Itemized Replies to Dr. N. Suciu's comments

This is the itemized response to the comments by Dr. N. Suciu, Nonlin. Processes Geophys. Discuss., 2, C494-C495, 2015, on our manuscript NPGD, 2, 1-22, 2015, entitled "Toward a practical approach for ergodicity analysis".

We would like to thank Dr. N. Suciu's for reviewing our manuscript in detail as well as for the constructive comments which enable us to greatly improve the quality of our manuscript. Dr. N. Suciu's thorough review and constructive comments, especially clearly pointing out the two approaches for ergodicity analysis, have not only enabled us to improve this manuscript but also opened us another window for our undergoing work on covariance ergodicity.

In the following, we present a detailed point-to-point response to each of the comments. We hope that the revisions in the manuscript and this response will be sufficient to address Dr. N. Suciu's concerns on this manuscript.

<u>General Comments</u>

*The discussion paper "**Toward a practical approach for ergodicity analysis**" by Wang et al. published in NPGD investigates the possibility that some precipitation time series behave as ergodic processes. During an informal exchange of views on this possibility with the second author, Cheng Wang, I did some comments on an earlier version of the paper one year ago (sent by e-mail on 09.10.2014). My comments focused on three issues: the definition of ergodicity, the methodology used to investigate whether this property holds for the time series investigated in the paper, and the interpretation of the results. After reading the paper, as well as a couple of articles listed in the bibliography, I shall try to present in the following a more complete formulation of these issues and my own view on a possible interpretation of the results.*

No response needed.

Since the time average $MT = \Sigma X(t)/T$, t=1,2,...,T, of a stationary process $X(t)$ is an *unbiased estimator* of the mean, the estimator MT is also *consistent* and the process is ergodic if and only if MT tends, in the mean square sense to the constant ensemble average $m=\langle X(t)\rangle$, that is, $\langle (MT - m)2 \rangle \rightarrow 0$ as $T \rightarrow \infty$[Yaglom, 1987, p. 214]. Instead of this rigorous definition, the authors use as an ergodicity criterion the limit $D(MT) \rightarrow 0$ as $T \rightarrow \infty$. The "variance" $D(MT)$, defined by their Eq. (1), is equivalent, after a rearrangement of terms, with $D(MT) = \Sigma\ M t2/\mathrm{T} - M T 2$, where t=1,2,...,T. This definition is neither a stochastic average, nor an estimation by time average (which would have been a moving average with averaging window equal to T). It seems that this unusual, and actually wrong, definition of ergodicity, as well as the approach for ergodicity analysis, have been borrowed from another paper of the first author, [Wang et al., 2009], which, however, is not cited in this discussion paper.

Response: Thanks for pointing out this issue. We have incorporated this suggested rigorous definition in the manuscript. The definition we used original is actually equivalent to the suggested one by the reviewer in calculation.

The ergodicity of a stationary process cannot be assessed in absence of some information about its probability distribution [Yaglom, 1987; Duan and Goldys, 2001; Oliveira et al., 2006; Suciu, 2014]. Without prior knowledge of this distribution one can at the best guess the next outcome of some stationary time series, provided that they are ergodic [e.g. Morvai and Weiss, 2005].

No response needed.

Nevertheless, some empirical investigations on ergodicity could help us to identify those time series which very likely are not ergodic. The present paper is an attempt in this direction. If a moving averaging is used instead of Eq. (1) to estimate the variance of the estimator *MT*, using it to assess the ergodicity of the time series presumes the ergodicity of the variance. Then, the results indicating ergodicity only tell us that the time series behave consistently with the variance-ergodicity assumption. Following my comments on the earlier version of the paper, the authors propose this interpretation of the results at the end of Section 3.3.

No response needed.

But the results presented in Figs. 2-4 rather indicate that the time series, even those identified as "ergodic" (Fig. 4) are not stationary, because the estimated mean is not constant. In this case the notion of ergodicity, as a property of stationary processes, is useless. More general ergodic properties, which do not require statistical stationarity, can be formulated for *processes having time average mean value and correlation function* [Yaglom, 1987, Sect. 26.6]. That means, processes for which the time integrals of the (time dependent) stochastic mean value and correlation divided by T converge to some finite limit as $T \rightarrow \infty$. Such properties, again, cannot be proved without information about the statistics of the process [see Yaglom, 1987, the four examples at the end of Sect. 26.6]. Nevertheless, we can follow empirical approaches similar to that for stationary processes described above.

Response: Thanks for the comment. Figures 2-4 have been updated as Figures 1-2 which plot the time average of each monthly precipitation data series, and Figure 5 which plots the trend of each monthly precipitation data series, respectively, in the revised manuscript.

First approach: Consider a process possessing both mean and correlation time averages. Let m0 be the time average of the variable stochastic average *m(t)*. Then, *MT* (defined above and considered in the present paper) is a consistent estimator of m0 if and only if the average spectral density of the centered process *Y(t) = X(t) - m(t)* is continuous in the origin. An equivalent formulation of this condition using the time average correlation function, similar to Slutsky theorem [Yaglom, 1987, Eq. (3.15a)], can be derived by using the equations (3.15a), (4.512) and (4.499) from [Yaglom, 1987]. The estimation of the time average correlation from a single realization of *Y(t)*, following [Yaglom, 1987, Eq. (4.505)], can be obtained by the autocorrelation function of the process *Y(t)*, which is precisely the "noise" extracted from *X(t)* with the automatic de-trending algorithm of Vamoș and Crăciun [2012]. These results can be readily obtained by the same codes used to prepare Fig. 4 of the paper, available online at http://www.ictp.acad.ro/vamos/trend/trendrcma.htm.

Response: Thanks a lot for teaching us this approach and providing the guidelines to implement it. We have incorporated this approach as Approach 3 in the revised manuscript. This approach together with the following second approach really helped us to come up with a series of rigorous ergodicity analysis for both mean ergodicity and co-variance ergodicity. To our understanding, this approach is suitable to analyze co-variance ergodicity, we therefore plan to implement this approach in our ongoing study on practical approaches for co-variance ergodicity analysis, which is also an extension of this present paper.

Second approach: Assuming only the existence of the time average m0, the consistency of its estimator *MT* is ensured if and only if the ergodic estimation of the mean of *Y(t)* is zero, i.e. *MT* (*Y(t)*) $\rightarrow 0$ as $T \rightarrow \infty$ [Yaglom, 1987, p. 486]. The centered process *Y(t)* can be estimated by the noise determined with the same automatic de-trending algorithm.

Response: Thanks a lot for suggesting us this approach. This approach has been implemented in the manuscript as Approach 2. We appreciate the handy tool of automatic trend algorithm developed by Dr. Suciu.

The scheduled empirical approaches can be used to reject the ergodicity hypothesis for time series which do not fulfill the sufficient and necessary condition from above. I would recommend to use both approaches, because some time series could be consistent to both the hypothesis of existence of time average mean value and that of existence of time average correlation while other series could be consistent to only one of these hypotheses. It would be also desirable to extend the time series with the RBF neural network approach described in Section 3.2., before using the automatic de-trending algorithm. Finally, it should be stressed again that the "ergodicity hypothesis" in this general empirical approach is in fact the hypothesis that the non-stationary time series possess mean and correlation time averages which can be consistently estimated through time averages.

Response: Thanks for the comment and suggestion.

Specific Comments

Page 1428: " Most studies of time series applications, such as in the fields of hydrology, hydrodynamics, and noise (Jiang and Zheng, 2005; Oliveira et al., 2006; Veneziano and Tabaei, 2004), discuss statistic characteristics simply by assuming time series having ergodicity without justifying this assumption with a rigorous approach."

-Statement inaccurate. In (Oliveira et al., 2006, p. 379, Eq. (12)) there is a rigorous ergodicity condition fulfilled by the fast decaying correlation in homogeneous turbulence.

Response: Thanks for pointing out this inaccurate statement. This sentence has been deleted from the manuscript.

Page 1428: "... ergodicity ... is a fundamental presumption for many time series problems (Ding and Deng, 1988; Fiori and Jankovi₵, 2005; Hsu, 2003; Liu, 1998; Mitosek, 2000; Wang et al., 2004)."

-Statement generally incorrect in the case of subsurface stochastic hydrology. Only in special cases of small fluctuations of the hydraulic conductivity the problem of estimating transport coefficients can be formulated in terms of processes, i.e., time series [Suciu, 2014, Sect. 5.3]. For instance in [Fiori and Jankovi₵, 2005; Hsu, 2003] random fields are used to model the transport and ergodicity (for random space functions) is ensured by increasing the dimension of contaminant source. See [Suciu, 2014] and references [95] and [100] therein for different meanings of the term "ergodicity" in subsurface hydrology.

Response: Thanks for the comment. While we state that ergodicity plays a fundamental role in MANY time series problems, which is supported by many reference including the serval listed in the present manuscript, we do not intend to exclude any case where ergodicity theory has unimportant roles to play. Therefore, we would like to remain the mentioned statement as is in the manuscript.

Page 1429: The definition of the second-order stationarity is not correct. The first moment cannot depend on time differences, it must be constant [see Yaglom, 1986, Chap. 1, Sect. 3].

Response: Thanks for pointing out this mistake. It has been corrected as shown in Line 100 in the revised manuscript.

page 1431: " If the ACF rapidly approaches 0 (i.e. falls into the stochastic domain), the time series is stationary; otherwise it is non-stationary (Cline and Pu, 1998, 1999)."

-False. The decay of ACF does not prove the stationarity. A counter-example: Even if the fractional Brownian motion has long tail, power-law correlations, which don't approach rapidly 0, it is stationary and variance-ergodic [Suciu, 2014, Sect. 5.3, p. 123].

-The reference to (Cline and Pu, 1998, 1999) for the ACF stationarity criterion is not correct. Neither stationarity nor ACF are mentioned in these papers.

-Instead, as seen for instance in (Chen and Rao, 2002), segmentation algorithms and autoregressive models are often used to construct stationarity tests.

Response: Thanks for correcting this ACF issue. The ACF method has been deleted from the manuscript and only ADF test remains. We agree that ACF is not as rigorous as ADF test for stationary test. We have seen using ACF for stationary test in literatures though.

Page 1433: "ADF test indicate that all the 20 individual monthly precipitation data series at Newberry are stationary."

-There are no results from ADF tests presented in the paper.

Response: The ADF test results are presented in Table 1 in the revised manuscript.

Page 1436: "A linear trend analysis is also performed following Vamos and Craciun (2012)"

-Wrong. The output of the automatic algorithm described in (Vamos and Craciun, 2012) is not a linear trend, as already shown in fig. 4.

Response: Thanks for pointing out this mistake. The statement has been corrected in the revised manuscript, as shown in Line 343 in the revised manuscript.

Page 1437: "Some researchers (Duan and Goldys, 2001; Koutsoyiannis, 2005; Liu, 1998), however, have pointed out that hydrological processes may have ergodic properties although no particular ergodicity analysis was performed in these works."

-Inaccurate. Duan and Goldys (2001, Theorem 4.1C) give a rigorous proof of ergodicity.

Response: Thanks for pointing out this issue. We have deleted the second half of the statement from the manuscript, see Line 368-369 in the revised manuscript.

Figure 2 and 3 are identical, even though the latter should have been obtained by using the RBF neural network.

Response: Thanks for pointing out this mistake. I somehow messed up Figures 2 and 3 during proofreading. These two figures have been updated.

Technical Corrections

Last row on page 1426: What is "the phase mean function"? An explanation is needed here.

Response: A note on "the phase mean function" is presented in Ln 42 of the revised manuscript.

First row on page 1427: the correlation function is the auto-covariance divided by the variance. Last row on page 1427: Should "... approaches have yet been available." be "... approaches have NOT yet been available."?

Response: Thanks for pointing out the mistakes. These two mistakes have been corrected in the test, as shown in Line 26 and 51 respectively.

Page 1428, end of the first paragraph: " process averaged over time behaves identical to the process averaged over space." It should be "... averaged over PHASE space.". Other suggestions: averages over the statistical ensemble, stochastic averages.

Response: Thanks for the suggestion. It has been incorporated in the manuscript, as shown in Line 62.

Third row from bottom on page 1428: "... used mainly in mathematical physics, e.g. dynamics,". What does it mean here "dynamics"?

Response: Thanks for pointing out this confusion. This sentence has been deleted from the text to avoid the possible unnecessary confusion.

Page 1429: The reference (Davis et al., 1994) cited here is not included at References.

Response: Thanks for pointing out this mistake. The reference has been added in the manuscript.

Figs. 2 and 3: The unit for estimated mean values is given in mm. That for variances should be mm2.

Response: Thanks for pointing out this mistake. It has been corrected.