

# ***Interactive comment on “Explanation of the values of Hack’s drainage basin, river length scaling exponent” by A. G. Hunt***

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The referee makes a number of points, but I refute them.

Referee’s statements

1. This paper attempts to derive with the help of percolation models the range of empirical estimates of the scaling exponent of river length, known as the Hack’s law exponent. I believe this an original and seductive idea. However, I would like first to clearly state that the repeated use of the expression ‘percolation theory’ seems to be rather misleading: what is really shown is a given agreement of a range of numerical values yielded by given percolation models with empirical estimates, not a theoretical explanation provided by an abstract theory.

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Response to 1. I do not understand why the values of the exponents are called “empirical estimates.” Consider the following two references and quotes therefrom, as well as the table copied from them (and put in a separate file): Optimal Path in Strong Disorder and Shortest Path in Invasion Percolation with Trapping Markus Porto, Shlomo Havlin, Stefan Schwarzer, and Armin Bunde Phys. Rev. Lett. 79, 4060 – Published 24 November 1997

“We present analytical and numerical results suggesting that the optimal path in an energy landscape in the strong disorder limit is in the universality class of the shortest path in invasion percolation with trapping. Our results imply that, in contrast to common belief, invasion percolation with trapping and regular percolation in  $d=3$  are in different universality classes.” LETTER TO THE EDITOR Invasion percolation: new algorithms and universality classes J. Phys. A: Math. Gen. 32 (1999) L521–L529. Printed in the UK PII: S0305-4470(99)09096-4 Adrian P Sheppard†, Mark A Knackstedt†, W V Pinczewski and Muhammad Sahimi§

“Second, using the new algorithm we measure the fractal properties of various IP models with an accuracy which is at least an order of magnitude better than the best previous estimates. This enables us to test various hypotheses and conclusively establish the true universality classes of the IP models.”

The shortest paths exponent for a homogeneous system (RP for random percolation), where the bonds are chosen randomly is  $D_{min}$  above,  $1.1307 \pm 0.0004$ . The optimal paths exponent for a system in the extreme heterogeneous limit is also known, though not quite as accurately,  $1.21 \pm 0.01$ . The argument is that these two exponent values are the bounds of the tortuosity exponent from percolation theory over the range of all possible levels of heterogeneity.

2. I would also like to call the attention of the author on a few facts: 2a there is presently no attempt to physically explain the observed fluctuations of the Hack’s exponent by the multifractal heterogeneity of the driving fields of the river flow, e.g. landscape,

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topography, soil, rainfall, etc., whereas this exponent should have a unique value in the framework of the Fractal Geometry;

Response to comment 2a. The flow path does not relate directly to any fractal heterogeneity as long as it is a path which generates the lowest net flow resistance, as defined by setting the cumulative distribution of all flow resistances less than a critical value equal to a percolation threshold. The optimal paths are then those with the least energy dissipation in a landscape with a wide range of local resistances, to current or to flow. This represents the universality of the fluid flow paths in the strong disorder limit. Note that Sahimi and Mukhopadhyay, in a different context, Sahimi, M., Mukhopadhyay, S.: Scaling properties of a percolation model with long-range correlations. Phys. Rev. E 54, 3870 (1996), have indeed shown that the percolation backbone fractal dimensionality can be changed by long-range correlations, but the optimal paths exponent seems insensitive to such correlations or, indeed variations in percolation model (see Table).

2b. this would be furthermore in agreement with the fact that the present estimates of the Hack's exponent does not rely on the (fractal) binary percolation model;

Response to 2b. The binary percolation model is the other limit of the argument, where heterogeneity can be neglected.

2c. unfortunately, the present manuscript argues that 'Hack's law can be understood using percolation theory : : : because of 1) the fractal structure of the percolation cluster';

Response to 2c. It has often been argued that the sinuosity of rivers and the self-similarity of drainage basins reflects properties of a fractal object. I do not know why this is "unfortunate."

2d. it is also unfortunate that no hint is given on how much non-binary and heterogeneous is the model used by Sheppard et al. (1999);

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Response to 2d. It is the strong disorder limit. See above.

2e there is therefore no obvious justification to take the scaling exponent of the aforementioned model as being the upper bound of the tortuosity exponent;

Response to 2e Since it is the strong disorder limit, yes there is.

2f there is an interesting attempt to physically explain the relevance of the percolation with the help of considerations based on erosion, but the latter is a 3D process -or at least a  $(2 + \_ )D$  process- and the considered percolation models are only 2D;

Response to 2f. The two – dimensionality of the percolation is exactly appropriate. A two-dimensional surface can be deformed into the third dimension, but this does not add alternate flow paths that are off the surface, which would be the only way to change the topology of the connections. Every point on the earth's surface can be defined by two angular coordinates; the third dimension is required only to describe locations above the surface or below it, but the connections of the rivers are on the surface.

2g furthermore percolation exponents are often strongly dimensionality dependent;

Response to 2g. They are, but the topology of percolation and the topology of the connections of the rivers are both two-dimensional, on the surface of the earth. There are only two coordinates necessary to describe any point on the earth's surface.

2h. there is no explanation on uncertainties or on the range of fluctuations of the empirical estimates of the Hack's exponent, e.g. is the range obtained with rms estimates?

Response to 2h. I have now accessed two original papers by Gray, including the one referenced. He does not generate ranges of Hack's exponent for a given data set, but assigns the uncertainty to the numerical prefactor. The variability that he obtains for the Hack's law exponent is through consideration of different data sets: his own, Taylor and Schwartz, (1952) and Langbein and others (1947) in different geographic regions. Taylor, A. B. and H. E. Schwarz, 1952, Unit hydrograph lag and peak flow related to

basin characteristics. Trans. Amer. Geophys. Union 33: 235-246. Langbein, W., and others, 1947, Topographic characteristics of drainage basins. U.S. Dept. Interior. Geological Survey Water-Supply Paper 968-C: 125-155. This is exactly the kind of variability addressed in the paper, so there is no need for modification of the paper to address this issue.

2i conclusions should be accordingly revised..

3. Overall, I would suggest a revision with further review.

I infer that the original conclusions of the paper do not need revision. All statements are appropriate as given, there is no doubt about the values of the exponents, nor in the appropriateness of their application on the surface of the earth. The conclusions of the paper that the claimed variability in Hack's law is consistent with the percolation exponents for the sinuosity of paths in the extremes of no disorder and the limit of strong disorder are justified. Whether the suggestion is ultimately borne out may require further research.

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Interactive comment on Nonlin. Processes Geophys. Discuss., 2, 1355, 2015.

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**Table 1.** Fractal dimensions for invasion percolation in two dimensions. Numbers in parenthesis are the estimates obtained from local  $D_f(M)$  analysis.

Model	$D_{\min}$	$D_b$
NTIP	$1.1293 \pm 0.0010$	$1.6422 \pm 0.0040$
Site TIP	$1.214 \pm 0.001$	$1.217 \pm 0.020$
Bond TIP	$1.2170 \pm 0.0007$	$1.217 \pm 0.001$
RP	$1.1307 \pm 0.0004$	$1.6432 \pm 0.0008$
Optimal paths	$1.21 \pm 0.01$	$1.21 \pm 0.01$

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Fig. 1.