Response to Dr. Esau:

1) "One difficulty appears in the discussion of the internal wave generation by the turbulence on the pages 339-340. Figure 3b does not show any internal waves generated by the turbulence. Moreover, the narrative discussion proposed at this place is not convincing. As the authors know, the IW spectrum is limited by the Brunt-Vaisala frequency. This frequency could be translated in certain wavelength at the cut-off scale, which could be larger than the interval of length scales of the turbulence in the DNS run. Thus, the problem of the IW generation requires careful analysis of the spectra of the components of motions. Moreover, it is not obvious why the turbulence should generate a monochromatic wave as Fig 3b seems to suggest."

Figure 3b shows the density oscillations in the middle of the pycnolcine obtained in DNS with initially excited IW without turbulence (black line) and with turbulence without initially excited IW (in color). Note that only density oscillations due to initially excited IW (due to initial condition (8-10)), without turbulence, can be characterized as a monochromatic wave. These oscillations are given for comparison to show that the turbulence-generated IWs are much weaker.

We obtained a power spectrum of IWs excited by turbulence only (without initially induced IW (8-10)) in the pycnocline by performing Fourier transform of the density oscillations at point x = 20, y = 10, and z = 8 (i.e. at the pycnocline center in the middle of the computational domain, shown in Fig. 3b in color) and at two other points with coordinates x = 10 and x = 30 and the same y and z, and then averaged the spectra obtained at these three different points. The resulting spectrum is shown below in Fig. 1:

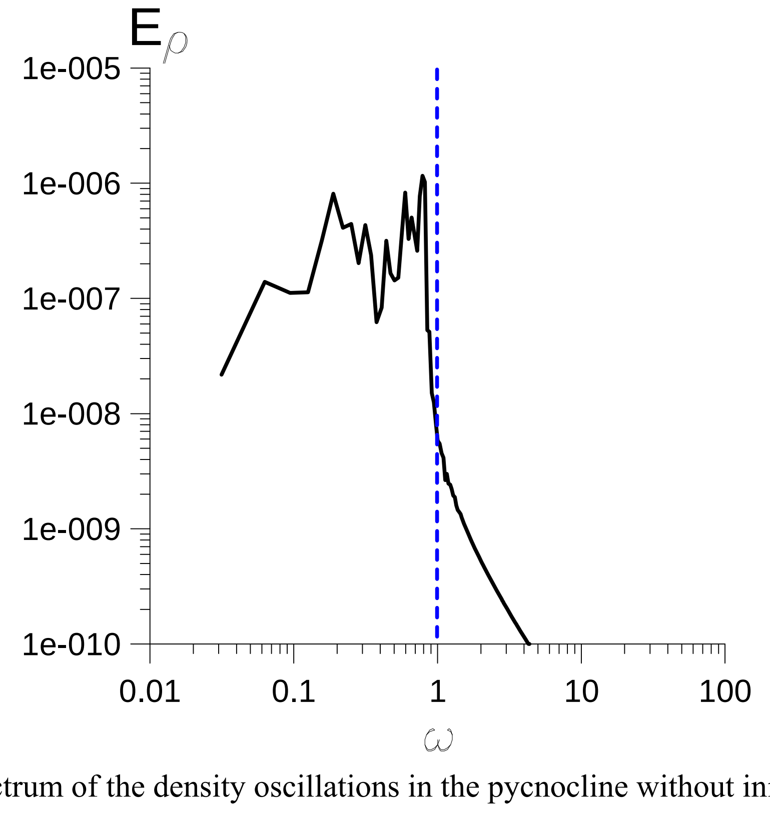

Fig. 1. Power spectrum of the density oscillations in the pycnocline without initially induced IW.

The vertical blue dashed line in Fig.1 denotes the maximum buoyancy frequency in the middle of the pycnocline, N_m = 1. The figure shows that IWs generated by turbulence are mostly at frequencies $\omega_1 \approx 0.8$ and $\omega_2 \approx 0.2$. Figure 2 below presents an enhanced view of the isopycnals of the density field in the vicinity of the pycnocline obtained in DNS in the vertical (x,z) plane at y = 0 at time t = 400. This is the same density field as in Fig. 3e, but shown with higher resolution over z-coordinate.

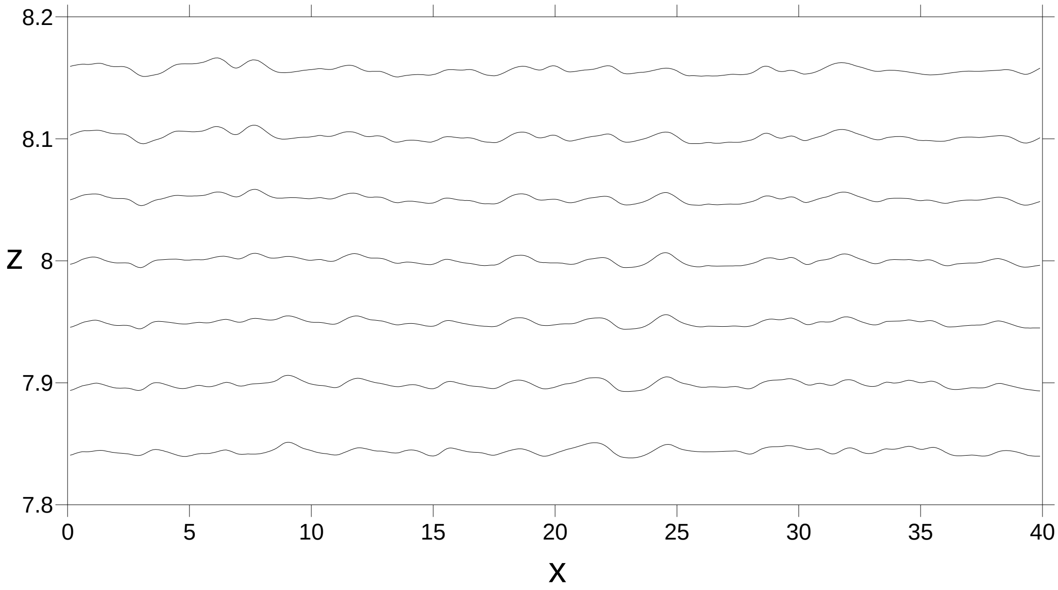

Fig. 2. Isopycnals of the density field obtained in DNS at time t = 400 without initially induced IW. The interval between lines is 0.05. The lowest line corresponds to ρ = 1.65.

Figure 2 shows that 1st and 2nd IW modes are present, and the most pronounced IW length is about 3 to 4 dimensionless units (i.e. the turbulence-generated IWs length is about 4 times larger than the pycnocline width L_0). Frequencies $\omega_1 \approx 0.8$ and $\omega_2 \approx 0.2$ correspond to the 1st and 2nd IW modes, respectively.

Thus, DNS results in Figs.1 and 2 show clearly that turbulence indeed generates IWs in the pycnocline. It is important to note, however, that the amplitude of these turbulence-generated IWs is by the order of magnitude smaller as compared to the internal wave induced due to initial condition (8-10). That's why the turbulence-generated IWs are not visible in Fig. 3e.

In order to answer the referee's criticism we included the following comment in the revised manuscript (p. 339, line 13):

"… The analysis of the frequency spectrum of the density oscillations in the pycnocline and the structure of isopycnals (not presented here) shows that mostly first- and second-mode IWs are generated by turbulence with corresponding frequencies $\omega_1 \approx 0.8$ and $\omega_2 \approx 0.2$ and wavelength $\lambda_t \approx 4$. "

Since the detailed discussion of the properties of IWs generated by turbulence is beyond the scope of the present paper, we did not include figures 1 and 2 above in the revised paper.

2) "Another difficulty appears in the wave-turbulence interaction discussion in the pages 342-343. The DNS runs revealed that the turbulence has only weak impact on the IW. It has been explained as the turbulence amplitude is too small to damp the IW. However, it is also clear that the IW and the turbulence have very different scales where the IW are much larger than the typical turbulent motions. Since the most effective interactions are between the motions of the same scale, the weakness of the interactions in the run could be just due to this scale separation. It would be reasonable to have another run with the IW of much shorter wavelength to check the interactions."

The objective of the present paper is to investigate the possibility of the enhancement of small-scale turbulence by strong, non-breaking IW. Therefore, spatial scales of turbulence and IWs are considered to be significantly separated. This is usually the case under typical stratification conditions in the ocean, where largest turbulence scale is at the order of dozen meters (due to forcing by the surface waves and their breaking) whereas IW length is larger than 100 m (and typically of the order of several hundred meters or larger, cf. e.g. Phillips (1977), Thorpe (2007)).

Our previous DNS results (Druzhinin et al. (2013)) show that weak IWs of short length (e.g. with λ = 4, or 2.5 times smaller as compared to the λ = 10 considered in the paper) are severely damped by turbulence. The results show that the damping rate of IWs with the amplitude two times less than the turbulence amplitude grows as $1/\lambda^2$ as wavelength λ is reduced.

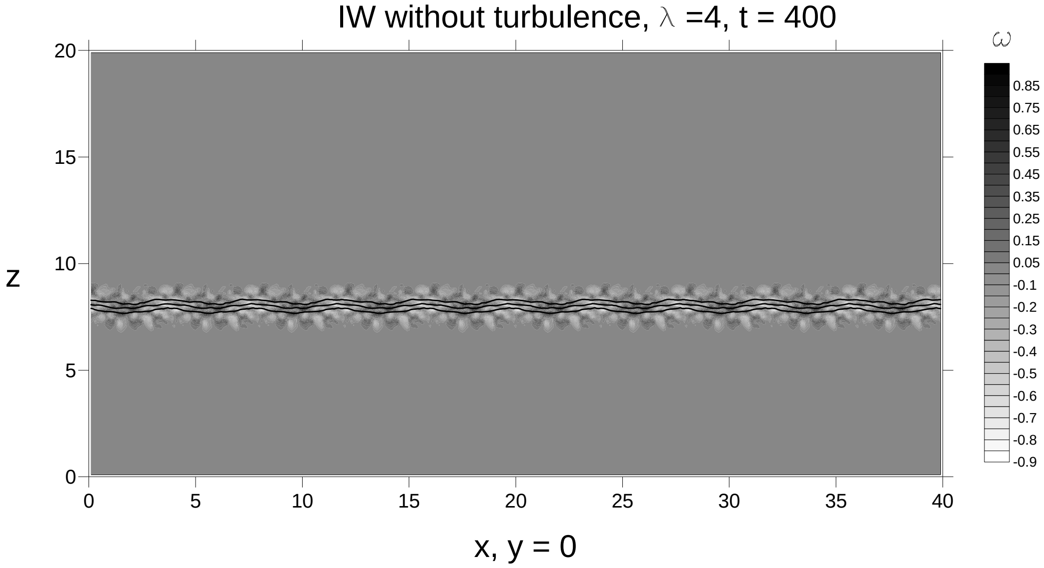

Fig. 3. Instantaneous distribution of the vorticity y-component ω (in grey scale) with imposed density contours (1.3, 1.5, 1.7) in the central (x,z)-plane at time moment t = 400 obtained in DNS of IW with length λ = 4 and amplitude $W_0 = 0.1$ without initially-induced turbulence.

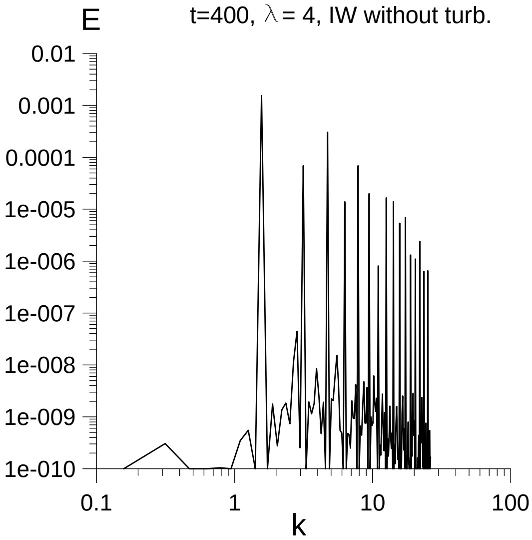

Fig. 4. Power spectrum in the IW with wavelength λ = 4 and amplitude $W_0 = 0.1$ obtained in DNS at z = 8 and time t = 400, without initially-induced turbulence.

On the other hand, if we consider large IW amplitudes and considerably reduce the IW length, as compared to λ = 10 considered in the paper, the wave slope also increases and IW becomes strongly non-linear and prone to breaking and/or consumed by viscous dissipation at sufficiently late times. In the present paper, we choose IW length λ = 10 and amplitude $W_0 = 0.1$. In this case, the initial turbulence integral length scale (about unity) is by the order of magnitude smaller than the IW length. The amplitude of the isopycnal displacement in IW is about $a \approx 0.2$, and the wave slope is about $ka = 2\pi a/\lambda \approx 0.12$ which may be regarded small enough to ensure that non-linear effects during the IW propagation in the pycnocline remain negligible. Spatial IW spectra (cf. Fig. 7 in the paper) also show that amplitudes of higher harmonics remain negligible as compared to the first harmonics amplitude.

We performed an additional DNS run to show the evolution of the IW excited due to the initial condition (8-10) with wavelength $\lambda = 4$ and amplitude $W_0 = 0.1$ *without initially induced turbulence*. In this case, although the IW amplitude is the same as in the case with $\lambda = 10$, the wave slope is about ka = 0.3 (i.e. 2.5 times larger as compared to the case $\lambda = 10$ in the paper). The instantaneous vorticity and density fields in the vertical (x, z) central plane and the power spectrum obtained in DNS at time $t = 400$ is shown in Figs. 3 and 4. Figures 3 and 4 show that the IW is strongly non-linear, as expected for such large wave slope, and considerable portion of the energy goes from the first harmonics to higher harmonics. The conclusion follows that such short-length, sufficiently strong IWs are not sustainable in our DNS.

The DNS study of the interaction of IWs and turbulence with comparable length scales, suggested by the referee, is certainly of interest. We can expect some stronger and interesting interaction effects here. But, as we assume, it would be also quite difficult to separate the waves and turbulence and elucidate the effect of IW on turbulence in this case. The applicability of the results to realistic natural oceanic conditions is also not quite clear where the scale separation between sustainable IWs and small-scale turbulence is a common feature (cf. e.g. Thorpe 2007).

The study of turbulence generation by strongly non-linear IW, when high harmonics generation, caused by non-linear effects, become significant, is also of interest. But this is a subject of a future study and not included in the present paper.

In order to answer the referee's criticism and justify the choice of the IW parameters we included the following comment in the revised text (p.336, line 13):

"…Previous DNS results by Druzhinin et al. (2013) show that weak IWs of short length (say, about 3 times smaller as compared to the $\lambda = 10$ considered in the present paper) are severely damped by turbulence. The results show that the damping rate of IWs with the amplitude two times less than the turbulence amplitude grows as $1/\lambda^2$. On the other hand, if we consider larger IW amplitudes and reduce the IW length, the wave slope increases so that strong, short-length IW become strongly non-linear and are prone to breaking and viscous dissipation."

Minor comments:

1) "The Re definition here (Eq. 4 and below in the text) is rather meaningless as it does not refer to the turbulence features of the fluid and the ability of the DNS to reproduce them. It is wrong to claim that if you double L0, your Re will also double. Traditional estimation, based on Taylor microscale, required L0 to be the integral scale of the turbulence and U0 the scale of TKE fluctuations, roughy it could be approximated as Re _ N^(4/3), which place your DNS in the class of Re _ 300 or even lower as turbulence decay with time, which is normal for such exercises."

The Reynolds number in the present paper (Eq. (4)) is based on L_0 (the pycnocline thickness), $T_0 = 1/N_0$ (where N_0 is the buoyancy frequency in the middle of the pycnocline) and the corresponding velocity scale, $U_0 = L_0 / T_0$. Since the time scale is defined as $T_0 = 1/N_0$ and the velocity scale $U_0 = L_0 / T_0 = L_0 N_0$, the Richardson number (4) in DNS identically equals unity, Ri = 1. This is convenient since the characteristics of IWs (eigenfunctions and dispersion relation $\omega(k)$ in Fig. 2a) remain the same for different parameters in DNS runs. However, we agree with the referee, that Re is *not* the turbulent Reynolds number. For the considered choice of the spectrum (13) with $k_f = 1$ the turbulence dimensionless integral length scale, L_t, at initialization is of order unity. Thus, the turbulent Reynolds number, Re_t, based on L_t and turbulence velocity amplitude $U_{t0} = 0.1$ at initialization and dimensionless viscosity 1/Re (with Re = 20000), is evaluated as $Re_t = L_t U_{t0} Re \approx 2000$.

Note however, that wavenumber k_f in spectum (13) is normalized by L_0, so if L_0 is doubled, than the dimensional initial turbulence integral length scale and hence the turbulent Reynolds number are also doubled provided the fluctuation amplitude, U_{t0}, is fixed.

Note also that in DNS with initially induced IW, turbulence TKE spectrum is characterized by the well-pronounced energy peak at the IW wavenumber $k = 2\pi/\lambda = 0.628$ (cf. Fig. 7). Thus, in this case, the turbulent length scale is actually determined by the IW length ($\lambda = 10$). Then the flow Reynolds number is estimated as $\mathrm{Re}_t = L_t\, U_{t0}\,\mathrm{Re} = \lambda\, U_{t0}\,\mathrm{Re} = 20000$ for the amplitude $U_{t0} = 0.1$.

In order to answer the referee's criticism we included the following comments in the revised text

(p.337, 16th line);
"…For the considered choice of the spectrum (13) with $k_f = 1$ the turbulence dimensionless integral length scale, L_t, at initialization is of order unity. Thus, the turbulent Reynolds number, Re_t, based on L_t and U_{t0}, is evaluated as $\mathrm{Re}_t = L_t\, U_{t0}\,\mathrm{Re} \approx 2000$."

(p. 344, attached to the last paragraph):
Note also that since the energy peak at the IW wavenumber $k = 2\pi/\lambda = 0.628$ in the TKE spectrum is most pronounced, the turbulent length scale, in this case, is actually determined by the IW length ($\lambda = 10$). Than the turbulent Reynolds number can be estimated as $\mathrm{Re}t = Lt\ U_{t0}\,\mathrm{Re} = \lambda\ U_{t0}\,\mathrm{Re} = 20000$ for the amplitude $U_{t0} = 0.1$.

2) "Eq. (17) is problematic. Is "j=3"? Otherwise it will be incompatible with Eq. (16)"

In the revised text we re-defined the velocity instantaneous deviation from the mean field as ($\widetilde{U}_i = U_i - \langle U_i \rangle$). We agree that the notation for the velocity deviation used in the original text (U_i') was confusing. Now eq.(17) is just the well-known definition of the TKE dissipation rate. It is computed in DNS as

$$\varepsilon = \frac{1}{\mathrm{Re}} \sum_i \left\langle \left(\frac{\partial \widetilde{U}_i}{\partial x}\right)^2 + \left(\frac{\partial \widetilde{U}_i}{\partial y}\right)^2 + \left(\frac{\partial \widetilde{U}_i}{\partial z}\right)^2 \right\rangle,\ i = x, y, z.$$

We changed the revised text accordingly (p. 338, eq.(17)).

3) "Page 343. The interesting discussion point that the turbulence survive longer in the vicinity of the pycnocline centre. Could it be because at this level the stability is the strongest and the turbulence has the largest horizontal scales so that the interactions between the shortest waves and the largest turbulence is more efficient? It would be interesting to have an analysis."

In the case with initially induced IW, mentioned by the referee, turbulence is maintained by the strain created by the IW field. Due to this IW strain field the vorticity field has maxima localized the vicinity of IW crests and troughs. Since, in the considered case, IW length is much (about 10 times) larger as compared to the pycnocline thickness, the IW-induced velocity field decreases exponentially with the distance from the pycnocline. So it is expected that the effect of the IW field on turbulence is most pronounced in the immediate vicinity of the pycnocline. Note that a similar enhancement of turbulence was observed by Tsai et al. (2015) in the vicinity of the waved water surface. Their DNS results show that turbulence is enhanced by the straining field of the surface wave in the vicinity of the water surface and this enhancement is most pronounced in the vicinity of the surface wave crests and troughs.

We added a comment in p. 344, 1st paragraph.

We are grateful to the referee for all comments and suggestions.