

First, I would like to thank Brian Watson for his comments. I address comment 2 first. The surface of the earth is defined by two coordinates, and any stream position may be defined by an ordered pair. Consider the case where the flow friction is a strongly varying function of position. Water from precipitation or from upstream can flow from one position on the x-y plane to another. Consider the case where $x = 0$ is at the lowest elevation (say a higher order stream), and $x = x_0$ is at a higher elevation, say at a divide. This is equivalent to a potential difference across a resistor network. There is a resistance between each pair of positions. By assumption, the distribution of these resistances is spread over a wide range of values. The global optimal water path connects a series of links, each from one grid position to the next, in such a way that the sum of the resistances along the path is the smallest possible value. This is done mathematically in critical path analysis by setting an integral over the resistance distribution, from the smallest value to a maximum value, equal to the critical bond fraction (in two dimensions) for percolation. The tortuosity of the resulting path is described by the two dimensional value of the optimal paths exponent. The medium itself is far above the percolation threshold, but the critical path defined in critical path analysis as above is right at the threshold.

For comment 4, my statement that the natural logarithm of the local resistances has a variance much larger than 1 is not meant to imply that the particular distribution is log-normal. It could have a log-uniform character, or any number of other forms. Also, it refers to resistance to water flow on the surface, and that does not have a clear or direct relationship with soil particle size distributions, since the surface can include various geological heterogeneity as well. However, it is of course known that long range-correlations in resistance distributions can change some fractal dimensionalities in percolation, in particular, that of the backbone (Sahimi and Mukhopadhyay, 1996; Hunt et al., 2014). However, the optimal paths exponent is not known to depend on such correlations (Sheppard et al., 1999), for which the most important input is that the relevant coordination number is very small (about 2.7), leading to a critical region, which is very wide compared to that of a homogeneous medium (Ghanbarian et al., 2015).

For comment 3, it is not envisioned that any particular regions, or types of region, need necessarily percolate. It is sufficient that the water paths do. If the medium is homogeneous (and with a small elevation gradient, for example), the percolation path is constructed by the paths that the water happens to choose, and these will be heavily influenced by where the heaviest precipitation actually falls. Water will eventually find a path across the system. But if it is primarily driven by precipitation fluctuations, then the local bonds will be selected at random. When such a path is eventually connected, the percolation threshold has been reached, and the shortest distance across the system through that connected cluster will have the lowest flow resistance, providing the dominant flow. All other paths will have greater resistance, and will be shorted out.

Thus, comment number 1 is also addressed; it is not the system that is at the threshold, it is the incipient path through the system that reaches the threshold, whether links through a homogeneous system are connected at random based on the occurrence of precipitation, or whether, in a heterogeneous medium, the connections provide the lowest possible resistance through the optimal paths.

References

Ghanbarian, B., H. Daigle, A. G. Hunt, R. P. Ewing, and M. Sahimi, 2015, Gas and solute diffusion in partially saturated porous media: Comparison with Lattice-Boltzmann simulations, *J. Geophys. Res. Solid Earth*, doi:10.1002/2014JB011645.

Hunt, A. G., R. P. Ewing, and B. Ghanbarian, 2014, Percolation Theory for Flow in Porous Media, Lecture Notes in Physics, Springer, Berlin.

Sahimi, M. and S. Mukhopadhyay, 1996, Scaling properties of a percolation model with long-range correlations, *Phys. Rev. E* **54**: 3870, doi: 10.1103/PhysRevE.54.3870.

Sheppard, A.P., M.A. Knackstedt, W.V. Pinczewski, and M. Sahimi, 1999, Invasion percolation: new algorithms and universality classes, *J. Phys. A: Math. Gen.* **32**: L521–L529.