Nonlin. Processes Geophys. Discuss., 2, 97–114, 2015
www.nonlin-processes-geophys-discuss.net/2/97/2015/
doi:10.5194/npgd-2-97-2015

Nonlinear Processes in Geophysics
Discussions
Open Access

Nonstationary time series prediction combined with slow feature analysis

G. Wang[1] **and X. Chen**[1,2]

[1]Key Laboratory of Middle Atmosphere and Global Environment Observations, Institute of Atmospheric Physics, Chinese Academy of Sciences, Beijing 100029, China
[2]Pingtan Meteorological Bureau of Fujian Province, Pingtan 350400, China

Received: 13 October 2014 – Accepted: 7 January 2015 – Published: 26 January 2015

Correspondence to: G. Wang (wgl@mail.iap.ac.cn)

Published by Copernicus Publications on behalf of the European Geosciences Union & the American Geophysical Union.

Abstract

Almost all climate time series have some degree of nonstationarity due to external driving forces ~~perturbations of~~ the observed system. Therefore, these external driving forces should be taken into account when reconstructing the climate dynamics. This paper presents a new technique of ~~combining~~ the driving ~~force~~ of a time series ~~obtained using~~ the Slow Feature Analysis (SFA) approach, then introducing ~~the driving force~~ into a predictive model to predict non-stationary time series. ~~In essence, the main idea~~ of the technique is to consider the driving forces as state variables and incorporate them into the ~~prediction~~ model. ~~To test the method, experiments~~ using a modified logistic time series and winter ozone data in Arosa, Switzerland, were conducted. The results showed improved ~~and effective~~ prediction skill.

1 Introduction

~~Studies have addressed the fact that the essential behavior of~~ the climate system is non-stationary (Trenberth, 1990; Tsonis, 1996; Yang and Zhou, 2005; Boucharel et al., 2009). However, lacking of any general theory for predicting non-stationary processes has become one of the main barriers in climate prediction ~~theories~~. To unravel this issue, in recent years, increasing effort has been devoted to devising methods to analyze and predict nonstationary time series (e.g. Hegger et al., 2000; Verdes et al., 2000; Wan et al., 2005; Wang and Yang, 2005; Yang et al., 2010). ~~The basic idea used~~ in such studies was to remove or reduce the nonstationarity of the predicted system using some mathematical techniques, thereby improving the prediction.

~~In fact, the~~ nonstationarity ~~is generated because of~~ the fact that the driving forces ~~perturbations of~~ the observed system change with time (Manuca and Savit, 1996). Consequently, the most effective way to remove the nonstationarity may be to incorporate all the driving forces in the reconstructed dynamical system, ~~considering~~ them as the state variables of that system when establishing a prediction equation ~~with~~ general circulation model~~s~~ (GCM). Based on this principle, lately a ~~compatible modeling path to GCM given by data-driven technique~~ was proposed to predict several artificial non-stationary time series with known external forces ~~and~~ achieved success in improving predictions when driving forces were included in some ideal ~~or~~ climate systems, such as the Lorenz system, ~~a logistic model, or global temperature over seasonal timescales including~~ the North Atlantic Oscillation (NAO), the Pacific Decadal Oscillation (PDO), the El Niño/Southern Oscillation (ENSO), and the North Pacific Index (NPI) variability (Wang et. al., 2012, 2013). However, a ~~shortcoming~~ of this technique is that it ~~joins~~ the assumed driving forces ~~in~~ the predictive model. Therefore, in the present study we considered the extraction of driving forces from the time series itself and established a predictive model by incorporating the reconstructed driving forces. As a result, the extraction of driving forces became the ~~principal contribution~~.

~~In recent years, one of the techniques~~ called Slow Feature Analysis ~~by Wiskott (2003) for extracting driving forces has been presented, this~~ technique has been applied to ~~non-stationary~~ time series with some success (Wiskott, 2003; Berkes and Wiskott, 2005; Gunturkun, 2010; Konen and Koch, 2011). In this paper, ~~by using the Slow Feature Analysis (SFA) approach developed by Wiskott (2003),~~ we ~~reconstructed~~ the driving force ~~of~~ a ~~given~~ time series, and then established ~~predictive models~~ that incorporated the driving forces. ~~This~~ paper is organized as follows: a brief description of the predictive technique is presented in ~~the following section~~. In ~~Sect.~~ 3, results are reported from applying the approach to a modified logistic time series and the total ozone data of Arosa, Switzerland. A summary is provided in ~~Sect.~~ 4.

2 Methodology

SFA is a method ~~for extracting~~ slowly varying driving forces from a quickly varying non-stationary time series. In this section we provide ~~an introduction to the improvement of a predictive model, including~~ a brief overview of SFA and its ~~use in~~ the extraction and reconstruction of the driving force from the time series. Let us assume that we

have a single variable time series $\{x(t)\}_{t=1,2,\ldots,n}$ from a dynamical system~~, the principle of SFA is presented in Wiskott (2003), but the basic steps of the technique are also provided here for convenience and completeness~~:

1. Embed the above time series into an m-dimensional space (also named the length of the m window), a phase trajectory in the m-dimensional space denoted as

$$X(t) = \{x(t), x(t-1), \ldots, x(t-(m-1))\}_{t=1,\ldots,N} \quad \text{or}$$
$$X(t) = \{x_1(t), x_2(t), \ldots, x_m(t)\}_{t=1,\ldots,N} \tag{1}$$

where $N = n - m + 1$.

2. Generate an expanded signal $\mathbf{H}(t)$ for a quadratic expansion, all monomials of degree one and two including mixed terms are used:

$$\mathbf{H}(t) = \Big\{x_1(t), \ldots, x_m(t), x_1^2(t), \ldots, x_1(t)x_m(t), \ldots \\ \ldots x_{m-1}^2(t), x_{m-1}(t)x_m(t), x_m^2(t)\Big\}_{t=1,\ldots,N}, \tag{2}$$

where $\mathbf{H}(t)$ is an $k \times N$ matrix and $k = m + m\,(m+1)/2$.
To simplify Eq. (2) as

$$\mathbf{H}(t) = \{h_1(t), h_2(t) \ldots, h_k(t)\}_{t=1,\ldots,N}. \tag{3}$$

The general objective of SFA is to extract slowly varying features from the time series $\{x(t)\}_{t=1,2,\ldots n}$, in other words, to find a set of coefficients, $W^* = (w_1^*, w_2^*, \ldots, w_K^*)$, to make the output signal $y^*(t) = W^* \cdot \mathbf{H}(t)$ satisfy

$$\langle \dot{y}^* \dot{y}^{*T} \rangle = \min_k \left\{ \left\langle \dot{y}_k \dot{y}_k^{\mathrm{T}} \right\rangle \right\}. \tag{4}$$

Here, $\dot{y}_k$ is first-order derivative, calculated by $\Delta y_k(t_i) = y_k(t_{i+1}) - y_k(t_i)$.

3. Normalize the expanded signal $\mathbf{H}(t)$, by an affine transformation to generate $\mathbf{H}'(t)$ with zero mean and unit covariance matrix:

$$\mathbf{H}'(t) = \{h'_1(t), h'_2(t)\ldots, h'_k(t)\}t = 1,\ldots,N \tag{5}$$

Where $\overline{h}'_j = 0$, $h'_j h'^{\mathrm{T}}_j = 1$, $h'_j(t) = (h_j(t) - \overline{h}_j)/S$, and $S = \frac{1}{k}\sqrt{\sum_{j=1}^{k}\left(h_j(t) - \overline{h}\right)^2}$

4. By means of the Schmidt algorithm, the function space Eq. (5) is orthogonalized as

$$\begin{aligned} z_1(t) &= h'_1(t) \\ z_j(t) &= h'_j(t) - \sum_{i=1}^{j-1} \frac{h'_{i+1}(t)\cdot z_i(t)}{\|z_i\|} z_i(t) \quad (j = 2,\ldots,K) \end{aligned} \tag{6}$$

which is also denoted as $\mathbf{Z}(t) = \{z_1(t), z_2(t)\ldots, z_k(t)\}_{t=1,\ldots N}$. Here, $z_i(t)\cdot z_j(t) = 0(i \neq j)$ and it guarantees that every variable of the output is uncorrelated.

5. Establish the covariance matrix of $\mathbf{Z}(t)$, denoted as $\mathbf{B} = (\dot{\mathbf{Z}}\dot{\mathbf{Z}}^{\mathrm{T}})_{K\times K}$. The k eigenvectors with smallest eigenvalues, λ_K, yield the normalized weight vectors with $\lambda_1 \leq \lambda_2 \leq \ldots\ldots \leq \lambda_k$, which can be easily found by principle component analysis. The smallest eigenvalue, λ_1, corresponding to the eigenvector $\boldsymbol{W}_1$ can satisfy Eq. (4), which represents the weight coefficient of the slowest varying component. Here, $\boldsymbol{W}_1$ has a free scale factor (presented as r), and then the slowest varying variable, or the driving forcing, can be obtained by the following equation:

$$y_1(t) = r\boldsymbol{W}_1 \cdot \mathbf{Z}(t) + c, \tag{7}$$

Where c is a given constant and $\{y_1(t)\}$ is the output signal of the slowest driving force obtained by Eq. (7).

~~Above is the main principle for SFA, following we started a test with~~ a logistic map

$$s_{t+1} = \mu_t s_t(1 - s_t) \tag{8}$$

with a given driving force parameter

$$\mu_t = 3.5 - 0.45\cos(3\pi t/1600)\exp(-t/2500) \tag{9}$$

To test the ability ~~of reconstructing~~ the driving force from this modified logistic map, we took a time series ~~consisting~~ of 5000 data points from this map. ~~Utilizing~~ the SFA algorithm on this time series with the ~~embedding~~ dimension chosen as 3, we constructed the driving force shown in Fig. 1, in which the dotted line represents the true driving force given by Eq. (9) and the solid line the reconstructed driving force ~~with~~ the SFA approach. ~~As can be seen, both the true and constructed driving force fit very well, and the correlation coefficient between the true and extracted driving force reached~~ 0.998. This suggests that SFA was able to extract the driving force from the observed time series in an unsupervised manner.

~~So~~ far we have two time series, one is original time series $\{x(t)\}$, the ~~other one~~ is the slowest driving force $\{y_1(t)\}$. ~~next~~ we demonstrate how to establish a predictive model that includes the driving force reconstructed by the SFA procedure described above. We present the basic principle to build the prediction model, for convenience, we assume a ~~non-stationary~~ process ~~composed~~ of two series, $\{x(t)\}_{t=1,2,\ldots n}$ and $\{y_1(t)\}$, with the former being the state variable time series and the latter ~~for~~ the reconstructed external driving force obtained through the SFA approach. The two time series were embedded in an $m_1 + m_2$ dimensional phase space with a selected time lagτ. The reconstructed phase trajectory using the embedding theorem of Takens (1981) is shown as:

$$\begin{aligned} \boldsymbol{E}(t) = &\{x(t), x(t-\tau),\ldots, x(t-(m_1-1)\tau);\; y_1(t), y_1(t-\tau),\ldots, \\ &y_1(t-(m_2-1)\tau)\}_{t=1,2,\ldots,N} \end{aligned} \tag{10}$$

Here, m_1 and m_2 are the given embedding dimensions for $\{x(t)\}$ and $\{y_1(t)\}$, respectively, and $N = n - (\max(m_1, m_2) - 1)\tau$ is the number of phase points on the trajectory.

Based on this trajectory, a predictive model ~~to predict~~ the future state of the system can be established as:

$$x(t+p) = \hat{f}_p\left(\overline{x}(t); \overline{y}_1(t)\right) + \varepsilon(t) \qquad (11)$$

Where p is the prediction time step (considered as 1 in the present study), $\varepsilon(t)$ is the fitting error, and $\hat{f}$ is assumed to be a quadratic polynomial in this study. The Takens embedding theorem is appropriate ~~only~~ for an autonomous dynamical system, we followed the method of Stark (1999) to embed the driving forces in the same state space for a nonstationary system. The next task is to find the cost function $\eta = \sum_{t=1}^{N}[f(x(t), y_1(t)) - x(t+1)]^2$ when it reaches its minimum value. For more details, refer to the studies of Farmer and Sidorowich (1987) and Casdagli (1989).

3 Experiments

We applied the prediction technique ~~introduced~~ above to perform some prediction experiments using several given non-stationary time series. The ~~first~~ experiment was performed with data from the modified logistic model given above.

3.1 Prediction experiments for ideal time series

~~Subsequent~~ prediction experiments were based on 5000 data points from the above verified logistic map (Eq. 8) with the assumed driving force (Eq. 9). The ~~preceding~~ 4800 data points were applied to establish the predictive model, and the ~~subsequent~~ 200 data points were used to test the prediction and estimate the correlation coefficient between the actual and predicted values as a function of the prediction time step. The embedding dimension of the verified logistic time series, named m_1, took values from 2 to 3, and the embedding dimension of the driving force time series, named m_2, was

set to either 0 (the driving force was not taken into account, and is referred to as the "stationary model" hereinafter) or 1 (the driving force extracted from the verified logistic map by SFA was taken into account, and is referred to as the "forcing model" hereinafter). The time lag τ was always ~~taken to be~~ 1. Figure 2 shows the prediction skill with and without the influence of the driving force, which was ~~extracted with~~ the SFA approach. ~~As can be seen, the~~ forcing model excelled over the stationary model~~, for~~ the fourth prediction step, the correlation coefficients ~~was~~ below 0.2 in the stationary model ~~and still~~ above 0.6 in the forcing model~~, the~~ average correlation ~~over~~ the prediction time step was improved, indicating that introducing the driving force extracted through the SFA approach into the prediction model ~~improved~~ can yield ~~an obvious~~ improvement in ~~their~~ accuracy.

3.2 Prediction experiment for total ozone

Many studies have sought to explain the variables involved in ozone dynamics, such as the Quasi-Biennial Oscillation (QBO), the 11 year solar cycle, ~~and~~ volcanic eruptions, El Niño Southern Oscillation ENSO, North Atlantic Oscillation NAO (e.g., Brasseur and Granier, 1992; Hood, 1997; Schmidt et al., 2010; Rieder et al., 2010). In this paper we focus~~ed~~ on prediction experiments with total ozone data. The total ozone data were from Arosa, Switzerland, ~~which has~~ the world's longest total ozone record. Homogenized total ozone data from 1927 to 2007 were obtained from the World Ozone and Ultraviolet Radiation Data Centre (WOUDC; http://www.woudc.org).

By using the SFA technique on Arosa's daily total ozone data in winter (from January to March) for the period 1927 to 2007, we obtained the first output of the driving force $\{y_1\}$ when the embedding dimension was chosen as 3, 5, 7, 9, 11, respectively (shown in Fig. 3). Note that the result did not change significantly with different embedding dimension values.

We established a prediction model ~~introduced above~~ for winter ozone data by incorporating the driving force constructed by SFA. The prediction was based on 7305 data points. Out of the 7305 data points, the first 7125 data points were used to build the predictive model, and the ~~last~~ 180 data points were used to test the prediction using root-mean-square error (RMSE) and the correlation coefficient between ~~actual~~ and predicted values. The time lag τ was taken to be 1, the embedding dimension of the total ozone data m_1 took values from 3 to 5, and the embedding dimension of the driving force time series m_2 was set to either 0 for the stationary model or 3 to 5 for the forcing model.

The experimental results for this case are listed in Table 1, Figs. 4 and 5. From Table 1, it can be seen that all RMSE values given by the forcing model were much lower than those by the stationary model. Figure 4 presents the correlation coefficients between the ~~actual~~ and predicted values~~, the~~ forcing model ~~excelled over~~ the stationary model, especially ~~on~~ the first two steps. ~~For~~ the first prediction step, the correlation coefficients reached 0.61 ~~in~~ the stationary model but 0.91 ~~in~~ the forcing model. ~~For~~ the 8th prediction step, the correlation coefficients ~~reached~~ 0.39 ~~in~~ the stationary model, but still 0.45 ~~in~~ the forcing model. ~~For~~ the 12th prediction step, the correlation coefficients ~~reached 0.22 in the stationary model, but 0.33 in the forcing model~~. ~~Clearly,~~ when the ~~input of~~ the reconstructed driving force is introduced ~~prediction~~ is dramatically improved. The average correlation over the prediction time step~~ range~~ is improved 50 % when the driving force extracted through SFA technique is included. Figure 5 illustrates the error between the prediction and observation. ~~It can be seen that all the~~ prediction error~~s for the forcing model were lower than those~~ for the stationary model. All these results indicate that the inclusion of the driving force constructed by the SFA approach into the prediction model largely improve~~d~~ the predictive skill of winter total ozone in Arosa. ~~Some s~~ensitivity analysis with different training/verifying lengths do not alter ~~the conclusions~~.

4 Discussion

In this study, we first ~~re~~constructed the driving forces of a time series based on the SFA approach, and then the~~se~~ driving forces were introduced into a predictive model.

~~In~~ doing so, we extend the study ~~of~~ Wang et. al. (2012, 2013) and present a novel technique ~~by incorporating driving forces reconstructed by SFA~~ to predict non-stationary time series. Unlike the former works by Wang et. al. (2012, 2013)~~, whose driving forces were assumed,~~ in this study the driving force ~~were~~ extracted from original time series. The experimental results obtained from a modified logistic time series and winter ozone data in Arosa ~~illustrated our model's effectiveness~~.

The driving force ~~re~~construction technique based on SFA represents a progress for climate causal relations. Such an approach may provide a compatible and direct window for studying causality ~~with~~ external driving forces. We ~~re~~constructed the driving forces with SFA and then combined these driving forces to establish the predictive model. Although we found this approach able to effectively improve the predictive ability, ~~in essence~~ the ~~re~~constructed driving force ~~information is just a time series without any physical sense.~~ In order to understand the real background of ~~this string of~~ figures, one has to further ~~explain~~ the physical ~~sense~~ behind it. One recommended method, provided by Verdes (2005), suggests using a measure called "transfer entropy" to analyze the causality~~,~~ another recommended method is named "convergent cross mapping" provided by Sugihara et al. (2012), which measures causality in nonlinear dynamic systems. Work in this area is in progress and will be reported in future publications.

Acknowledgements. This research was supported by the National Natural Science Foundation of China under Grant 41275087.

References

Berkes, P. and Wiskott, L.: Slow feature analysis yields a rich repertoire of complex cell properties, J. Vision, 5, 579–602, 2005.

Boucharel, J., Dewitte, B., Garel, B., and du Penhoat, Y.: ENSO's non-stationary and non-Gaussian character: the role of climate shifts, Nonlin. Processes Geophys., 16, 453–473, doi:10.5194/npg-16-453-2009, 2009.

Brasseur, G. and Granier, C.: Mount Pinatubo aerosols, chlorofluorocarbons and ozone depletion, Science, 257, 1239–1242, 1992.

Casdagli, M.: Nonlinear prediction of chaotic time series, Phys. D., 35, 335–356, 1989.

Farmer, J. D. and Sidorowich, J.: Predicting chaotic time series, Phys. Rev. E., 59, 845–848, 1989.

Gunturkun, U.: Sequential reconstruction of driving-forces from nonlinear nonstationary dynamics, Phys. D., 239, 1095–1107, 2010.

Hegger, R., Kantz, H., Matassini, L., and Schreiber, T.: Coping with non-stationarity by overembedding, Phys. Rev. E., 84, 4092–4101, 2000.

Hood, L.: The solar cycle variation of total ozone: dynamical forcing in the lower stratosphere, J. Geophys. Res., 102, 1355–1370, 1997.

Konen, W. and Koch, P.: The slowness principle: SFA can detect different slow components in non-stationary time series, Int. J. Innovative Computing and Applications, 3, 3–10, 2011.

Manuca, R. and Savit, R.: Stationarity and nonstationarity in time series analysis, Phys. D, 99, 134–161, 1996.

Rieder, H. E., Staehelin, J., Maeder, J. A., Peter, T., Ribatet, M., Davison, A. C., Stübi, R., Weihs, P., and Holawe, F.: Extreme events in total ozone over Arosa – Part 2: Fingerprints of atmospheric dynamics and chemistry and effects on mean values and long-term changes, Atmos. Chem. Phys., 10, 10033–10045, doi:10.5194/acp-10-10033-2010, 2010.

Schmidt, H., Brasseur, G., and Giorgetta, M.,: Solar cycle signal in a general circulation and chemistry model with internally generated quasi-biennial oscillation, J Geophys Res., 115, D00I14, doi:10.1029/2009JD012542, 2010.

Stark, J.: Delay embeddings for forced systems: deterministic forcing, J. Nonlinear Sci., 9, 255–332,1999.

Sugihara, G., May, R., Ye, H., Hsieh, C., Deyle, E., Fogarty, M., and Munch, S.: Detecting causality in complex ecosystems, Science, 338, 496–500, doi:10.1126/Science.1227079, 2012.

Takens, F.: Detecting Strange Attractors in Turbulence, Dynamical Systems and Turbulence, Heidelberg, Springer-Verlag, 366–381, 1981.

Trenberth, K. E.: Recent observed inter-decadal climate changes in the Northern Hemisphere, B. Am. Meteorol. Soc., 7, 988–993, 1990.

Tsonis, A. A.: Widespread increases in low-frequency variability of precipitation over the past century, Nature, 382, 700–702, 1996.

Verdes, P. F.: Assessing causality from multivariate time series, Phys. Rev. E., 72, 026222, doi:10.1103/PhysRevE.72.026222, 2005.

Verdes, P. F., Parodi, M. A., Granitto, P. M., Navone, H. D., Piacentini, R. D., and Ceccatto, H. A.: Predictions of the maximum amplitude for solar cycle 23 and its subsequent behavior using nonlinear methods, Sol. Phys., 191, 419–425, 2000.

Wan, S., Feng, G., Zhou, G., Wan, B., Qin, M., and Xu, X.: Extracting useful information from the observations for the prediction based on EMD method, Acta Meteorol. Sin., 63, 516–525, 2005.

Wang, G. and Yang, P.: A compound reconstructed prediction model for nonstationary climate process, Int. J. Climatol., 25, 1265–1277, 2005.

Wang, G., Yang, P., Zhou, X., Swanson, K., and Tsonis, A.: Directional influences on global temperature prediction, Geophys. Res. Lett., 39, L13704, doi:10.1029/2012GL052149, 2012.

Wang, G., Yang, P., and Zhou, X.: Nonstationary time series prediction by incorporating external forces, Adv. Atmos. Sci., 30, 1601–1607, 2013.

Wiskott, L.: Estimating driving forces of nonstationary time series with slow feature analysis, arXiv.org e-Print archive, available at: http://arxiv.org/abs/cond-mat/0312317/, last access: 12 December 2003.

Yang, P. and Zhou, X.: On nonstationary behaviors and prediction theory of climate systems, Acta Meteorol. Sin., 63, 556–570, 2005

Yang, P., Wang, G., Bian, J., and Zhou, X.: The prediction of non-stationary climate series based on empirical mode decomposition, Adv. Atmos. Sci., 27, 845–854, doi:10.1007/s00376-009-9128-x, 2010.

Discussion Paper | Discussion Paper | Discussion Paper | Discussion Paper |

Table 1. RMSE comparison of the prediction experiments (unit: Dobson units).

	1	2	3	4	5	6	7	8	9	10
Stationary model	0.80	0.88	0.90	0.94	0.96	0.99	1.03	1.02	1.04	1.05
Forcing model	0.62	0.55	0.62	0.74	0.87	0.93	0.97	0.98	1.01	1.01

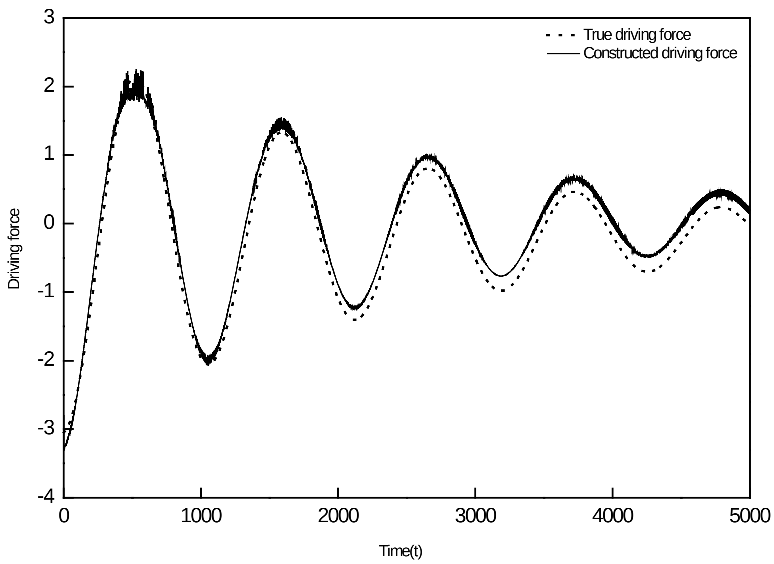

Figure 1. The true and ~~re~~constructed driving force.

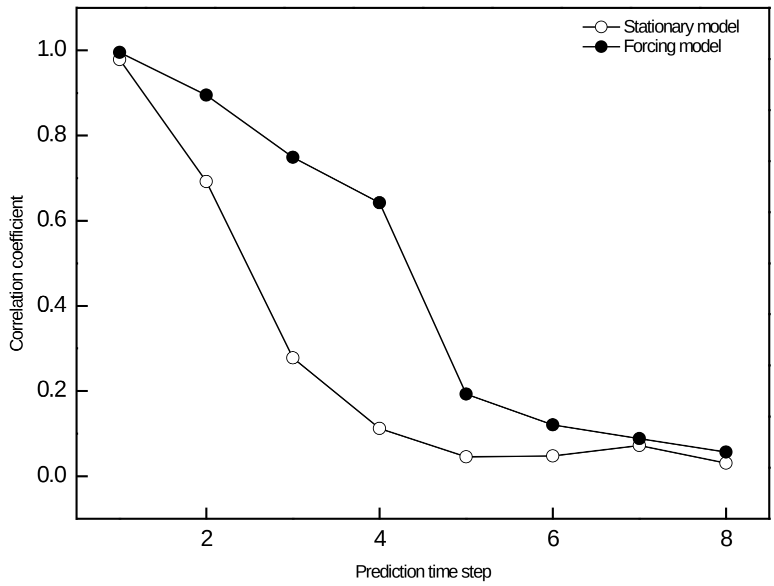

Figure 2. ~~Prediction skill comparison~~ combined with or without driving force.

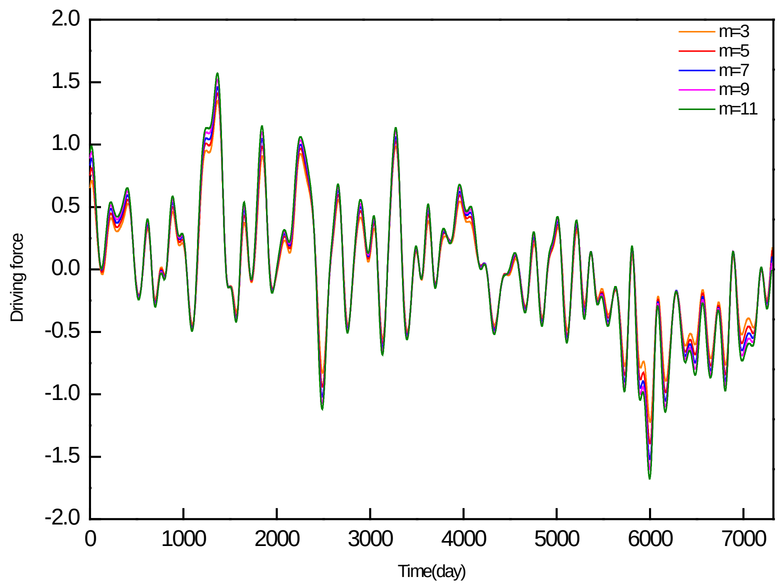

Figure 3. The slowest driving force with different embedding dimension for total ozone data.

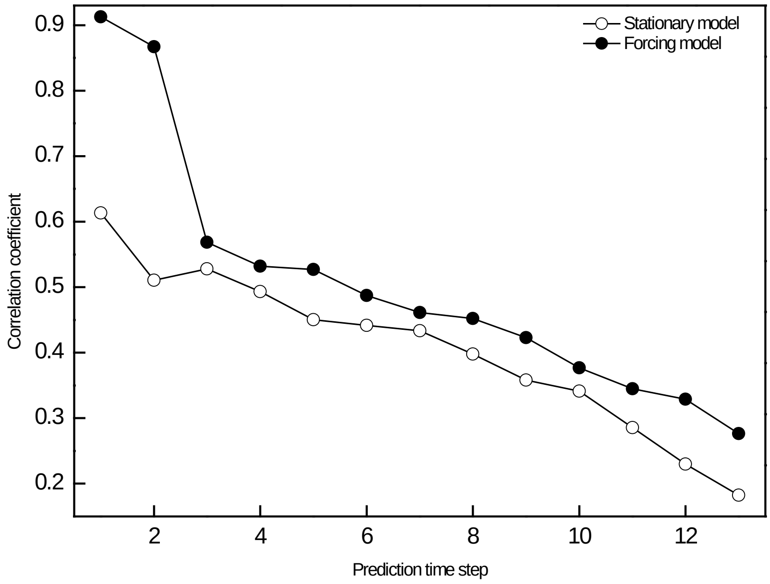

Figure 4. ~~Prediction skill comparison~~ combined with or without driving force.

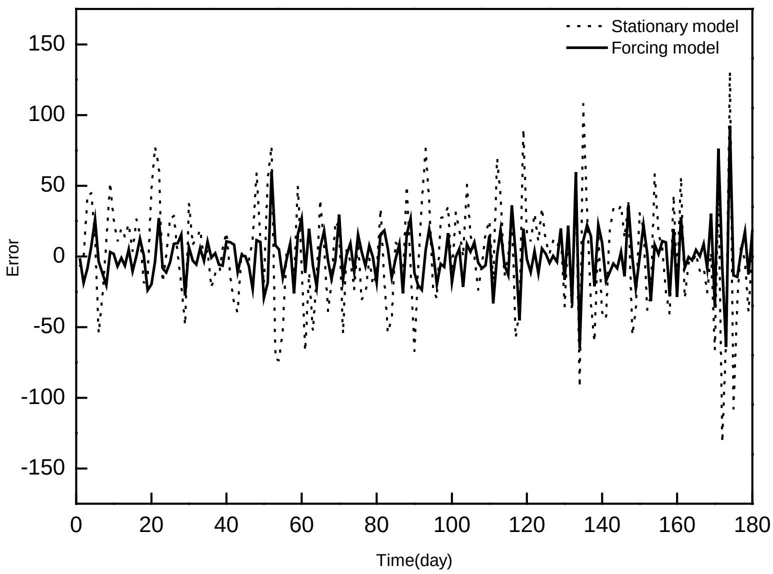

Figure 5. Error (Dobson Units) at prediction steps with or without forcing input.