

Interactive comment on “Insight into earthquake sequencing: analysis and interpretation of time-series constructed from the directed graph of the Markov chain model” by M. S. Cavers and K. Vasudevan

Anonymous Referee #1

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The paper can be considered an advancement of the previous paper by the same authors published in Pageoph (172, 225, 2015). The novelty with respect to that one is the nonlinear analysis of the state-to-state transition frequency of the modified Markov chain model of earthquake sequencing shown in Fig. 2a. The paper needs to be revised taking into account the following issues: 1) Since the $\Delta T = 9$ days, it would be useful to add the sampling time (9 days) in the x-axis title of Fig. 2a.

We will add the sampling time into the x-axis title of Figure 2a of the revised manuscript.

2) At page 9: the authors say that they add a white noise to time-series of state-to-state transition frequencies of the modified Markov chain model. Actually the term “white” refers to the spectral properties of the noise. But the statistical distribution could be of different types, generally uniform or Gaussian. So, what type of statistical distribution is the white noise considered by the authors?.

The white noise considered in this paper is Gaussian. We will state this clearly in the revised manuscript.

3) The authors show Fig.2a and Fig. 2b. The two figures furnish different representation of the same Markov chain. The first can be considered “dynamic”, because it shows the time evolution of the transition from one state to another in consecutive time intervals of 9 days each. The second can be considered “static” because it shows the transition probabilities from one state to another but considering the whole earthquake sequence occurred during the whole observation period. We could also say that the first representation is local, the second one is global. Are the two representations equivalent? How is it possible to derive one from the other?

They are not equivalent. We can go from the time-series data to transition-frequency matrix (see page 7 for details). However, we cannot go from the time-frequency matrix to time-series without additional information such as the catalogue and the record-breaking statistics of recurrences.

4) There is an error in the denominator of the first formula of Eq. 2, η_i and not η_{ij}

We will correct this in the revised manuscript.

5) No comment is given on the different IMFs. They are only shown but no features of each IMF component are described and highlighted.

We will highlight features of the different IMFs in the revised manuscript and also, attempt to provide an interpretation of them.

6) I do not understand the transition probability matrix associated to (or derived from) the IMFs, since these last assume positive and negative values. If the state-to-state transition frequencies represent the total weights (which are positive) and to this time series the transition probability matrix is associated, what does exactly mean the negative values of IMFs in the context of the weights of the Markov chain? And what is the transition probability matrix associated with a function that assumes positive but also negative values?

The referee makes an excellent point. The problem could be a scaling issue. We are currently thinking this over.

7) Figure 4 is not clear. The HHT is for the time series shown in Fig. 2a? Why the time axis is in seconds and not days? Why the y-axis is in Hz? The comments given are not satisfactorily and seem quite vague. I would see, instead, a periodic trend at low frequencies (after they have clarified which frequency is actually involved in the HHT) and a higher power at 900 and 950 interval (colored in red) that needs to be explained.

We'll correct the annotation in the revised manuscript.

8) At page 12, the authors say: "An expression similar to Eq. (7) can be derived if we know the optimal time-interval for the Markov chain model. Since we know the optimal time-interval, we introduce a sequence of state-to-state transition frequencies, with $N_{(sstf,k)}(\tau)$ referring to the number of state-to-state transitions over the k th window for the optimal time-interval, as is shown in Fig. 5e" Is that optimal time-interval 9 days?

This should be clearly stated. The sketch shown in Fig. 5e to explain the way to obtain the $N_{(sstf,k)}(\tau)$ from the earthquake sequence is not clear. The sketch should explain step by step how to pass from an earthquake sequence to the sequence of the state-to-state transition frequencies. I mean, if the derivation of Fig. 5d from Fig. 5b is clear, not clear is that from Fig. 5b to Fig. 5e.

We'll create a new Figure 5e and relabel the current Figure 5e as Figure 5f. We 'll add details in the text of the revised manuscript.

9) At page 12, the authors say: "First, $N_{(sstf,k)}(\tau)$ is not necessarily an integer number for any k th window". This sentence seems in contradiction with what the authors said few lines before "with $N_{(sstf,k)}(\tau)$ referring to the number of state-to-state transitions". In this last sentence the word "number" is generally understood by the reader as an integer number with a specific cardinality. Furthermore, the sketch provided by the authors in Fig. 5e represents the $N_{(sstf,k)}(\tau)$ as integer number. So, it is necessary an explanation and clarification. Actually it is not clear what is the timing of this new sequence.

We plan to modify figure captions and text to incorporate the referee's comments.

10) At page 12, the authors say: "It is the weight associated with the edge of the directed graph that plays an important role. Since we have used a modified Markov chain model which includes the influence of the event recurrences in the record-breaking sense, the above expression includes their weights as well in the computation of $N_{sstf(k)}$ " First of all, there should be an error in the notation, because few lines before for the same quantity the symbol used was $N_{(sstf,k)}(\tau)$. Then, it is not clear how the weights are included

within the computation of the number of state-to-state transition frequencies. More explications (with visual examples) is necessary at this point.

We thank the referee in pointing this mistake to us and we stand corrected in the revised manuscript.

11) The x-axis of Fig. 6 shows the “Dyadic exponent”: but, shouldn’t it be the counting time tau? And in this case, what is the measure unit of tau?

We will modify the figure caption and explain the changes in the text of the revised manuscript.

12) The calculation of the Allan factor and Fano factor as done in the papers by Telesca et al. on the analysed global earthquake catalogue is necessary to see if any relationship could be stated with the results obtained with the new formulation proposed by the authors.

We’ll generate a new figure and incorporate it the revised manuscript.