour of these variations is similar at the three heights. The w'
2 component fluctuation presents an analogous behaviour. The fractal dimension values
3 are in a range between 1.30 to nearly 1.00. We have found that during the diurnal hours
4 the fractal dimension is bigger than at night (Tijera, 2012). We have no theoretical
5 reason to explain this result, but a possible explanation of why this happens could be
6 that fractal dimension is related with atmospheric stability and with the intensity of
7 turbulence. It is well known that the intensity of turbulence grows up as solar radiation
8 increases, producing instability close to the ground, mainly in the hours of noon.
9 Therefore, one of the possible reasons of the increase of FD is the instability of the
10 turbulent flow. In the other hand, during the nights a strong atmospheric stability
11 usually exists, so the fractal dimension is usually smaller than during the diurnal hours.
12 Thus, stable stratification decreases the fractal dimension.

13 In Fig. 4 it is observed a different behaviour in the integral scale for u' horizontal and
14 w' vertical component, it is not clear how it influences on the diurnal cycle. Sometimes
15 during the night and the noon the scales increase or decrease in a stochastic way and this
16 is due to the chaotic structured motion of the atmospheric fluid that we refer as
17 turbulence. It is observed in the own integral scale many eddies of many different
18 lengths. The integral scale for u' component varies between around 100 m on their
19 minor scales, until above 1500 m for its mayor scales. The integral scales for w'
20 component are slightly lower than u' component as it is indicated in Fig. 4. It is shown
21 that these vertical scales can reach sizes between a few tens of meters until 1000 m in
22 some occasions. It is observed in each component that the greater the height the greater
23 the integral scale. Usually, scales at 32 m are greater on average than those at 13 m and
24 the latter higher than at 5.8 m

25 Although the mental model of turbulence as eddies of various sizes is useful, it is
26 difficult to obtain a correlation between the integral scale and fractal dimension in the
27 atmosphere if we consider values throughout the whole day. However, it is much easier
28 to find a relationship between the integral scale and fractal dimension of horizontal and
29 vertical components of the wind velocity if we separate the hours of the day and night,
30 and hence analyze the influence of diurnal and night cycle over these parameters.
31 Daylight hours are from 6-18 UTC and the night from 18-6 UTC. These data set are
32 analyzed in the three studied heights. Fig. 5 shows the variations of the integral scale
33 versus fractal dimension at the level of 5.8 m for horizontal component. As it can be



1 appreciated in Fig. 5 in the diurnal hours the average values of the integral scale versus
2 the fractal dimension can be adjusted to the straight regression line given by the linear
3 equation that appears on the top left of the graph. During those hours these values of the
4 integral scale increase from a few tens of meters until 400 m with increasing values of
5 the fractal dimension until 1.25. During the nights the average values of the integral
6 scale decreases with the increase in the fractal dimension. These values also fit a
7 straight regression line as it is indicated in Fig. 5. One of the possible explanations for
8 this behaviour is that during the diurnal hours the average values of the integral scale
9 increase due to the unstable stratification. During the nights, the existence of the stable
10 stratification decreases the integral scale with an increase in fractal dimension until the
11 approximate value of 1.2. This tendency appears also in the other two heights, at 13 m
12 and at 32 m as it is shown in Fig. 6. Although during the diurnal cycle at 32 m the linear
13 fit it is not so evident, the maximum scales are in the 1.15-1.20 range of the fractal
14 dimension as it is illustrated in Fig. 6 (c).

15 For the vertical component in the three studied levels, the behaviour is slightly different.
16 The averages values have fitted a quadratic function as it is indicated in Fig. 7. During
17 the diurnal hours the averages values of integral scale reach maximum scales around
18 the value of the fractal dimension of 1.15 at the three heights. From this value the
19 integral scale decreases when fractal dimension increases. These maximum integral
20 scales depend on the height. At the level of 5.8 m their sizes reach 50 m in average and
21 the scattering of the values shows higher values that could reach 100 m. At the level of
22 13 m the values are about 100 m, the dispersion of these scales can reach sizes of 200 m
23 and at the height of 32 m their larger average scales are approximately around 200 m
24 and due to the variances of the data set could reach sizes of 400 m. From the value of
25 the fractal dimension value of 1.15, the scales decrease until a few meters.

26 Throughout the night the average values of the integral scales decrease with the increase
27 of the fractal dimension in a parabolic way as it is indicated in Fig. 7. This happens due
28 to the stable stratification that occurs at nights. This behavior during the diurnal and
29 night hours for w' component of the integral scale is similar to the results obtained for
30 u' component, although the fits of averages values are parabolic and not linear. In all
31 these cases our R^2 values and confidence levels are high as it is indicated in Fig. 7.

32
33



1 **4.2 Relationship between integral scale and Bulk Richardson number**

2 Among the numerous parameters existing to characterize the degree of stratification in
3 the atmosphere we will use the Bulk Richardson number. The interpretation of this
4 number has already been mentioned in the previous section. Here, we analyze how the
5 integral scale of each one of the u' horizontal and w' vertical components varies with
6 the Bulk Richardson number in diurnal and night cycle in the studied period. These
7 results are shown in Fig. 8 for horizontal component and in Fig. 9 for vertical
8 component. During the daylight hours appear the three kinds of stratification: unstable,
9 neutral and stable as it is shown in the three graphs on the left side of Fig. 8 and the Fig.
10 9, each one corresponds to the different heights. In the stratification unstable and neutral
11 the integral scales are higher than the integral scales under the influence of the stable
12 stratification. At 5.8 m for the horizontal component these scales vary between 200 m
13 and values slightly higher than 400 m and in the case of neutral stratification could
14 increase until 600 m. This same behaviour occurs in the other two studied heights 13 m
15 and 32 m although their scales are slightly higher as it is illustrated in Fig. 8. During the
16 nights it is observed the biggest stability due to positive values of the Bulk Richardson
17 number.

18 The same results are obtained for the integral scales of vertical component, although
19 their sizes are smaller. At 5.8 m during the diurnal hours the average values reach about
20 50 m and during the night hours their values are below 50 m. At 13 m and 32 m in the
21 diurnal hours the average values could reach about 150 m and 200 m and at the night
22 hours are below 100 m and 200 m respectively.

23

24 **4.3 Analysis of the anisotropy with the integral scale**

25 In the last section we study the relationship between the integral scales of the horizontal
26 and vertical components at different heights: 5.8 m, 13 m and 32 m. In Fig. 10 we
27 represent the integral scale of u' component versus the integral scale of w' component at
28 three studied heights and we find linear relations with the averages values of these
29 scales. All integral scales measured during the period of study from 14 to 21 of
30 September appear in this figure. The linear fits obtained are acceptable, with high R^2
31 values at 13 m and 32 m, as it is indicated in Fig. 10. The linear regression appears on
32 the top left of each graph: at 5.8 m $L_{intu}(5.8\text{ m}) = 1.46 L_{intw}(5.8\text{ m}) + 178$, at 13 m



1 $L_{intu}(13\text{ m})=0.957 L_{intw}(13\text{ m})+275.6$ and at 32 m $L_{intu}(32\text{ m})=0.646L_{intw}(32\text{ m}) +370$,
2 being L_{intu} and L_{intw} the average values of the integral scale for horizontal and vertical
3 component respectively.

4 The data in Fig. 10 appear quite scattered and the average values could be representative
5 to find relationships between these scales. This scatter is due to the large number of
6 uncontrolled variables, nonlocal disturbance, the presence of waves, horizontal
7 inhomogeneity, low frequency disturbances, etc. These graphs are showing that the
8 scale measured at 32 m is nearly always larger than the integral scale measured at 5.8 m.
9 On the basis of the results obtained, we find slight differences between these
10 components, thus there is anisotropy in atmospheric turbulent flows. In isotropic
11 turbulence the integral scales of both components should be the same at the same
12 height. Only under certain conditions and over limited scales is isotropy a property of
13 turbulence in the stratified atmosphere (Thorpe, 2005).

14

15 **5 Conclusions**

16 In this paper algorithms have been developed to calculate the fractal dimension and the
17 integral scale using wind velocity data from the convective boundary layer. We present
18 some results related to the time evolution of both fractal dimension and integral scale.
19 As well as how the day and night cycle affects the relationship between the fractal
20 dimension and integral scale and their behavior versus Bulk Richardson number. The
21 different levels of stratification help us to understand the relations between the fractal
22 dimension and integral scale. The stratification in the atmosphere has showed some
23 degree of the influence with the most of the integral scales. The main conclusions of
24 this study are as follows.

25 Although all data appear quite scattered in this work, the averages values of these
26 magnitudes show interesting results. During the diurnal hours the averages values of the
27 integral scale of the horizontal component increases with the increase in fractal
28 dimension until around 1.25 at 5.8 m and 13 m height. At these heights we have found
29 linear fits between these magnitudes with high coefficients of correlation. While at 32 m
30 the linear fit is not so evident, the maximum scales are in the 1.15-1.20 range of the
31 fractal dimension. One of the possible explanations for this behaviour is that during the
32 diurnal hours the average values of the integral scale increase due to the unstable



1 stratification. During the night hours the average values of the integral scale decreases
2 with the increase in the fractal dimension. These values also fit a straight regression
3 line at the three analysed heights. During the nights the existence of the stable
4 stratification decreases the integral scale with an increase in fractal dimension until the
5 approximate value of 1.2.

6 For the vertical component of the integral scale the results are similar, even though with
7 slight differences. The averages values have fitted a quadratic function. During the
8 diurnal hours the averages values of integral scale reach maximum around the value of
9 the fractal dimension of 1.15 in the three heights. From this value the integral scale
10 decrease with the increase of the fractal dimension until a few meters. The different
11 degree of stratification along diurnal hours will be reflected in that different behaviour
12 from the value of 1.15. At nights when stability is normally major the integral scale
13 decrease with increasing the fractal dimension of a parabolic way

14

15 In the unstable and neutral stratification the integral scales are higher than the integral
16 scales under the influence of the stable stratification.

17 To characterize the anisotropy of turbulent flows we have used the comparison of
18 integral scales of horizontal and vertical component showing that the scale of u'
19 component is almost always larger than the scale of the w' component at the same
20 height.

21

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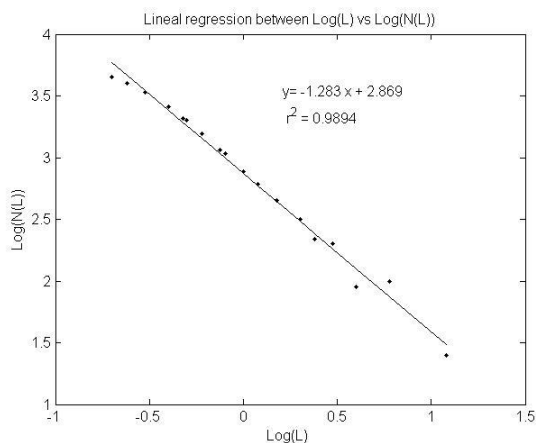
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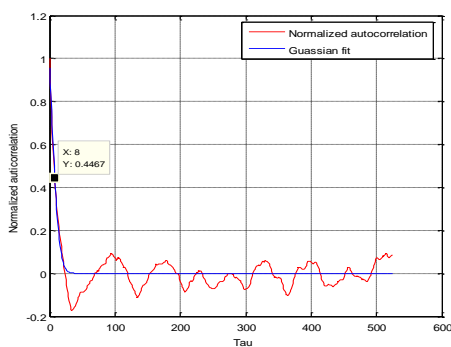
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1

2 **Figure 1.** Example of linear regression between number of not empty boxes and length
 3 side of the box. The slope (d) is the fractal dimension of the w' component, $d = 1.28 \pm$
 4 0.03 for a example of the w' component of the wind velocity.

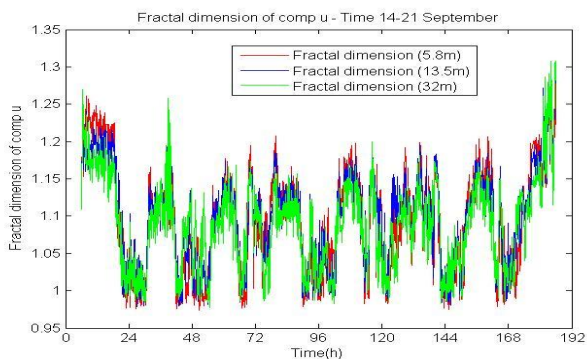
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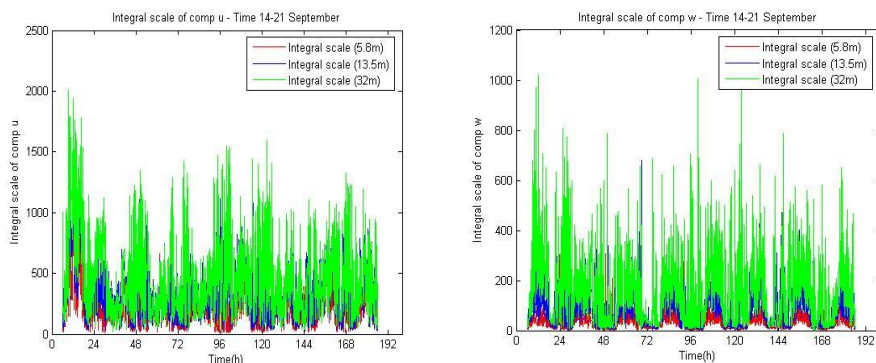
7 **Figure 2.** Gaussian fit for a data series of wind velocities u' component that allows us
 8 to calculate the integral scale.

9



1

2 **Figure 3.** Variation of the fractal dimension versus time for the u' component
3 fluctuation at the three heights, showing the influence of the diurnal cycle.

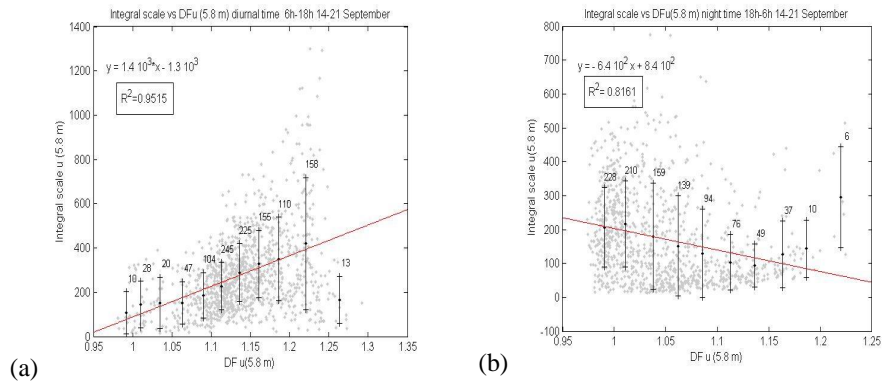


4 **Figure 4.** Variation of the integral length scale of horizontal and vertical components
5 versus time at the three heights.

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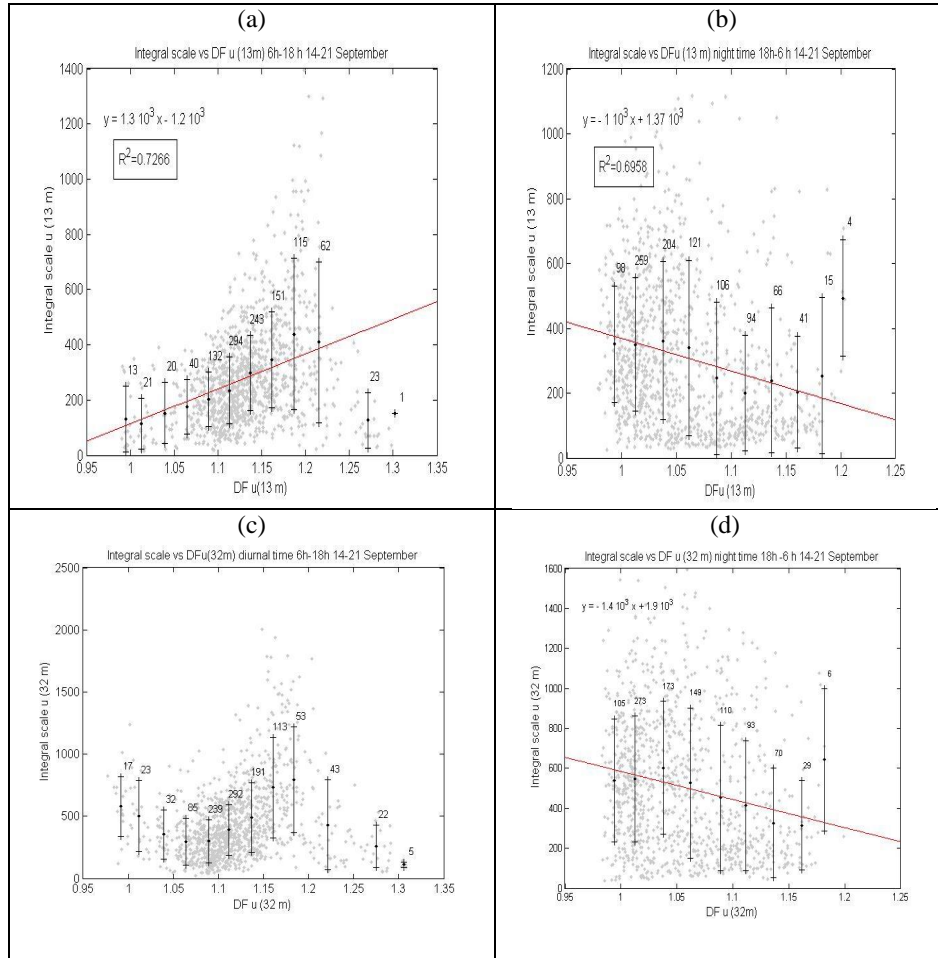


1 **Figure 5.** Variations of the integral scale versus the fractal dimension of u component
 2 of the wind velocity at 5.8 m. (a) diurnal hours 6h-18 h, (b) night hours 18 h - 6h. On
 3 the top left of the each graph it is indicated the linear regression of the averages values
 4 (a) $L_{intu}(5.8\text{ m}) = 1.4 \cdot 10^3 \cdot DF_u(5.8\text{ m}) - 1.3 \cdot 10^3$ (b) $L_{intu}(5.8\text{ m}) = -6.4 \cdot 10^2 \cdot DF_u(5.8\text{ m}) +$
 5 $8.4 \cdot 10^2$, being L_{intu} and DF_u the integral scale and fractal dimension for u component
 6 respectively.

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2 **Figure 6.** Variations of the integral scale versus the fractal dimension of the u
 3 component at 13 m and 32 m. (a) and (c) diurnal hours, (b) and (d) night hours. In the
 4 same manner that in the figure 5 the linear fits are: (a) $L_{intu}(13\text{ m}) = 1.3 \cdot 10^3 DF_u(13\text{ m}) -$
 5 $1.2 \cdot 10^3$, (b) $L_{intu}(13\text{ m}) = -1 \cdot 10^3 DF_u(13\text{ m}) + 1.37 \cdot 10^3$, (d) $L_{intu}(32\text{ m}) = -1.4 \cdot 10^3 DF_u(32$
 6 $\text{ m}) + 1.9 \cdot 10^3$

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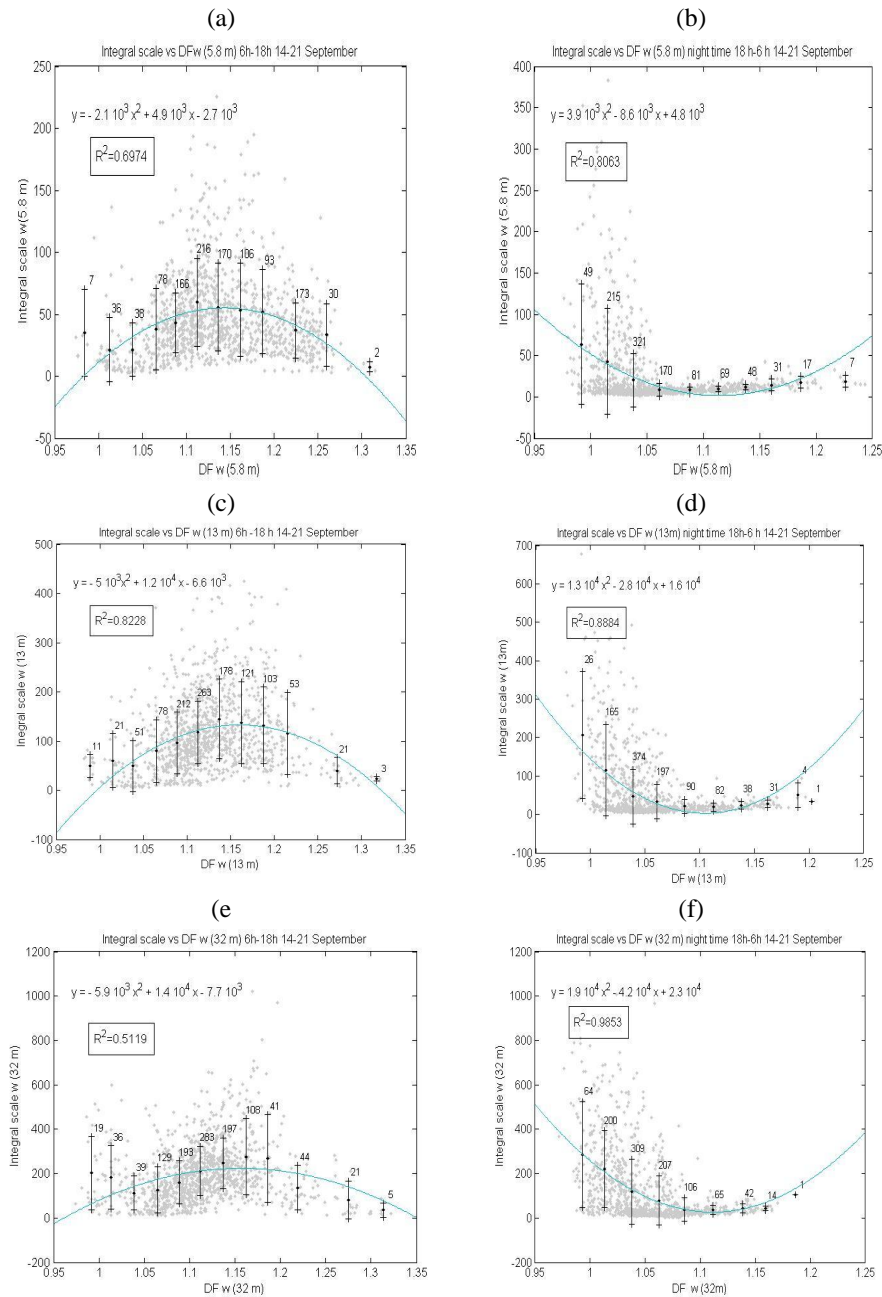
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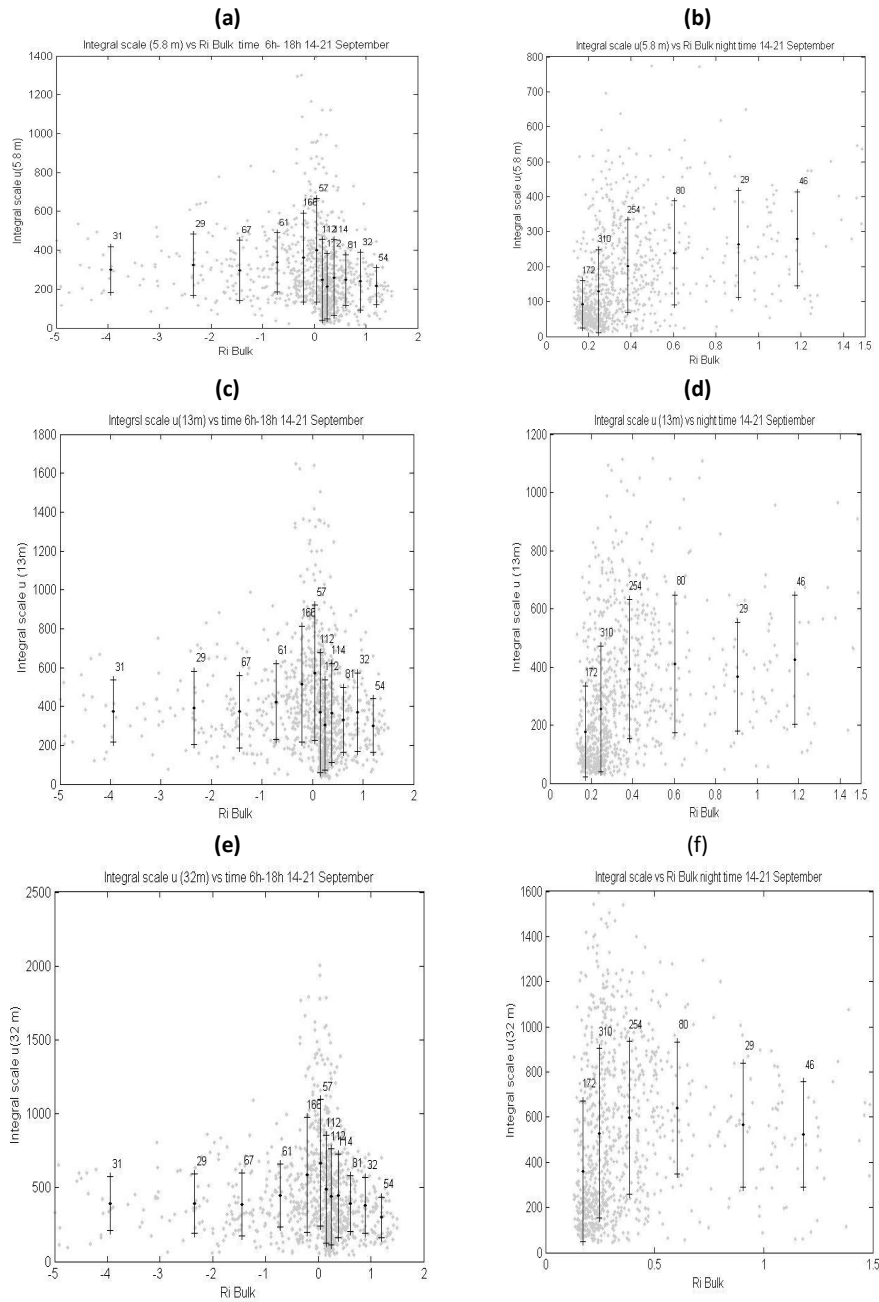
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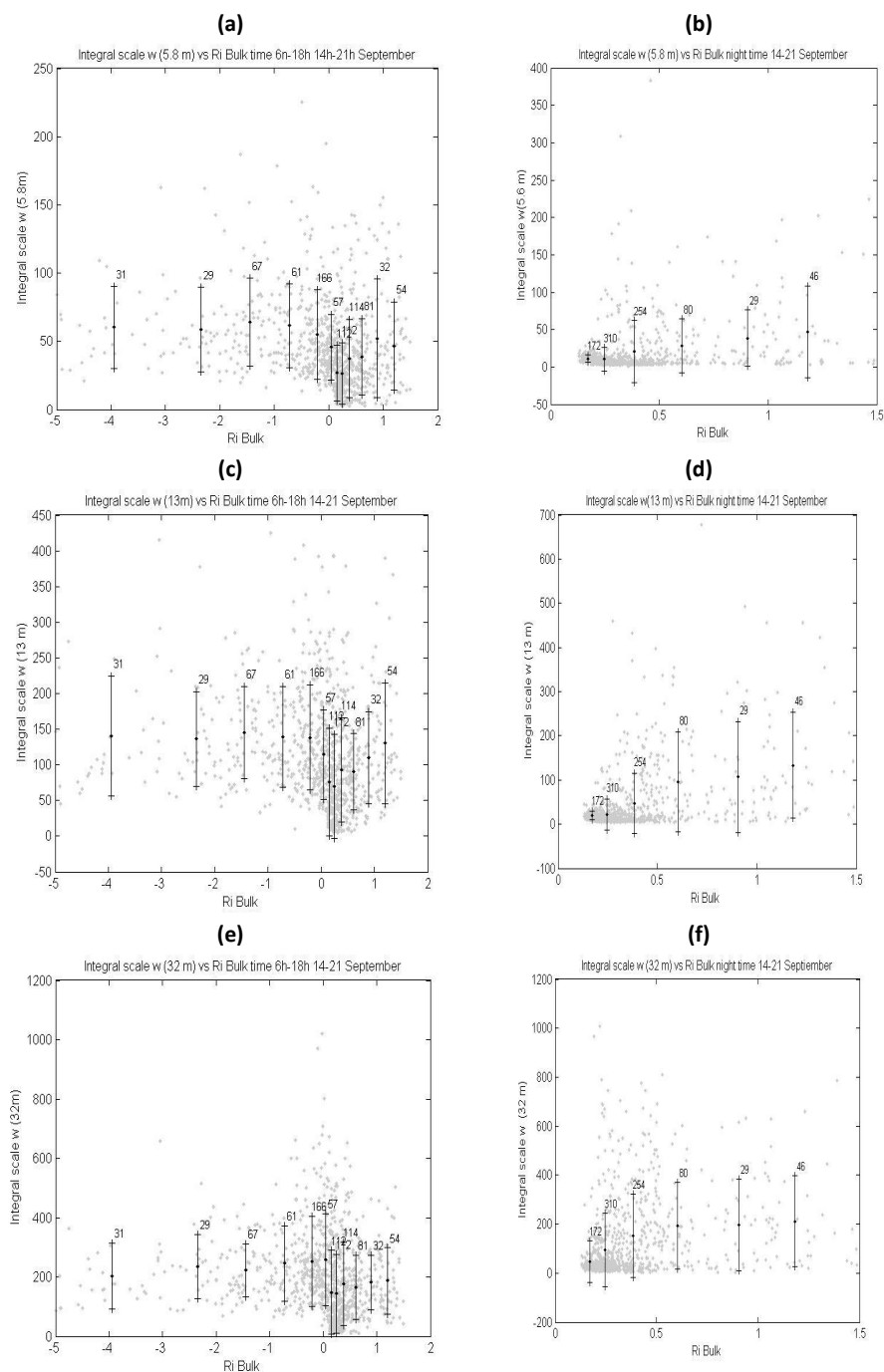


2 **Figure 7.** Variations of the integral scale versus the fractal dimension of the w`
 3 component at 5.8 m, 13 m and 32 m. (a), (c) y (e) diurnal hours, (b), (d) y (f) night.
 4 hours. The fits to a quadratic function of the averages values appear on the top left of
 5 the each graph, being y variable L_{intw} and x variable DF_w .
 6



1 **Figure 8.** Integral length scales of u' component plotted against the Bulk Richardson number
 2 at 5.8 m, 13 m and 32 m. (a), (c) y (e) diurnal hours, (b), (d) y (f) night hours.

3



1 **Figure 9.** Integral length scales of w' component plotted against the Bulk Richardson
 2 number at 5.8 m, 13 m and 32 m. (a), (c) y (e) diurnal hours, (b), (d) y (f) night hours

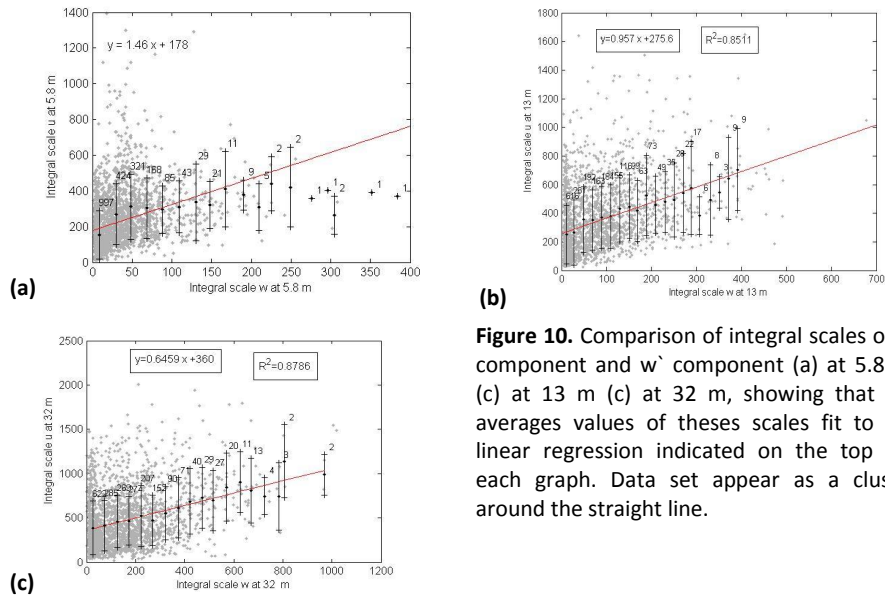


Figure 10. Comparison of integral scales of u' component and w' component (a) at 5.8 m, (b) at 13 m, (c) at 32 m, showing that the averages values of these scales fit to the linear regression indicated on the top left each graph. Data set appear as a cluster around the straight line.