4 Concluding remarks

4.1 Summary

Text left as in original version of the paper, but order of Sections 4.1 and 4.2 interchanged.

4.2 Discussion of the results

Text left as in original version of the paper, but order of Sections 4.1 and 4.2 interchanged.

4.3 Broader context

The results of this paper have to be viewed in the broader perspective of the hierarchy of climate models already mentioned repeatedly in its preceding sections (Schneider and Dickinson, 1974; Ghil, 1994, 2001, 2015; Dijkstra and Ghil, 2005; Ghil and Robertson, 2000). Recall that this hierarchy ranges from simple, conceptual ODE models (e.g., Stommel, 1961; Lorenz, 1963; Källèn et al., 1979), like the one formulated and analyzed herein—through intermediate models of varying complexity (e.g., Claussen et al., 2002; Ganopolski and Calov, 2011)—all the way up to full-scale GCMs (e.g., IPCC, 2013, and references therein).

Within this hierarchy, the role of the simple models, sometimes referred to as "toy" models, is to provide insight and help understand the behavior of the more complex models, as well as of the climate system itself. The role of the intermediate models is to refine these insights and bridge the gap between the toy models and the GCMs (Ghil, 2001; Claussen et al., 2002): on the one hand, they are still simple enough to allow a fairly thorough analysis of their behavior, on the other they may be detailed enough for a direct comparison with the GCMs and with increasingly more plentiful and accurate observational data sets.

Finally, GCMs allow an extensive comparison with the observations and can thus help invalidate (Popper, 1959) the theories suggested by toy models, whether "validated" (i.e., not refuted) by intermediate ones or contradicted by the latter. Still, when GCM results are at variance with those of simpler models, it is not always the former that are correct, and it is a careful analysis of the observations that ultimately decides which results are closer to the elusive truth; see discussion in Dijkstra (2007) and Ghil (2015).

The results of our toy model suggest that vegetation might play a larger role in climatic variability — whether in apparent jumps between two or more types of near-stationary states or in oscillatory behavior — than heretofore suspected. In particular, it might contribute to more-or-less regular, but not necessarily simply periodic variability.

It is clear, for instance, that the clouds' contribution to planetary albedo is larger than that of vegetation (IPCC, 2013). It is also clear that our model's change in land albedo between its bare and vegetated state is unrealistically large, cf. Crucifix and Hewitt (2005). But the role of clouds has been explored over the last few decades across the entire hierarchy of climate models. While much remains to be done to gain a complete understanding of cloud-radiation interactions, greater attention to the role of vegetation in the evolution of planetary albedo seems to be well worth the effort. In particular, since the time scale of changes in vegetation is considerable slower than that for changes in cloud cover, the former might play a greater role in low-frequency climate oscillations.

Recall also that, in the early days of energy balance models (EBMs; see Ghil, 2001, 2015, and references therein), even larger and more unrealistic albedo differences between low- and high-temperature surfaces were used in simple, albeit infinite-dimensional models. Still, the EBMs' suggestion of multiple equilibria being possible in the climate system on long time scales has led to a rich literature on bifurcations — more recently and excitingly called "tipping points" (Lenton et al., 2008) — and their potential role in both past and future climate evolution.

While some Earth models of intermediate complexity do indeed show multiple equilibria, these appear to be mostly of local relevance, for instance in the Sahel. Brovkin et al. (2003), though, found no support for the co-existence of multiple equilibria in northern high latitudes. Moreover, when global oscillations do appear in such models, they tend to be attributed to cyclicity in the ocean circulation.

Our paper is only trying to make a case for the possibility of vegetation playing a more important role than contemplated heretofore and does not claim

in the least to have definitively proven that this is so. A similar argument about local versus global effects has been made with respect to the oceans' thermohaline circulation. Recall that the Stommel (1961) paper — much quoted recently in the context of multiple equilibria and symmetry breaking in the meridional overturning of the Atlantic or even global ocean — was originally written to explain seasonal changes in the overturning of "large semienclosed seas (e.g. Mediterranean and Red Seas)"; see, for instance, Dijkstra & Ghil (2005).

There is no better way of concluding this broader assessment of our toy model's results than by citing Karl Popper: "Science may be described as the art of systematic oversimplification." It might be well to remember this statement, given an increasing tendency in the climate sciences to rely more and more on GCMs, to the detriment of simpler models in the hierarchy.

Additional references

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