

Response to the Referee #1

(Referee comments in black; response from the authors in blue)

This paper investigates the possibility of correlations between the integral scale of stratified turbulent flows in the boundary layer and parameters characterizing topological features of the wind velocity field, such as the fractal dimension, and its stability properties studied through the Richardson number. Using data of the SABLES-98 campaign, at moderate temporal resolution, authors present here a series of results which provide interesting insights on the dynamics of the atmospheric flow in the site under study. The outcome of the paper is relevant for the audience of Nonlin.Processes in along the lines detailed in the following.

We thank the referee for his positive reviews and constructive comments for our paper. We greatly appreciate the fact that our work is deemed acceptable for publication in NPG with some revisions. In the following, we respond point by point to the referee 1 remarks (in blue).

1 line 5-6 page 2. The text says: "The aim of this paper is to bridge the considerable gap that exists between the fractal dimension and the integral scale"

We agree with referee #1 to clarify this sentence, and have it replaced by the following in this paper:

The aim of this paper is to investigate the possible correlations between the integral scale of the turbulent stratified flows in the Atmospheric Boundary Layer and parameters characterizing topological features of the wind velocity field, such as fractal dimension and its stability properties, studied through the Bulk Richardson number. We are aware that there is a lack of investigations between the integral scale and fractal dimension.

2 line 22-23 page 2. The text is not clear, is it a 5 minutes running average used to remove the mean field? This point needs to be better explicated in the text.

We decided to change the sentence line 23-24 page 2 “ In this paper we have carried out the necessary transformation to get the mean wind series in short intervals, namely 5 minutes “. It is not a 5 minutes running average. The new sentence reads as:

In this work, the series of wind velocities in the three directions x, y and z recorded by the anemometer are divided in series of non-overlapping five minutes length. Each of these series applies the necessary rotations to get the x-axis in the mean wind direction (mean v is zero) and zero mean vertical velocity (w vertical component)(Kaimal and Finnigan, 1994).

3 line 11-14 page 3. The sentence within these lines is not at all clear, please rephrase it.

We rephrased the sentence line 11-14 page 3 by:

Turbulent motion of the atmospheric flows occurs through a broad range of scales from the smallest ones that are usually defined as the scales at which the motion dissipates into heat due to the viscosity of the fluid until the larger scales corresponding to the integral scale.

4 line 2 page 4. The Richardson number provides a measure of how a turbulent flow is prone to develop instabilities. I thus disagree with referring to this parameter as a number that characterizes the degree of stratification in the atmosphere. This definition (used in many places in the paper) is misleading and it is very important to modify it in the text not to drive the wrong message that stratified flows cannot be unstable. Indeed the Richardson number is used to identify unstable regimes also in strongly stratified flows and strong instabilities (and turbulence) can develop in the direction of the gravity, characterized a vertical Froude number

of order ~ 1 as described in Billant&Chomaz, "Self-similarity of strongly stratified inviscid flows," Phys. Fluids 13, 1645–1651 (2001).

We replaced the sentence line 2 page 4 by:

As well as the relationship between the integral scale with the Bulk Richardson number, which provides a measure of the degree of stability in the flow, and how this turbulent flow is prone to develop instabilities. It is also used as a criterion for the existence or non-existence of turbulence in a stably stratified environment (a large positive value over a critical threshold, is indicative of a decaying turbulence or a completely non-turbulent) (Arya, 2001).

5 line 22 page 4. The text says: "...the integral scale of the mean wind direction u ". Here the authors probably means the horizontal direction but the way this is written sounds wrong. Please rephrase it.

We change the sentence line 22 23 page 4 by:

In this paper we focus on calculating the integral scales for horizontal and vertical component fluctuations u' and w' .

7 line 1-2 page 6. See comment ## 2 and please specify better the way the data have been processed. This is important for the readers to be able to reproduce (and therefore validate) the results presented in the paper.

We change the sentence line 1-2 page 6 by:

In this work, data from sonic anemometers measured at a sampling rate of 20 Hz installed at 5.8 (~ 6), 13 and 32 m are analyzed. 5-minute non-overlapping series are used to evaluate the different parameters.

8 line 3-4 page 6. Authors claim they resolve vertical integral scales from 1m to 1000m, which puzzles me a bit. Indeed, I am wondering how is it possible to detect(using the Taylor hypothesis) vertical scales larger than the height(s) at which the sonic anemometer are placed? An answer to this point should be included in the report and a compelling explanation integrated in the text.

We have included the following explanation in line 5 of how vertical integral scales are possible to detect larger than the heights at which the anemometer are placed:

We detect vertical scales over a broad range of scales from 1 m to 1000 m. The integral scales here are calculated based on the autocorrelation function, the mean wind velocity and integral time scale, and each of them can be expected to vary significantly. As the integral scale are the larger scales of turbulent flows it is possible to detect vertical scales larger than heights at which the sonic anemometer are located.

9 line 20 page 6. Details on how the potential temperature has been estimated should be included in the text.

We have included in the line 21 how the potential temperature has been estimated.

The potential temperature has been estimated as relative to ground level by using the following formula: $\Delta\theta = \Delta T + \Gamma\Delta z$, $\Gamma = 0.0098 \text{ K m}^{-1}$ (Arya 2001)

10 line 12 page 7. The text says: "Thus, stable stratification decreases the fractal dimension." Authors acknowledge they cannot provide rigorous arguments to explain the variation of the fractal dimension with the height. As a consequence the statement above sounds a bit too strong, unless they propose some solid argument to support it.

As we cannot provide rigorous arguments to explain the variation of the fractal dimension with the height, we have removed from the text the statement "Thus, stable stratification decreases the fractal dimension."

11 line 14-27 page 7. The text within these lines is poorly written and its meaning is a bit confusing. I suggest to re-written it from scratch and perhaps make it a bit more concise.

We will clarify the text within the line 14-27 page 7 by:

In Fig 4 it is observed how the integral scale varies versus time at the three heights. There are some questions that have not been clarified yet in the literature. For example: How does the diurnal and night cycle influence on the integral scale? Which is the mechanism responsible for the growth of this integral scale? It has been observed in previous works that under certain conditions the turbulent flows self – organize and develop large-scale structures that take place through an inverse cascade that occurs in stably stratified anisotropic flows (with or without rotation) (Smith and Waleffe, 2002, Marino et al., 2014). The inverse cascade mechanism might also be responsible for the growth of the integral scale in the stratified atmosphere. It is a fundamental issue that should be clarified in a future research. As it is indicated in Fig 4 the integral scale for u' component varies between around 100 m on their smaller scales, until above 1500 m for its larger scales. The integral scales for w' component are slightly lower than for u' component. It is shown that these vertical scales can reach sizes between a few tens of meters until 1000 m in some occasions. It is observed, for each of them, that the greater is the height at which is located the sonic, the greater is the integral scale in the turbulent flow. Usually, at 32 m these scales are, on average, greater than those of the 13 m and the latter higher than at 5.8 m height.

12 There is a long standing debate in literature on whether an inverse cascade occurs in stably stratified anisotropic flows (with or without rotation). The inverse cascade mechanism, if any, might also be responsible for the growth of the integral scale in the stratified atmosphere. I thus suggest the authors to address this point, perhaps when they re-arrange the text (line 14-27) as indicated in the previous comment (## 11). On this note/topic I suggest to cite these two papers:

L. Smith and F. Waleffe, "Generation of slow large scales in forced rotating stratified turbulence," J. Fluid Mech. 451, 145–168 (2002).

R. Marino, P.D. Mininni, D. Rosenberg, and A. Pouquet, "Large-scale anisotropy in stably stratified rotating flows". Phys. Rev. E 90, 023018 (2014).

This comment is appreciated and the suggestions and the references of the two papers have been incorporated in the point (## 11)

13 Conclusions should definitely be rearranged from line 16-24 to achieve a better clarity of the text.

We clarify the text within the line 16-24 page 10 by:

We have calculated the fractal dimension and the integral scale of the horizontal and vertical components using wind velocity data from sonic anemometers at three different heights: 5.8 m, 13 m and 32 m. The numerical results show light significant differences on the diurnal and night cycle when the variation of the integral scale is analyzed versus the fractal dimension. Atmospheric stratification is analyzed for the three heights through the Bulk Richardson number, finding the three classical types of stratification along the diurnal cycle. It would be interesting for future works to study the growth of the integral scale in stratified flows and if it could be due to the inverse cascade on both diurnal and nighttime cycles. The main conclusions of this study are as follows.

14 Units are missed in some plots, for instance in figure 2 " τ " is indicated but it is not clear whether the lag is given in seconds, minutes or hours. Please double-check all the figures and add the units (on the axes/legend) when it feels needed.

Thank you for reminding the units in some plots. We have solved these missed units for the different figures.

Response to the Referee #2

We thank the reviewer 2 for the comments and the possible suggestions that can help to improve the paper. Although some of them we do not agree, below we respond point-by-point to the reviewers' comments (colored in blue)

Let me first recall that over the last 10 years, the statistical characterization of velocity turbulent fluctuations in the atmosphere is a widely studied topic. Numerous results from experiments and from numerical simulations are published in the literature.

It is true that there are numerous papers that have been published in the last 10 years over statistical characterization of velocity turbulent fluctuations in the atmosphere but not many of them establish the relation between the fractal dimension and the integral scale at changing the atmospheric stratification, which is the objective of the present paper.

The fractal dimension, or box-counting dimension D_0 , is a statistical property of turbulent signals, providing a measure of the degree of roughness, and it is thought to be a 'universal' property of it (see also many text books on the topic of the relation among the generalized fractal dimensions and the singularity spectrum of a turbulent signal). This means, in particular, that it should not depend on the value of the integral scale of the flow, something that is clearly not universal but depends on the geometry of the system, and on its stability. If the authors differently think the such a relation exist, they should give a solid basis to their working hypothesis. A linear regression can be applied to any pair of observables.

We do not consider as a universal property the fractal dimension of u' and w' components of wind velocity, in fact we find that this magnitude depend on integral scale and stratification. This is deduced from the experimental data. During diurnal cycle the average values of the integral scale increase versus the fractal dimension for the u' horizontal component due to the instability of stratification into the ABL, and these values can be adjusted to the straight regression line with a R^2 acceptable. At nighttime hours due to the atmosphere stability the average values of the integral scale decrease with the increase in the fractal dimension. These values also fit a straight line as it is indicated in Fig 5 and Fig 6. For the w' vertical component have fitted a quadratic function as it is indicated in Fig 7. In all these cases the R^2 values and confidence levels are high. Find a theoretical background of why this happens at the three heights is not easy, but the fact to show it can be interesting for the NPG readers.

We also think that if the fractal dimension is a statistical property of turbulent signals providing a measure of the degree of roughness, the atmospheric stability somehow softens the irregular flow and instability increases such irregularity as is indicated in Fig 3. At night hours the fractal dimension decreases while it increase in noon as is observed in Fig 3.

The computation of the fractal dimension is in principle an easy task, provided there exists an extended range of scales where there is scaling. The authors extract the fractal dimensions at fixed heights in the ABL with no discussion about the evolution of the mixed layer, and kind of flow they expect to measure at different heights depending on the stratification. For example: similar to homogeneous and isotropic 3D turbulence during convection, similar to homogeneous stratified flow at night, etc.. Even if the diurnal cycle is never really stationary, there are stages of quasi-stationarity when statistical properties can be assessed.

It is true that we do not discuss about the evolution of the mixed layer but this is out of the scope of our work. With regards to the kind of flow we expect to measure at different heights, we have to say that during diurnal hours it is expected to have a flow of similar characteristics, while at night, especially for strong stable stratification, it can exist different degrees of stability at these 3 heights affecting then to the flow characteristics. However these topics are also out of the scope of the present work, where we want to underline the relationships between integral scales and fractal dimension and also between integral scales and the degree of stability, evaluated from the bulk Richardson number.

Finally, I find that the manuscript contains limited novelty with respect to Ref. "Tijera M., Maqueda G., Yaque C., and Cano J.: Analysis of fractal dimension of the wind speed and its relations with turbulent and stability parameters, Intech, Fractal Analysis 6 and Chaos in Geosciences, 29-46, 2012"

Although this work deals with the fractal dimension of the velocity components, there are significant differences with the cited paper because, both papers study different parameters of the turbulent flows. In the paper that you just quoted, the fractal dimension is analyzed versus the potential temperature differences, the turbulent kinetic energy, the friction velocity and Bulk Richardson number; but do not study the behavior of the wind fluctuations integral scale versus the fractal dimension at changing atmospheric stratification neither separates its diurnal cycle (including the nocturnal behaviour) to better analyze and understand the influence of diurnal and nocturnal hours.

1 Influence of Atmospheric Stratification on the Integral 2 Scale and Fractal Dimension of Turbulent Flows.

3
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12 13 **Abstract**

14 In this work the relation between integral scale and fractal dimension and the type of
15 stratification in fully developed turbulence is analyzed. Integral scale corresponds to
16 that in which energy from larger scales is incoming into turbulent regime. One of the
17 aims of this study is the understanding of the relation between the integral scale and the
18 Bulk Richardson number, which is one the most widely used indicators of stability close
19 to the ground in atmospheric studies. This parameter will allow us to verify the
20 influence of the degree of stratification over the integral scale of the turbulent flows in
21 the Atmospheric Boundary Layer (ABL).The influence of the diurnal and night cycle in
22 the relationship between the fractal dimension and integral scale is also analyzed.
23 Fractal dimension of wind components is a turbulent flow characteristic as it has been
24 shown in previous works, where its relation to stability was highlighted. Fractal
25 dimension and integral scale of the horizontal (u') and vertical (w') velocity fluctuations
26 have been calculated using the mean wind direction as framework. The scales are
27 obtained using sonic anemometer data from three elevations 5.8 m, 13 m and 32 m
28 above the ground measured during the SABLES- 98 field campaign. In order to
29 estimate the integral scales a method that combines the normalized autocorrelation
30 function and the best gaussian fit ($R^2 \geq 0.70$) has been developed. Finally, by comparing,

1 at the same height, the scales of u' and w' velocity components it is found that almost
2 always the turbulent flows are anisotropic.

3

4 **1 Introduction**

5 The aim of this paper is to investigate the possible correlations between the integral
6 scale of the turbulent stratified flows in the Atmospheric Boundary Layer and
7 parameters characterizing topological features of the wind velocity field, such as fractal
8 dimension and its stability properties, studied through the Bulk Richardson number. We
9 are aware that there is a lack of investigations between the integral scale and fractal
10 dimension. The size of the integral scale of the horizontal and vertical components and
11 fractal dimension of wind velocity near the earth's surface in boundary layer are
12 determined. Also, these magnitudes are compared between them and versus other
13 parameters such as the Bulk Richardson number. It is assumed that the turbulence is the
14 primary agent that causes changes in the boundary layer. In turbulent flows it is
15 observed that time series of meteorological variables as wind velocity, temperature,
16 pressure and other atmospheric mechanical magnitudes fluctuate in a disordered way
17 with peaks extremely sharp and irregular space and time variations. The complicated
18 nature of these series indicates that the motion of the air is turbulent. If we take a good
19 look at the variety of fluctuations of different periods and amplitudes observed in them
20 we could explain the complicated structure of turbulence. The irregularity of the time
21 series obey to the existence of different size and time scales and also to the nonlinear
22 transfer of energy that exists between them in the turbulent flows (Monin and Yaglom,
23 1971).

24 The irregular behavior of these flows is also due to waves and turbulence that are often
25 superimposed on a mean wind (Stull, 1998). If we filter the mean wind and waves in the
26 appropriate range we will only have turbulence. Some previous works present results
27 about this procedure (Tijera et al., 2008). In this work, the series of wind velocities in
28 the three directions x , y and z recorded by the anemometer are divided in series of non-
29 overlapping five minutes length. Each of these series applies the necessary rotations to
30 get the x – axis in the mean wind direction (mean v is zero) and zero mean vertical
31 velocity (w vertical component) (Kaimal and Finnigan, 1994). We filter horizontal and
32 vertical mean wind velocity obtaining the time series of fluctuations of the velocity in
33 both directions ($u' = u - \bar{u}$, $w' = w - \bar{w}$).

1 When we observe these time series such as wind velocity, they vary in an irregular
2 shape and in spite of their complexity presents a self-similarity structure (Frisch,1995) .
3 This is a common property of the fractals, so that wind velocity could be considered as a
4 fractal magnitude. The modern physical notion of fractals is largely known due to
5 Mandelbrot (1977, 1985), but the mathematical notion of curves lines or sets having
6 noninteger dimensions is much older (Hausdorff, 1919, Besicovitch, 1929). An analysis
7 that compare the Hausdorff dimension and Kolmogorov capacities of self-similar
8 structure with non integer fractal dimensions (Kolmogorov capacity or box counting
9 dimension) was presented by Vassilicos (Vassilicos and Hunt, 1991). The wind
10 velocity versus time are irregular curves of this type, with noninteger dimensions. These
11 values correspond to the fractal dimension. A way of measuring the complexity of these
12 series is by means of fractal dimension. The Fractal Dimension of wind components is a
13 characteristic of turbulent flow as it has been shown in previous works where its
14 relation to stability was highlighted (Tijera et al., 2012)

15 In this paper the integral scale of u' and w' component are compared. The scales are
16 calculated using sonic anemometer data from three elevations 5.8 (~ 6), 13 and 32 m
17 above the ground at the main tower site of the Sables 98 field campaign. [Turbulent
18 motion of the atmospheric flows occurs through a broad range of scales from the
19 smallest ones that are usually defined as the scales at which the motion dissipates into
20 heat due to the viscosity of the fluid until the larger scales corresponding to the integral
21 scale.](#) The integral scale can be defined in several ways: the larger scale of the flow, the
22 scale above which the Fourier transform has a slope inferior a $-5/3$ slope, as which the
23 turbulent kinetic energy (TKE) is maximum. Micrometeorological studies have found
24 integral scale varying in a huge range, from around a hundred to a thousand meters
25 (Teunissen, 1980, Kaimal and Finnigan, 1994).

26 We study the anisotropy of the turbulent atmospheric flows in these scales comparing
27 integral scale of fluctuations of the velocity component along of the mean wind
28 direction and the vertical component at three different levels above the ground (5.8 m,
29 13.5 m, 32 m).

30

31 **2 Theoretical background**

32 The irregular behavior of the atmospheric turbulent fluxes in the boundary layer at large
33 Reynolds number leads us to be interested in calculating their fractal dimension. Fractal

1 dimension could help us to classify the irregularity of these flows. The more irregular
2 the flow the greater its fractal dimension. Turbulent flows are characterized by the
3 formation of many eddies of different length scales. These irregularities are due to the
4 superimposition of eddies of different sizes and it is related with a broad range of scales
5 which exist in turbulence. These scales vary from the smallest scales as dissipative scale
6 to larger scale as integral scale. This paper is concerned with the analysis of the
7 relationship between the integral scale and fractal dimension. As well as the relationship
8 between the integral scale with the Bulk Richardson number, which provides a measure
9 of the degree of stability in the flow, and how this turbulent flow is prone to develop
10 instabilities. It is also used as a criterion for the existence or non-existence of turbulence
11 in a stably stratified environment (a large positive value over a critical threshold, is
12 indicative of a decaying turbulence or a completely non-turbulent) (Arya, 2001).

13 In this section we describe the methodology applied to calculate the fractal dimension
14 and the integral scale. The estimation of the fractal dimension of time series has been
15 the most commonly used criteria to measure their chaotic structure, there exist different
16 works in that direction (Grassberger and Procaccia, 1982, Shirer et al, 1997). One of the
17 methods most commonly used to estimate to fractal dimension of atmospheric flows has
18 been the mean slope method through box-counting dimension using mean slopes of the
19 graph of $\ln N(L)$ versus $\ln(L)$ for small ranges of L , where $N(L)$ is the number of the
20 boxes of side L necessary to cover the different points that have been registered in the
21 physical space (velocity-time) (Falconer, 2000, Peitgen et al., 2004). As $L \rightarrow 0$ then
22 $N(L)$ increases, N meets the following relation:

$$23 \quad N(L) \cong kL^{-d} \quad (1)$$

24 The value d is the box-counting dimension that is an approximation of the Hausdorff
25 dimension and is calculated approximately by means of least-square-fitting of the
26 representation of $\log N(L)$ versus $\log L$ obtaining the straight line regression given by
27 the following equation:

$$28 \quad \log N(L) = \log k - d \log L \quad (2)$$

29 The fractal dimension d will be given by the slope of this equation as is shown in the
30 Fig.1.

31 In this paper we focus on calculating the integral scales for horizontal and vertical
32 component fluctuations u' and w' , and we studied their variations with respect to the

1 fractal dimension and with the Bulk Richardson number, a turbulent parameter of
2 stability.

3 These integral scales have been estimated using the normalized autocorrelation function
4 and a Gaussian fit. The velocity autocorrelation function as a function of τ (lags
5 number) for u' component is:

$$6 \quad R(\tau) = \frac{\overline{u'(t)u'(t+\tau)}}{\overline{u'^2(t)}} \quad (3)$$

7 Integral time scale is:

$$8 \quad T_L = \int_0^{\infty} R(\tau) d\tau \approx \int_0^{\tau'} R(\tau) d\tau \quad (4)$$

9 The integral time scale provides a measure of the scales of eddies in the x direction of
10 a flow field. In the Eq. (2) we observed that τ' denotes the last lag in the data series. In
11 boundary layer observations this time scale can be related to a length by multiplying the
12 mean wind velocity by time scale. This requires the assumption of frozen turbulence
13 known as Taylor's hypothesis (Panofsky and Dutton, 1984). The integral length scale
14 can be defined as:

$$15 \quad \lambda = \bar{v} T_L \quad (5)$$

16 The used method is based on Gaussian fit of the normalized autocorrelation function
17 $R(\tau)$ and we calculated the value of τ that verifies the following equation:

$$18 \quad \tau - \int_0^{\tau} R(\tau) d\tau = \int_{\tau}^{\tau'} R(\tau) d\tau \quad (6)$$

19 The Fig. 2 shows the Gaussian fit for an example of a data series of wind velocities with
20 τ that verifies Eq. 6. This value allows us to calculate the integral time scale
21 multiplying it by the time interval between each lag.

22

23 **3 Description of Data**

24 The data set was recorded in the Research Centre for the lower Atmosphere (CIBA in
25 the Spanish acronymus), located in Valladolid province (Spain) and were measured in
26 the experimental campaign Sables-98. This research centre was set up primarily to study
27 the atmospheric boundary layer. The campaign took place from 10th to 27th September
28 1998 (Cuxart et al., 2000). This experimental site is a quite flat and homogeneous

1 which forms a high plain of nearly 200 Km², surrounded by crop fields and some small
2 bushes strewn over ground. Duero river flows along the SE border of the high plain.
3 The synoptic conditions during the period of study of eight consecutive days (from 14
4 to 21 September) were controlled by a high pressure terrain system which produces
5 thermal convection during the diurnal hours and from moderate to strong stable
6 stratification during the nights.

7 In this work, data from sonic anemometers measured at a sampling rate of 20 Hz
8 installed at 5.8 (~ 6), 13 and 32 m are analyzed. 5 - minute non - overlapping series are
9 used to evaluate the different parameters. At a rate of 20 data points per second, sonic
10 anemometers can resolve integral scales between about 10 m to 2000 m of u' horizontal
11 component and 1 m to 1000 m of the w' vertical component, depending on the height in
12 which the sonic anemometer is positioned and at the wind speeds typically measured in
13 the Sables-98 experiment. We detect vertical scales over a broad range of scales from 1
14 m to 1000 m. The integral scales here are calculated based on the autocorrelation
15 function, the mean wind velocity and integral time scale, and each of them can be
16 expected to vary significantly. As the integral scale are the larger scales of turbulent
17 flows it is possible to detect vertical scales larger than heights at which the sonic
18 anemometer are located.

19

20 **4 Results**

21

22 **4.1 Fractal Dimension, integral scale and stability of stratification.**

23 In this paper we analyze the influence of stability of stratification on fractal dimension
24 and integral scale. Different atmospheric surface-layers data are separated into thermal
25 and dynamics stability classes based on a dimensionless parameter such as the Bulk
26 Richardson number Ri_B . This parameter represents the ratio of the production or
27 destruction of turbulence by buoyancy and by wind shear strain that is caused by
28 mechanical forces in the atmosphere:

$$29 \quad Ri_B = \frac{g}{\theta} \frac{\Delta\bar{\theta}\Delta z}{(\Delta u)^2} \quad (7)$$

1 where g is the gravity acceleration and $\bar{\theta}$ the average potential temperature at the
2 reference level, the term $\frac{g}{\bar{\theta}}$ is known as the buoyancy parameter. Ri_B is positive for
3 stable stratification, negative for unstable stratification and approximate zero for neutral
4 stratification (Arya, 2001). The way to calculate this number is described next:

5 1. Calculation of the mean potential temperatures at height $z = 32$ m, and close to the
6 surface $z = 5.8$ m, namely $\bar{\theta}_{32}$ and $\bar{\theta}_{5.8}$ respectively. Being $\Delta\bar{\theta} = \bar{\theta}_{32} - \bar{\theta}_{5.8}$. The
7 potential temperature has been estimated as relative to ground level by using the
8 following formula: $\Delta\theta = \Delta T + \Gamma\Delta z$, $\Gamma = 0.0098 \text{ K m}^{-1}$ (Arya 2001)

9 2. Obtaining of \bar{u}_z the mean wind velocity module at the height $z = 32$ m and $z = 5.8$
10 m, denoted by \bar{u}_{32} and $\bar{u}_{5.8}$ respectively, where $\Delta\bar{u} = \bar{u}_{32} - \bar{u}_{5.8}$.

11 Once the values of $\Delta\bar{\theta}$, $\Delta\bar{u}$ and Δz have been obtained by means of Eq. (7) we
12 calculate the Bulk Richardson number in the layer between 32m and 5.8m.

13 In Fig. 3 we present the variation of the fractal dimension of the u' horizontal
14 component of the velocity fluctuations along time at the three considered heights: 5.8 m,
15 13 m and 32 m. The behaviour of these variations is similar at the three heights. The w'
16 component fluctuation presents an analogous behaviour. The fractal dimension values
17 are in a range between 1.30 to nearly 1.00. We have found that during the diurnal hours
18 the fractal dimension is bigger than at night (Tijera, 2012). We have no theoretical
19 reason to explain this result, but a possible explanation of why this happens could be
20 that fractal dimension is related with atmospheric stability and with the intensity of
21 turbulence. It is well known that the intensity of turbulence grows up as solar radiation
22 increases, producing instability close to the ground, mainly in the hours of noon.
23 Therefore, one of the possible reasons of the increase of FD is the instability of the
24 turbulent flow. In the other hand, during the nights a strong atmospheric stability
25 usually exists, so the fractal dimension is usually smaller than during the diurnal hours.

26 In Fig 4 it is observed how the integral scale varies versus time at the three heights.
27 There are some questions that have not been clarified yet in the literature. For example:
28 How does the diurnal and night cycle influence on the integral scale? Which is the
29 mechanism responsible for the growth of this integral scale? It has been observed in
30 previous works that under certain conditions the turbulent flows self – organize and

1 develop large-scale structures that take place through an inverse cascade that occurs in
2 stably stratified anisotropic flows (with or without rotation) (Smith and Waleffe, 2002,
3 Marino et al., 2014). The inverse cascade mechanism might also be responsible for the
4 growth of the integral scale in the stratified atmosphere. It is a fundamental issue that
5 should be clarified in a future research. As it is indicated in Fig 4 the integral scale for
6 u' component varies between around 100 m on their smaller scales, until above 1500 m
7 for its larger scales. The integral scales for w' component are slightly lower than for u'
8 component. It is shown that these vertical scales can reach sizes between a few tens of
9 meters until 1000 m in some occasions. It is observed, for each of them, that the greater
10 is the height at which is located the sonic, the greater is the integral scale in the
11 turbulent flow. Usually, at 32 m these scales are, on average, greater than those of the
12 13 m and the latter higher than at 5.8 m height.

13 Although the conceptual model of turbulence as eddies of various sizes is useful, it is
14 difficult to obtain a correlation between the integral scale and fractal dimension in the
15 atmosphere if we consider values throughout the whole day. However, it is much easier
16 to find a relationship between the integral scale and fractal dimension of horizontal and
17 vertical components of the wind velocity if we separate the hours of the day and night,
18 and hence analyze the influence of diurnal and night cycle over these parameters.
19 Daylight hours are from 6-18 UTC and the night from 18-6 UTC. These data set are
20 analyzed in the three studied heights. Fig. 5 shows the variations of the integral scale
21 versus fractal dimension at the level of 5.8 m for horizontal component. As it can be
22 appreciated in Fig. 5 in the diurnal hours the average values of the integral scale versus
23 the fractal dimension can be adjusted to the straight regression line given by the linear
24 equation that appears on the top left of the graph. During those hours these values of the
25 integral scale increase from a few tens of meters until 400 m with increasing values of
26 the fractal dimension until 1.25. During the nights the average values of the integral
27 scale decreases with the increase in the fractal dimension. These values also fit a
28 straight regression line as it is indicated in Fig. 5. One of the possible explanations for
29 this behaviour is that during the diurnal hours the average values of the integral scale
30 increase due to the unstable stratification. During the nights, the existence of the stable
31 stratification decreases the integral scale with an increase in fractal dimension until the
32 approximate value of 1.2. This tendency appears also in the other two heights, at 13 m
33 and at 32 m as it is shown in Fig. 6. Although during the diurnal cycle at 32 m the linear

1 fit it is not so evident, the maximum scales are in the 1.15-1.20 range of the fractal
2 dimension as it is illustrated in Fig. 6 (c).

3 For the vertical component in the three studied levels, the behaviour is slightly different.
4 The averages values have fitted a quadratic function as it is indicated in Fig. 7. During
5 the diurnal hours the averages values of integral scale reach maximum scales around
6 the value of the fractal dimension of 1.15 at the three heights. From this value the
7 integral scale decreases when fractal dimension increases. These maximum integral
8 scales depend on the height. At the level of 5.8 m their sizes reach 50 m in average and
9 the scattering of the values shows higher values that could reach 100 m. At the level of
10 13 m the values are about 100 m, the dispersion of these scales can reach sizes of 200 m
11 and at the height of 32 m their larger average scales are approximately around 200 m
12 and due to the variances of the data set could reach sizes of 400 m. From the value of
13 the fractal dimension value of 1.15, the scales decrease until a few meters.

14 Throughout the night the average values of the integral scales decrease with the increase
15 of the fractal dimension in a parabolic way as it is indicated in Fig. 7. This happens due
16 to the stable stratification that occurs at nights. This behavior during the diurnal and
17 night hours for w' component of the integral scale is similar to the results obtained for
18 u' component, although the fits of averages values are parabolic and not linear. In all
19 these cases our R^2 values and confidence levels are high as it is indicated in Fig. 7.

20

21 **4.2 Relationship between integral scale and Bulk Richardson number**

22 Among the numerous parameters existing to characterize the degree of stratification in
23 the atmosphere we will use the Bulk Richardson number. The interpretation of this
24 number has already been mentioned in the previous section. Here, we analyze how the
25 integral scale of each one of the u' horizontal and w' vertical components varies with
26 the Bulk Richardson number in diurnal and night cycle in the studied period. These
27 results are shown in Fig. 8 for horizontal component and in Fig. 9 for vertical
28 component. During the daylight hours appear the three kinds of stratification: unstable,
29 neutral and stable as it is shown in the three graphs on the left side of Fig. 8 and the Fig.
30 9, each one corresponds to the different heights. In the stratification unstable and neutral
31 the integral scales are higher than the integral scales under the influence of the stable
32 stratification. At 5.8 m for the horizontal component these scales vary between 200 m

1 and values slightly higher than 400 m and in the case of neutral stratification could
2 increase until 600 m. This same behaviour occurs in the other two studied heights 13 m
3 and 32 m although their scales are slightly higher as it is illustrated in Fig. 8. During the
4 nights it is observed the biggest stability due to positive values of the Bulk Richardson
5 number.

6 The same results are obtained for the integral scales of vertical component, although
7 their sizes are smaller. At 5.8 m during the diurnal hours the average values reach about
8 50 m and during the night hours their values are below 50 m. At 13 m and 32 m in the
9 diurnal hours the average values could reach about 150 m and 200 m and at the night
10 hours are below 100 m and 200 m respectively.

11

12 **4.3 Analysis of the anisotropy with the integral scale**

13 In the last section we study the relationship between the integral scales of the horizontal
14 and vertical components at different heights: 5.8 m, 13 m and 32 m. In Fig. 10 we
15 represent the integral scale of u' component versus the integral scale of w' component at
16 three studied heights and we find linear relations with the averages values of these
17 scales. All integral scales measured during the period of study from 14 to 21 of
18 September appear in this figure. The linear fits obtained are acceptable, with high R^2
19 values at 13 m and 32 m, as it is indicated in Fig. 10. The linear regression appears on
20 the top left of each graph: at 5.8 m $L_{intu}(5.8\text{ m}) = 1.46 L_{intw}(5.8\text{ m}) + 178$, at 13 m
21 $L_{intu}(13\text{ m}) = 0.957 L_{intw}(13\text{ m}) + 275.6$ and at 32 m $L_{intu}(32\text{ m}) = 0.646 L_{intw}(32\text{ m}) + 370$,
22 being L_{intu} and L_{intw} the average values of the integral scale for horizontal and vertical
23 component respectively.

24 The data in Fig. 10 appear quite scattered and the average values could be representative
25 to find relationships between these scales. This scatter is due to the large number of
26 uncontrolled variables, nonlocal disturbance, the presence of waves, horizontal
27 inhomogeneity, low frequency disturbances, etc. These graphs are showing that the
28 scale measured at 32 m is nearly always larger than the integral scale measured at 5.8 m.
29 On the basis of the results obtained, we find slight differences between these
30 components, thus there is anisotropy in atmospheric turbulent flows. In isotropic
31 turbulence the integral scales of both components should be the same at the same

1 height. Only under certain conditions and over limited scales is isotropy a property of
2 turbulence in the stratified atmosphere (Thorpe, 2005).

3 4 **5 Conclusions**

5 We have calculated the fractal dimension and the integral scale of the horizontal and
6 vertical components using wind velocity data from sonic anemometers at three different
7 heights: 5.8 m, 13 m and 32 m. The numerical results show light significant differences
8 on the diurnal and night cycle when the variation of the integral scale is analyzed versus
9 the fractal dimension. Atmospheric stratification is analyzed for the three heights
10 through the Bulk Richardson number, finding the three classical types of stratification
11 along the diurnal cycle. It would be interesting for future works to study the growth of
12 the integral scale in stratified flows and if it could be due to the inverse cascade on both
13 diurnal and nighttime cycles. The main conclusions of this study are as follows.

14 Although all data appear quite scattered in this work, the averages values of these
15 magnitudes show interesting results. During the diurnal hours the averages values of the
16 integral scale of the horizontal component increases with the increase in fractal
17 dimension until around 1.25 at 5.8 m and 13 m height. At these heights we have found
18 linear fits between these magnitudes with high coefficients of correlation. While at 32 m
19 the linear fit is not so evident, the maximum scales are in the 1.15-1.20 range of the
20 fractal dimension. One of the possible explanations for this behaviour is that during the
21 diurnal hours the average values of the integral scale increase due to the unstable
22 stratification. During the night hours the average values of the integral scale decreases
23 with the increase in the fractal dimension. These values also fit a straight regression
24 line at the three analysed heights. During the nights the existence of the stable
25 stratification decreases the integral scale with an increase in fractal dimension until the
26 approximate value of 1.2.

27 For the vertical component of the integral scale the results are similar, even though with
28 slight differences. The averages values have fitted a quadratic function. During the
29 diurnal hours the averages values of integral scale reach maximum around the value of
30 the fractal dimension of 1.15 in the three heights. From this value the integral scale
31 decrease with the increase of the fractal dimension until a few meters. The different
32 degree of stratification along diurnal hours will be reflected in that different behaviour

1 from the value of 1.15. At nights when stability is normally major the integral scale
2 decrease with increasing the fractal dimension of a parabolic way.

3 In the unstable and neutral stratification the integral scales are higher than the integral
4 scales under the influence of the stable stratification.

5 To characterize the anisotropy of turbulent flows we have used the comparison of
6 integral scales of horizontal and vertical component showing that the scale of u'
7 component is almost always larger than the scale of the w' component at the same
8 height.

9

10 **Acknowledgements**

11 This research has been funded by the Spanish Ministry of Science and Innovation
12 (projects CGL2009-12797-C03-03). The GR35/10 program (supported by Banco
13 Santander and UCM) has also partially financed this work through the Research Group
14 “Micrometeorology and Climate Variability” (No 910437). Thank to participant teams
15 in SABLES-98 for the facilities with the data.

16

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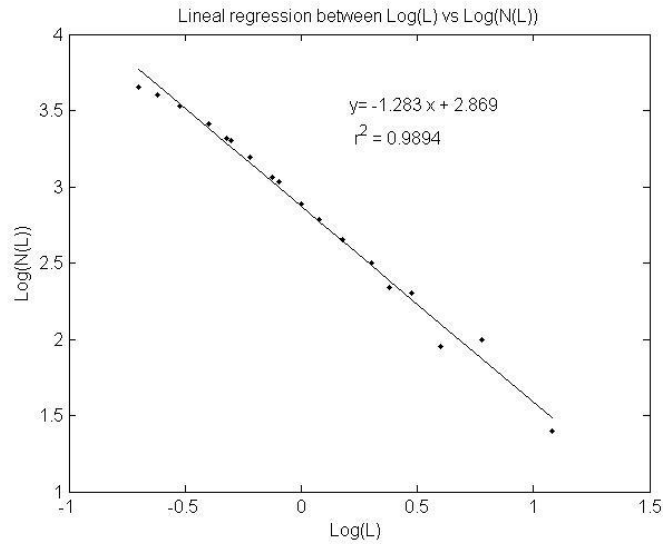
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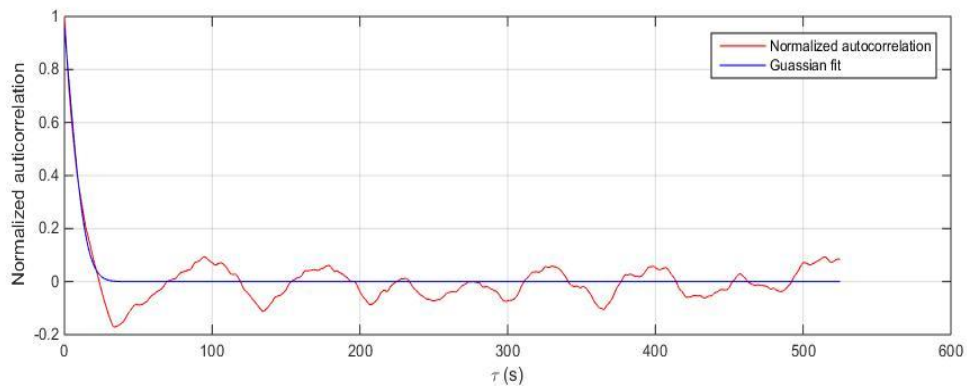
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2 **Figure 1.** Example of linear regression between number of not empty boxes and length
 3 side of the box. The slope (d) is the fractal dimension of the w' component, $d = 1.28 \pm$
 4 0.03 for a example of the w' component of the wind velocity.

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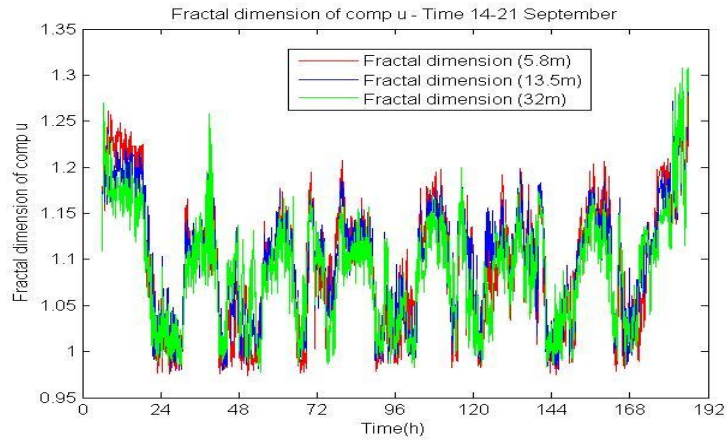


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7 **Figure 2.** Gaussian fit for a data series of wind velocities u' component that allows us
 8 to calculate the integral scale.

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2 **Figure 3.** Variation of the fractal dimension versus time for the u' component
 3 fluctuation at the three heights, showing the influence of the diurnal cycle.

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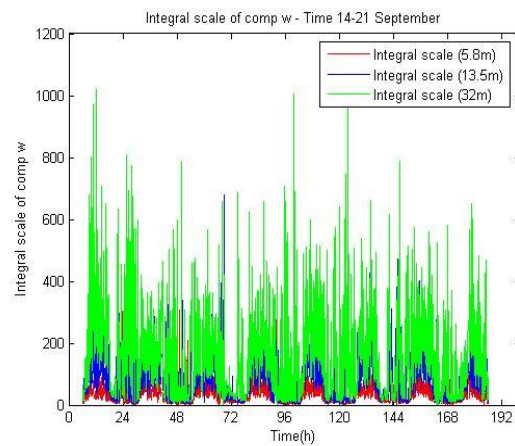
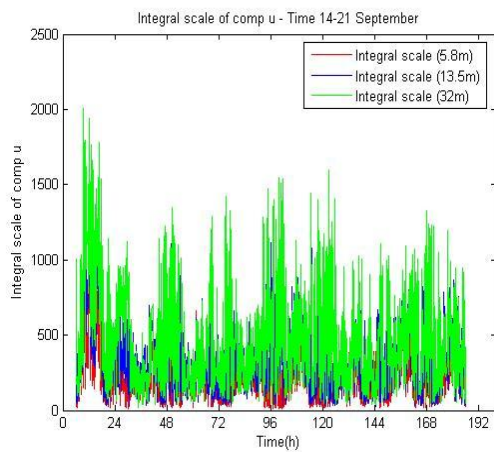
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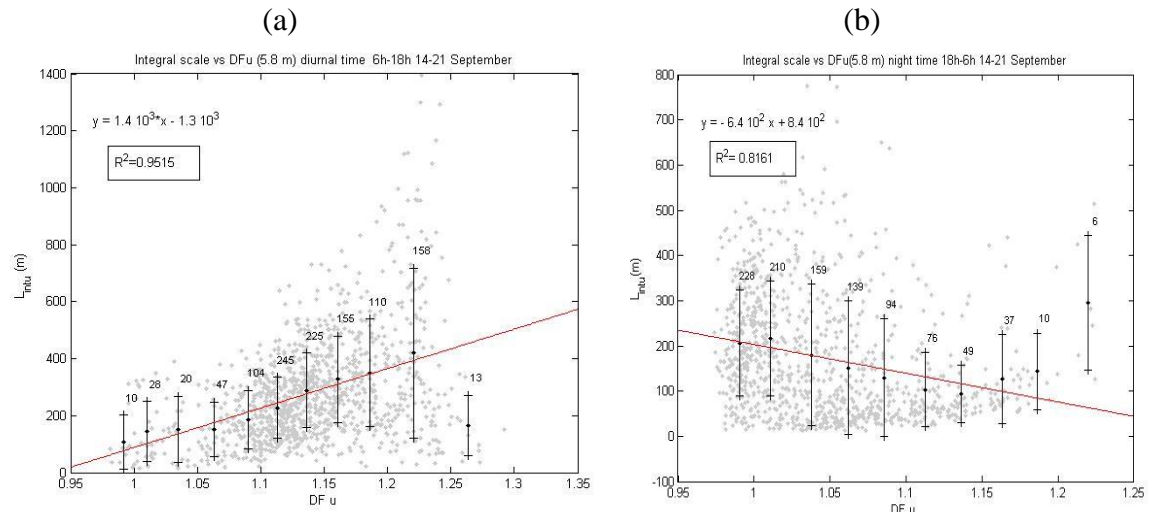
10 **Figure 4.** Variation of the integral length scale of horizontal and vertical components
 11 versus time at the three heights.

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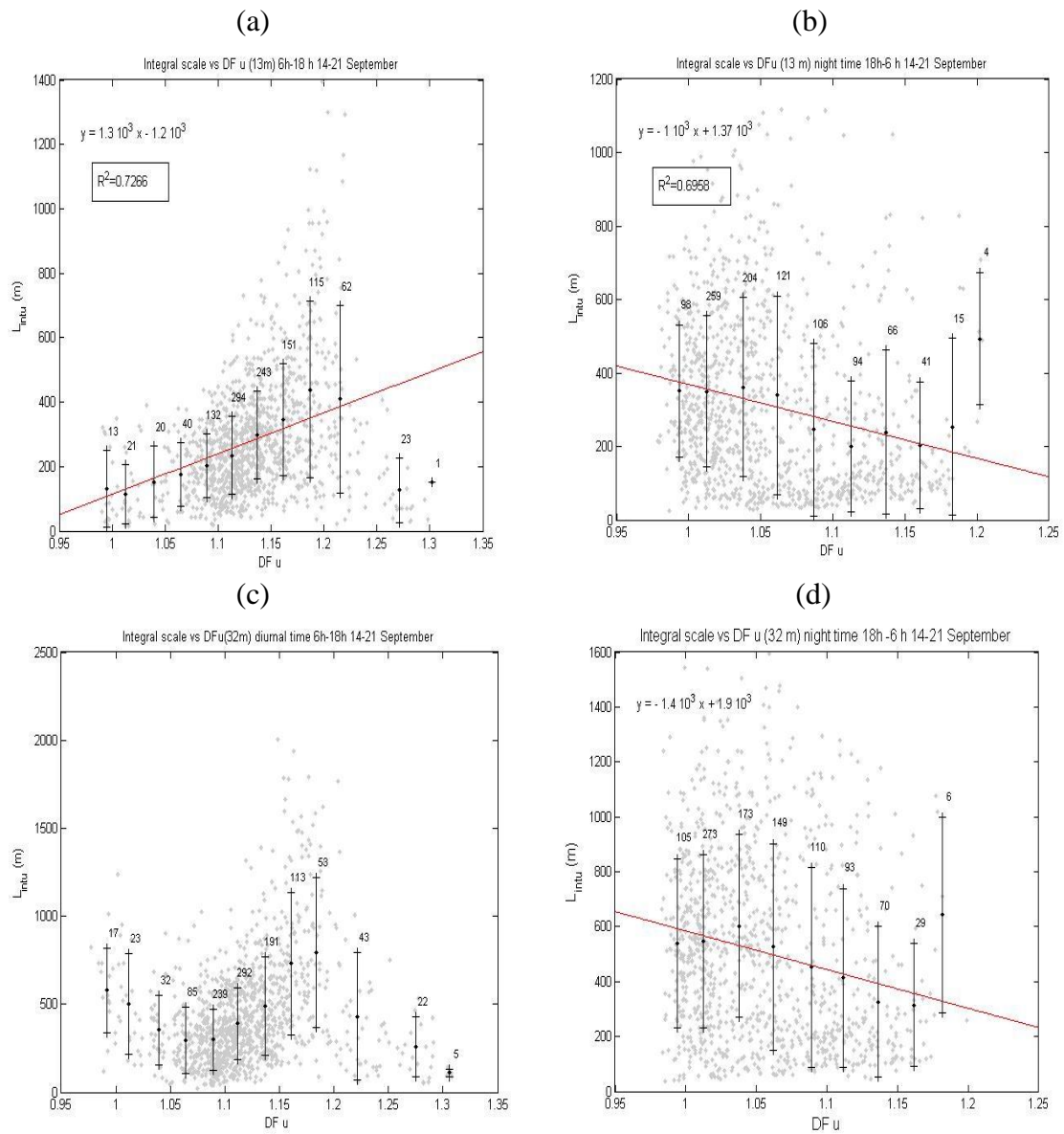
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1 **Figure 5.** Variations of the integral scale versus the fractal dimension of u component
 2 of the wind velocity at 5.8 m. (a) diurnal hours 6h-18 h, (b) night hours 18 h - 6h. On
 3 the top left of the each graph it is indicated the linear regression of the averages values
 4 (a) $L_{intu}(5.8\text{ m}) = 1.4 \cdot 10^3 DF_u(5.8\text{ m}) - 1.3 \cdot 10^3$ (b) $L_{intu}(5.8\text{ m}) = -6.4 \cdot 10^2 DF_u(5.8\text{ m}) +$
 5 $8.4 \cdot 10^2$, being L_{intu} and DF_u the integral scale and fractal dimension for u component
 6 respectively.

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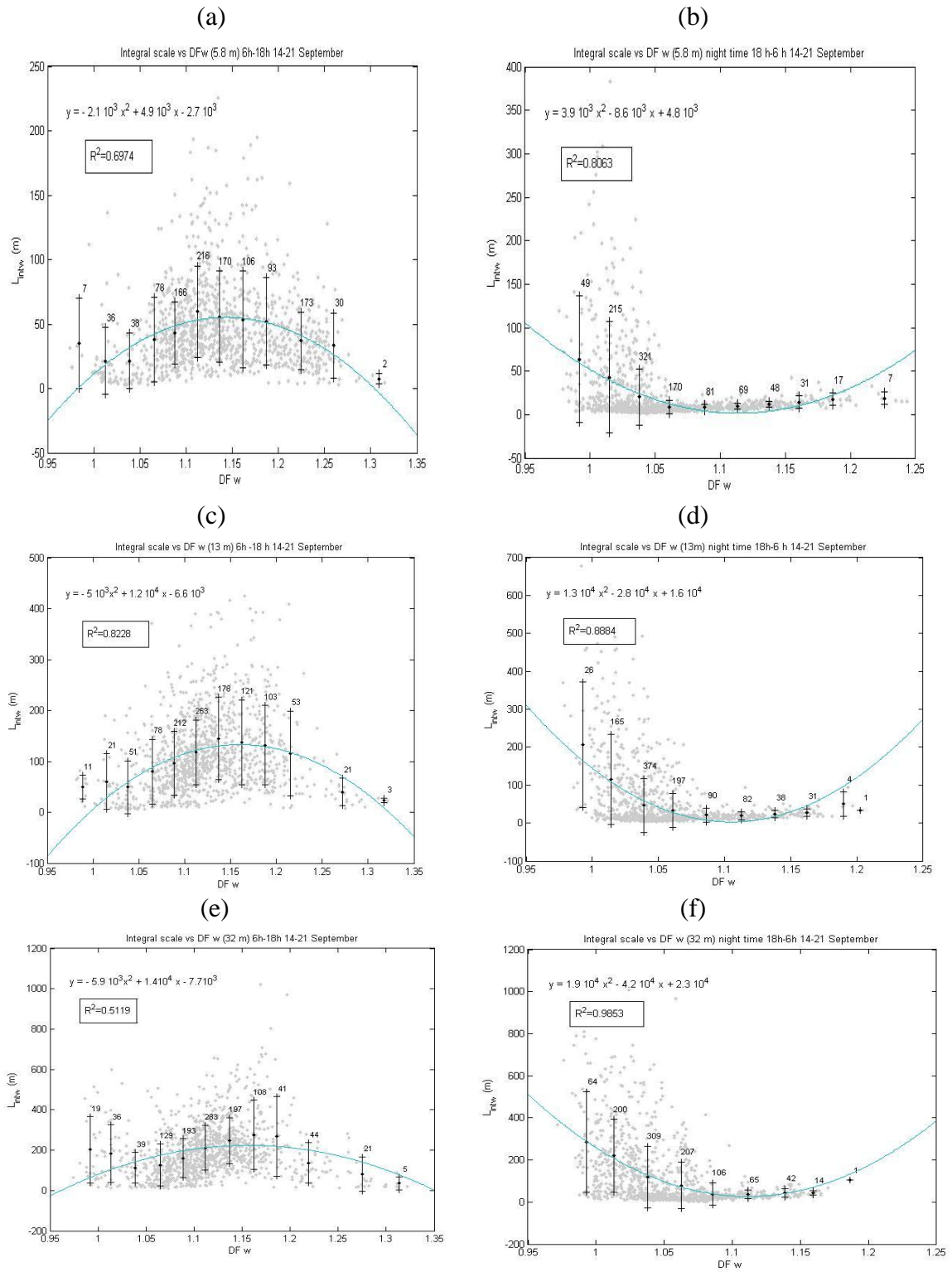
2 **Figure 6.** Variations of the integral scale versus the fractal dimension of the u'
 3 component at 13 m and 32 m. (a) and (c) diurnal hours, (b) and (d) night hours. In the
 4 same manner that in the figure 5 the linear fits are: (a) $L_{intu}(13\text{ m}) = 1.3 \cdot 10^3 DF_u(13\text{ m}) -$
 5 $1.2 \cdot 10^3$, (b) $L_{intu}(13\text{ m}) = -1 \cdot 10^3 DF_u(13\text{ m}) + 1.37 \cdot 10^3$, (d) $L_{intu}(32\text{ m}) = -1.4 \cdot 10^3 DF_u(32$
 6 $\text{m}) + 1.9 \cdot 10^3$

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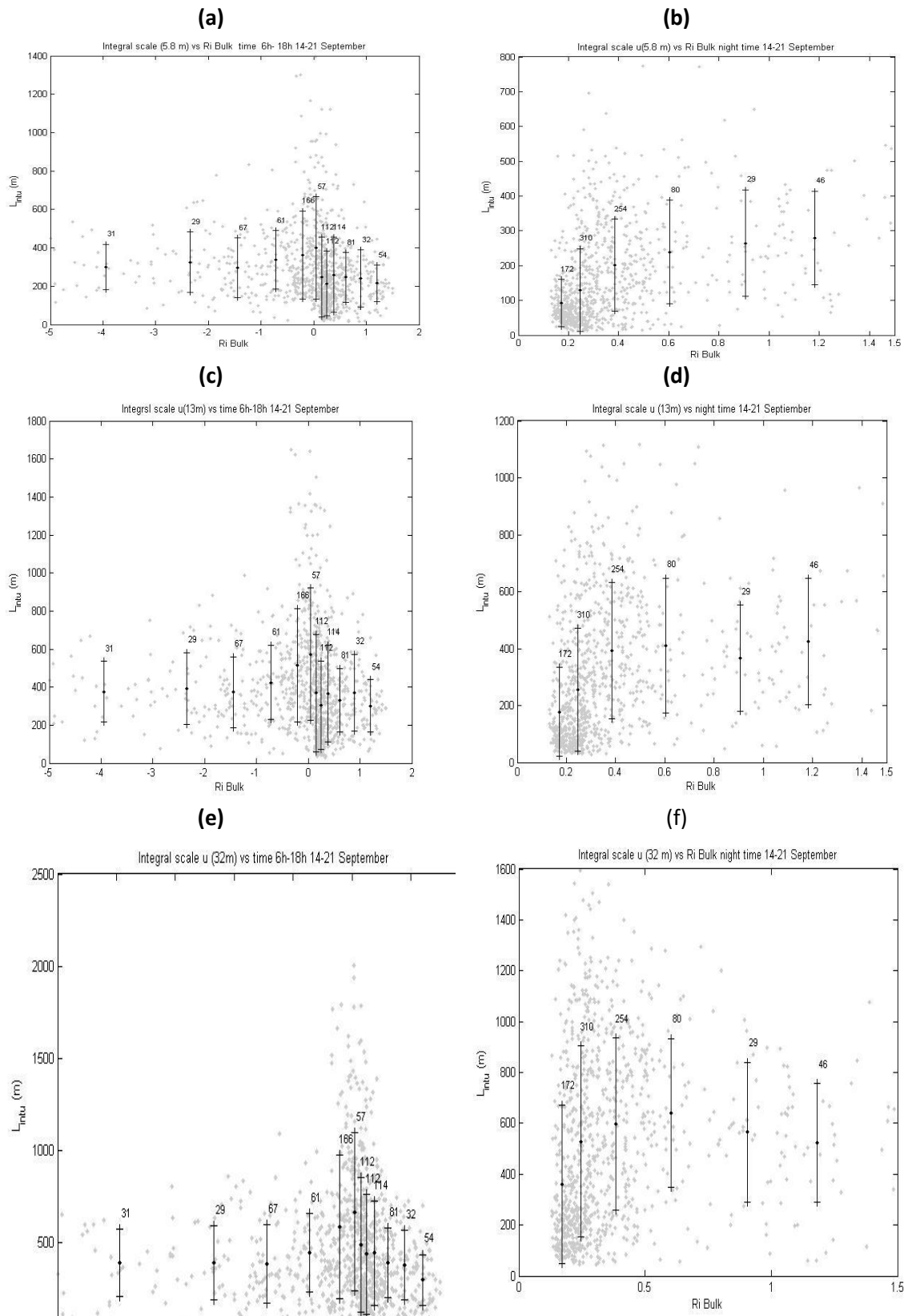
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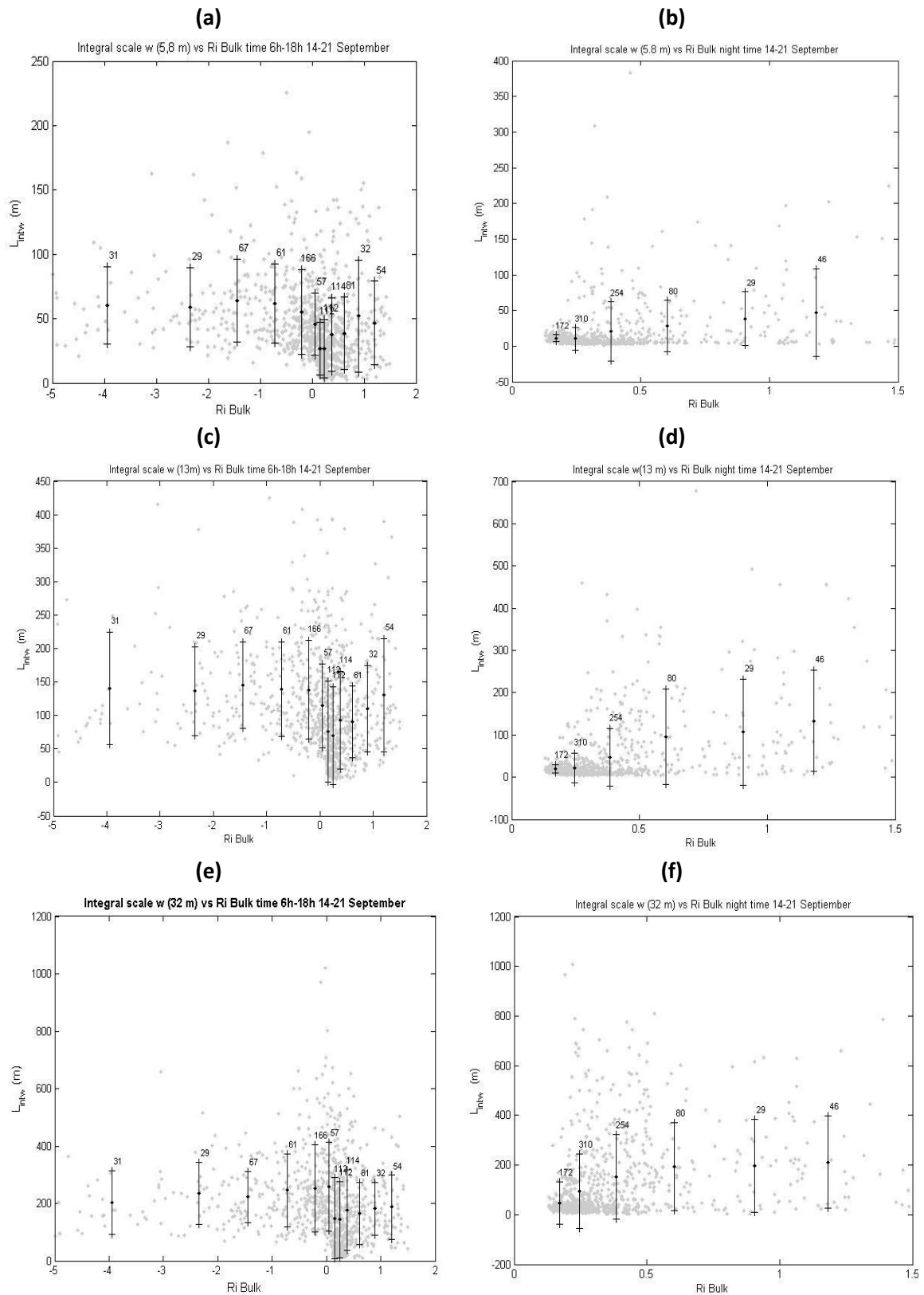
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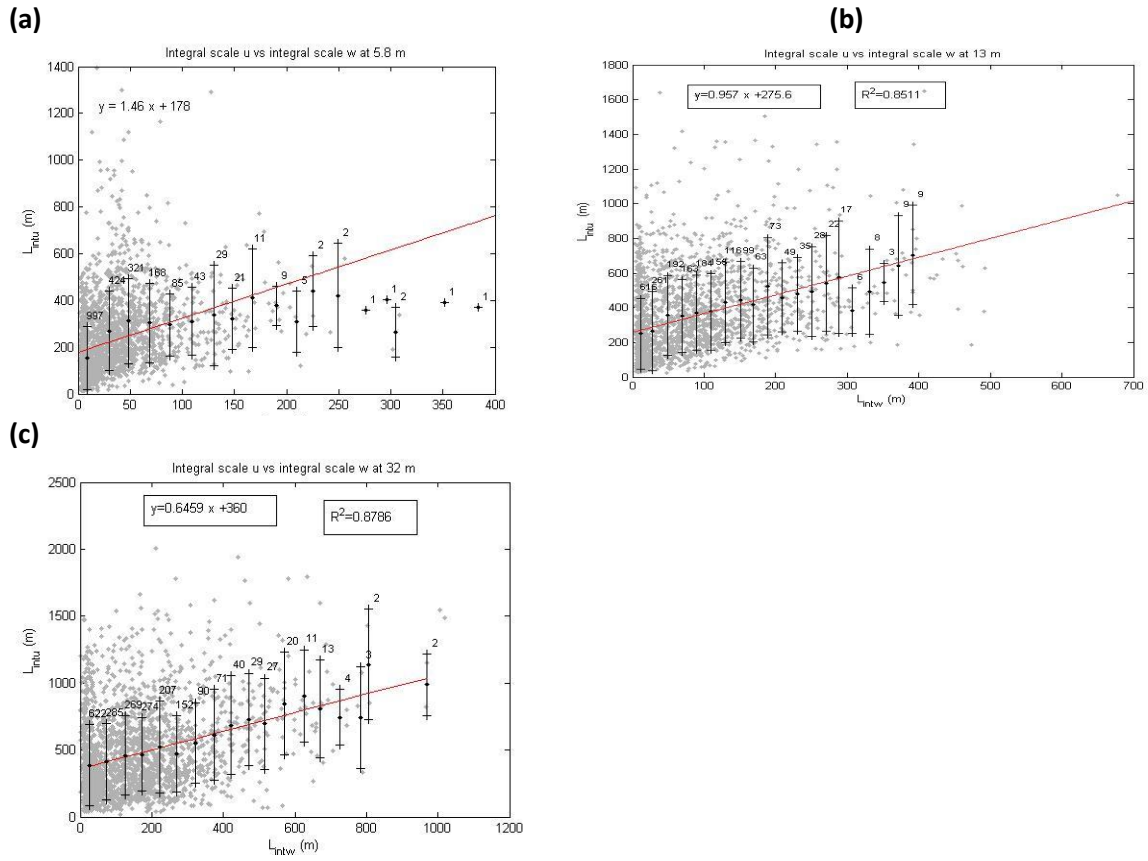
1 **Figure 7.** Variations of the integral scale versus the fractal dimension of the w`
 2 component at 5.8 m, 13 m and 32 m. (a), (c) y (e) diurnal hours, (b), (d) y (f) night.
 3 hours. The fits to a quadratic function of the averages values appear on the top left of
 4 the each graph, being y variable L_{intw} and x variable DF_w .



1 **Figure 8.** Integral length scales of u' component plotted against the Bulk Richardson
 2 number at 5.8 m, 13 m and 32 m. (a), (c) y (e) diurnal hours, (b), (d) y (f) night hours.



1 **Figure 9.** Integral length scales of w' component plotted against the Bulk Richardson
 2 number at 5.8 m, 13 m and 32 m. (a), (c) y (e) diurnal hours, (b), (d) y (f) night hours



1 **Figure 10.** Comparison of integral scales of u component and w component (a) at 5.8
 2 m, (c) at 13 m (c) at 32 m, showing that the averages values of these scales fit to the
 3 linear regression indicated on the top left each graph. Data set appear as a cluster around
 4 the straight line.

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