Responses to Prof. Pielke's comments:

> This is an excellent analysis and is ready to accept in my view after consideration of the one comment below. I have read the paper in the past and am pleased that it is moving forward to a final form.
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> My one comment is with respect the "butterfly effect of the first kind" and the "butterfly effect of the second kind". In the text it is written
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> "The 5DLM and 6DLM collectively suggest different roles for small-scale processes (i.e., stabilization vs. destabilization), consistent with the following statement by Lorenz (1972): If the flap of a butterfly's wings can be instrumental in generating a tornado, it can equally well be instrumental in preventing a tornado."
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> The paper writes in regards to this subject
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> "For example, although the butterfly effect (of the first kind) with dependence of solutions on initial conditions appears in the 3DLM within the range between r = 25 and 40, it does not exist in the 5DLM. Therefore, to examine whether or not small perturbations can alter large-scale structure (i.e., the butterfly effect of the second kind), a model containing proper representations of multiscale processes and their nonlinear interactions is required."
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> I would like to see the author discuss this difference and conclusions further. Lorenz used the butterfly analog for two reasons - its appearance visually in solution space and the sensitivity to initial conditions finding. The sensitivity to initial conditions is the basis for the statement that "the flap of a butterfly's wings can be instrumental in generating a tornado." However, this claim is based on a gross overstretch of the realism of the Lorenz model to multi-scale weather dynamics.
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> This claim should be further discussed in the paper (I suggest adding to the conclusion). With a model that contains the dissipation of coherent fluid motions into heat at even the finest spatial scales, there would not be a way for "a butterfly's disturbance of the fluid" to be transferred as a coherent signal such that it could affect the development of a tornado. This excellent paper could serve the role of clarifying this misinterpretation.

Thanks for your valuable comments and encouragement.

Our ultimate goal is to determine under which conditions increasing resolutions can improve the predictions in weather/climate models. Based on our recent studies with the 5DLM (e.g, Shen 2014), our hypothesis is that system's stability in the LMs, with a finite number of modes, can be improved with additional modes that provide negative nonlinear feedback associated with additional dissipative terms. However, since new modes can also introduce additional heating term(s), the competing role of the heating term(s) with nonlinear terms and/or with dissipative terms deserves to be examined so that the conditions under which solutions become more stable or chaotic can be better understood. This is the focus of this study. To verify the above hypothesis of whether the nonlinear feedback loop can be extended to provide nonlinear negative feedback to stabilize the solutions, we have started deriving higher-order LMs with three additional modes which are selected based on the analysis of the Jacobian term, $J(\Psi, \theta)$. A paper is being prepared for publication (Yoo and Shen, 2015; See Table 1 in the attached pdf file). Currently, we have been working to implement the trajectory separation method to a weather/climate model to perform the stability analysis. The tool and the weather/climate model will be used to examine the impact of small-scale processes on the solution's stability in a future study.

References:

Yoo, E. and B.-W. Shen, 2015: On the extension of the nonlinear feedback loop in 7D, 8D and 9D Lorenz models. (in preparation)

Table 1: Fourier modes used in our high-order LMs (e.g., Shen 2014a, 2015; Yoo and Shen, 2015) and the models by Curry (1978) and Lucarini and K. Fraedrich (2009). Note that $M_4 = \psi_1(1,3)$, $M_5 = \Theta_2(1,3)$, and $M_6 = \Theta_2(0,4)$.

model	Ψ	Θ	Θ	rc	References
5DLM	$\psi_1(1,1)$	$\Theta_2(1,1)$, $\Theta_2(1,3)$	$\Theta_2(0,2)$, $\Theta_2(0,4)$	42.9	Shen (2014)
6DLM	$\psi_1(1,1)$, $\psi_1(1,3)$,	$\Theta_2(1,1)$ $\Theta_2(1,3)$	$\Theta_2(0,2)$ $\Theta_2(0,4)$	41.1	Shen(2015)
7DLM	$\psi_1(1,1)$	$\Theta_2(1,1)$, $\Theta_2(1,3)$, $\Theta_2(1,5)$	$\Theta_2(0,2)$, $\Theta_2(0,4)$, $\Theta_2(0,6)$,	~116.9	Yoo and Shen (2015, in preparation)
8DLM	$\psi_1(1,1)$, $\psi_1(1,3)$	$\Theta_2(1,1)$, $\Theta_2(1,3)$, $\Theta_2(1,5)$	$\Theta_2(0,2)$, $\Theta_2(0,4)$, $\Theta_2(0,6)$	~105 (TBD with the eLE analysis)	Yoo and Shen (2015)
9DLM	$\psi_1(1,1)$, $\psi_1(1,3)$, $\psi_1(1,5)$	$\Theta_2(1,1)$, $\Theta_2(1,3)$, $\Theta_2(1,5)$	$\Theta_2(0,2)$, $\Theta_2(0,4)$, $\Theta_2(0,6)$	~105 (TBD with the eLE analysis)	Yoo and Shen (2015)
14DLM	$\psi_1(1,1)$, $\psi_1(1,3)$, $\psi_1(2,2)$, $\psi_1(2,4)$, $\psi_1(3,1)$, $\psi_1(3,3)$	$\Theta_2(1,1)$, $\Theta_2(1,3)$, $\Theta_2(2,2)$, $\Theta_2(2,4)$, $\Theta_2(3,1)$, $\Theta_2(3,3)$	$\Theta_2(0,2)$, $\Theta_2(0,4)$	rc~43	Curry (1978)
10EQs	$\psi_1(1,1)$ $\psi_1(2,2)$	$\Theta_2(1,1)$, $\Theta_2(2,2)$	$\Theta_2(0,2)$, $\Theta_2(0,4)$	n/a	Lucarini and K. Fraedrich (2009)