



Nonstationary time series prediction

G. Wang and X. Chen

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Nonstationary time series prediction combined with slow feature analysis

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Abstract

Almost all climate time series have some degree of nonstationarity due to external driving forces perturbations of the observed system. Therefore, these external driving forces should be taken into account when reconstructing the climate dynamics. This paper presents a new technique of combining the driving force of a time series obtained using the Slow Feature Analysis (SFA) approach, then introducing the driving force into a predictive model to predict non-stationary time series. In essence, the main idea of the technique is to consider the driving forces as state variables and incorporate them into the prediction model. To test the method, experiments using a modified logistic time series and winter ozone data in Arosa, Switzerland, were conducted. The results showed improved and effective prediction skill.

1 Introduction

Studies have addressed the fact that the essential behavior of the climate system is non-stationary (Trenberth, 1990; Tsonis, 1996; Yang and Zhou, 2005; Boucharel et al., 2009). However, lacking of any general theory for predicting non-stationary processes has become one of the main barriers in climate prediction theories. To unravel this issue, in recent years, increasing effort has been devoted to devising methods to analyze and predict nonstationary time series (e.g. Hegger et al., 2000; Verdes et al., 2000; Wan et al., 2005; Wang and Yang, 2005; Yang et al., 2010). The basic idea used in such studies was to remove or reduce the nonstationarity of the predicted system using some mathematical techniques, thereby improving the prediction.

In fact, the nonstationarity is generated because of the fact that the driving forces perturbations of the observed system change with time (Manuca and Savit, 1996). Consequently, the most effective way to remove the nonstationarity may be to incorporate all the driving forces in the reconstructed dynamical system, considering them as the state variables of that system when establishing a prediction equation with general

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circulation models (GCM). Based on this principle, lately a compatible modeling path to GCM given by data-driven technique was proposed to predict several artificial non-stationary time series with known external forces and achieved success in improving predictions when driving forces were included in some ideal or climate systems, such as the Lorenz system, a logistic model, or global temperature over seasonal timescales including the North Atlantic Oscillation (NAO), the Pacific Decadal Oscillation (PDO), the El Niño/Southern Oscillation (ENSO), and the North Pacific Index (NPI) variability (Wang et. al., 2012, 2013). However, a shortcoming of this technique is that it joins the assumed driving forces in the predictive model. Therefore, in the present study we considered the extraction of driving forces from the time series itself and established a predictive model by incorporating the reconstructed driving forces. As a result, the extraction of driving forces became the principal contradiction.

In recent years, one of the techniques called Slow Feature Analysis by Wiskott (2003) for extracting driving forces has been presented, this technique has been applied to non-stationary time series with some success (Wiskott, 2003; Berkes and Wiskott, 2005; Gunturkun, 2010; Konen and Koch, 2011). In this paper, by using the Slow Feature Analysis (SFA) approach developed by Wiskott (2003), we reconstructed the driving force of a given time series, and then established predictive models that incorporated the driving forces. This paper is organized as follows: a brief description of the predictive technique is presented in the following section. In Sect. 3, results are reported from applying the approach to a modified logistic time series and the total ozone data of Arosa, Switzerland. A summary is provided in Sect. 4.

2 Methodology

SFA is a method for extracting slowly varying driving forces from a quickly varying non-stationary time series. In this section we provide an introduction to the improvement of a predictive model, including a brief overview of SFA and its use in the extraction and reconstruction of the driving force from the time series. Let us assume that we

have a single variable time series $\{x(t)\}_{t=1,2,\dots,n}$ from a dynamical system, the principle of SFA is presented in Wiskott (2003), but the basic steps of the technique are also provided here for convenience and completeness:

1. Embed the above time series into an m -dimensional space (also named the length of the m window), a phase trajectory in the m -dimensional space denoted as

$$\begin{aligned} X(t) &= \{x(t), x(t-1), \dots, x(t-(m-1))\}_{t=1,\dots,N} \quad \text{or} \\ X(t) &= \{x_1(t), x_2(t), \dots, x_m(t)\}_{t=1,\dots,N} \end{aligned} \quad (1)$$

where $N = n - m + 1$.

2. Generate an expanded signal $\mathbf{H}(t)$ for a quadratic expansion, all monomials of degree one and two including mixed terms are used:

$$\begin{aligned} \mathbf{H}(t) &= \left\{ x_1(t), \dots, x_m(t), x_1^2(t), \dots, x_1(t)x_m(t), \dots \right. \\ &\quad \left. \dots, x_{m-1}^2(t), x_{m-1}(t)x_m(t), x_m^2(t) \right\}_{t=1,\dots,N}, \end{aligned} \quad (2)$$

where $\mathbf{H}(t)$ is an $k \times N$ matrix and $k = m + m(m+1)/2$.

To simplify Eq. (2) as

$$\mathbf{H}(t) = \{h_1(t), h_2(t), \dots, h_k(t)\}_{t=1,\dots,N}. \quad (3)$$

The general objective of SFA is to extract slowly varying features from the time series $\{x(t)\}_{t=1,2,\dots,n}$, in other words, to find a set of coefficients, $\mathbf{W}^* = (w_1^*, w_2^*, \dots, w_k^*)$, to make the output signal $y^*(t) = \mathbf{W}^* \cdot \mathbf{H}(t)$ satisfy

$$(\dot{y}^* \dot{y}^{*T}) = \min_k \left\{ \left(\dot{y}_k \dot{y}_k^T \right) \right\}. \quad (4)$$

Here, \dot{y}_k is first-order derivative, calculated by $\Delta y_k(t_i) = y_k(t_{i+1}) - y_k(t_i)$.

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Above is the main principle for SFA, following we started a test with a logistic map

$$s_{t+1} = \mu_t s_t (1 - s_t) \quad (8)$$

with a given driving force parameter

$$\mu_t = 3.5 - 0.45 \cos(3\pi t / 1600) \exp(-t / 2500) \quad (9)$$

5 To test the ability of reconstructing the driving force from this modified logistic map, we took a time series consisting of 5000 data points from this map. Utilizing the SFA algorithm on this time series with the embedding dimension chosen as 3, we constructed the driving force shown in Fig. 1, in which the dotted line represents the true driving force given by Eq. (9) and the solid line the reconstructed driving force with the SFA approach. As can be seen, both the true and constructed driving force fit very well, and the correlation coefficient between the true and extracted driving force reached 0.998. This suggests that SFA was able to extract the driving force from the observed time series in an unsupervised manner.

15 So far we have two time series, one is original time series $\{x(t)\}$, the other one is the slowest driving force $\{y_1(t)\}$, next we demonstrate how to establish a predictive model that includes the driving force reconstructed by the SFA procedure described above. We present the basic principle to build the prediction model, for convenience, we assume a non-stationary process composed of two series, $\{x(t)\}_{t=1,2,\dots,n}$ and $\{y_1(t)\}$, with the former being the state variable time series and the latter for the reconstructed external driving force obtained through the SFA approach. The two time series were embedded in an $m_1 + m_2$ dimensional phase space with a selected time lag τ . The reconstructed phase trajectory using the embedding theorem of Takens (1981) is shown as:

$$E(t) = \{x(t), x(t - \tau), \dots, x(t - (m_1 - 1)\tau); y_1(t), y_1(t - \tau), \dots, y_1(t - (m_2 - 1)\tau)\}_{t=1,2,\dots,N} \quad (10)$$

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Here, m_1 and m_2 are the given embedding dimensions for $\{x(t)\}$ and $\{y_1(t)\}$, respectively, and $N = n - (\max(m_1, m_2) - 1)\tau$ is the number of phase points on the trajectory.

Based on this trajectory, a predictive model to predict the future state of the system can be established as:

$$x(t + \rho) = \hat{f}_\rho(\bar{x}(t); \bar{y}_1(t)) + \varepsilon(t) \quad (11)$$

Where ρ is the prediction time step (considered as 1 in the present study), $\varepsilon(t)$ is the fitting error, and \hat{f} is assumed to be a quadratic polynomial in this study. The Takens embedding theorem is appropriate only for an autonomous dynamical system, we followed the method of Stark (1999) to embed the driving forces in the same state space for a nonstationary system. The next task is to find the cost function $\eta = \sum_{t=1}^N [f(x(t), y_1(t)) - x(t + 1)]^2$ when it reaches its minimum value. For more details, refer to the studies of Farmer and Sidorowich (1987) and Casdagli (1989).

3 Experiments

We applied the prediction technique introduced above to perform some prediction experiments using several given non-stationary time series. The first experiment was performed with data from the modified logistic model given above.

3.1 Prediction experiments for ideal time series

Subsequent prediction experiments were based on 5000 data points from the above verified logistic map (Eq. 8) with the assumed driving force (Eq. 9). The preceding 4800 data points were applied to establish the predictive model, and the subsequent 200 data points were used to test the prediction and estimate the correlation coefficient between the actual and predicted values as a function of the prediction time step. The embedding dimension of the verified logistic time series, named m_1 , took values from 2 to 3, and the embedding dimension of the driving force time series, named m_2 , was

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set to either 0 (the driving force was not taken into account, and is referred to as the “stationary model” hereinafter) or 1 (the driving force extracted from the verified logistic map by SFA was taken into account, and is referred to as the “forcing model” hereinafter). The time lag τ was always taken to be 1. Figure 2 shows the prediction skill with and without the influence of the driving force, which was extracted with the SFA approach. As can be seen, the forcing model excelled over the stationary model, for the fourth prediction step, the correlation coefficients was below 0.2 in the stationary model and still above 0.6 in the forcing model, the average correlation over the prediction time step was improved, indicating that introducing the driving force extracted through the SFA approach into the prediction model improved can yield an obvious improvement in their accuracy.

3.2 Prediction experiment for total ozone

Many studies have sought to explain the variables involved in ozone dynamics, such as the Quasi-Biennial Oscillation (QBO), the 11 year solar cycle, and volcanic eruptions, El Niño Southern Oscillation ENSO, North Atlantic Oscillation NAO (e.g., Brasseur and Granier, 1992; Hood, 1997; Schmidt et al., 2010; Rieder et al., 2010). In this paper we focused on prediction experiments with total ozone data. The total ozone data were from Arosa, Switzerland, which has the world’s longest total ozone record. Homogenized total ozone data from 1927 to 2007 were obtained from the World Ozone and Ultraviolet Radiation Data Centre (WOUDC; <http://www.woudc.org>).

By using the SFA technique on Arosa’s daily total ozone data in winter (from January to March) for the period 1927 to 2007, we obtained the first output of the driving force $\{y_1\}$ when the embedding dimension was chosen as 3, 5, 7, 9, 11, respectively (shown in Fig. 3). Note that the result did not change significantly with different embedding dimension values.

We established a prediction model introduced above for winter ozone data by incorporating the driving force constructed by SFA. The prediction was based on 7305 data points. Out of the 7305 data points, the first 7125 data points were used to build the

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predictive model, and the last 180 data points were used to test the prediction using root-mean-square error (RMSE) and the correlation coefficient between actual and predicted values. The time lag τ was taken to be 1, the embedding dimension of the total ozone data m_1 took values from 3 to 5, and the embedding dimension of the driving force time series m_2 was set to either 0 for the stationary model or 3 to 5 for the forcing model.

The experimental results for this case are listed in Table 1, Figs. 4 and 5. From Table 1, it can be seen that all RMSE values given by the forcing model were much lower than those by the stationary model. Figure 4 presents the correlation coefficients between the actual and predicted values, the forcing model excelled over the stationary model, especially on the first two steps. For the first prediction step, the correlation coefficients reached 0.61 in the stationary model but 0.91 in the forcing model. For the 8th prediction step, the correlation coefficients reached 0.39 in the stationary model, but still 0.45 in the forcing model. For the 12th prediction step, the correlation coefficients reached 0.22 in the stationary model, but 0.33 in the forcing model. Clearly, when the input of the reconstructed driving force is introduced prediction is dramatically improved. The average correlation over the prediction time step range is improved 50 % when the driving force extracted through SFA technique is included. Figure 5 illustrates the error between the prediction and observation. It can be seen that all the prediction errors for the forcing model were lower than those for the stationary model. All these results indicate that the inclusion of the driving force constructed by the SFA approach into the prediction model largely improved the predictive skill of winter total ozone in Arosa. Some sensitivity analysis with different training/verifying lengths do not alter the conclusions.

4 Discussion

In this study, we first reconstructed the driving forces of a time series based on the SFA approach, and then these driving forces were introduced into a predictive model.

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In doing so, we extend the study of Wang et. al. (2012, 2013) and present a novel technique by incorporating driving forces reconstructed by SFA to predict non-stationary time series. Unlike the former works by Wang et. al. (2012, 2013), whose driving forces were assumed, in this study the driving force were extracted from original time series.

The experimental results obtained from a modified logistic time series and winter ozone data in Arosa illustrated our model's effectiveness.

The driving force reconstruction technique based on SFA represents a progress for climate causal relations. Such an approach may provide a compatible and direct window for studying causality with external driving forces. We reconstructed the driving forces with SFA and then combined these driving forces to establish the predictive model. Although we found this approach able to effectively improve the predictive ability, in essence the reconstructed driving force information is just a time series without any physical sense. In order to understand the real background of this string of figures, one has to further explain the physical sense behind it. One recommended method, provided by Verdes (2005), suggests using a measure called "transfer entropy" to analyze the causality, another recommended method is named "convergent cross mapping" provided by Sugihara et al. (2012), which measures causality in non-linear dynamic systems. Work in this area is in progress and will be reported in future publications.

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Table 1. RMSE comparison of the prediction experiments (unit: Dobson units).

	1	2	3	4	5	6	7	8	9	10
Stationary model	0.80	0.88	0.90	0.94	0.96	0.99	1.03	1.02	1.04	1.05
Forcing model	0.62	0.55	0.62	0.74	0.87	0.93	0.97	0.98	1.01	1.01

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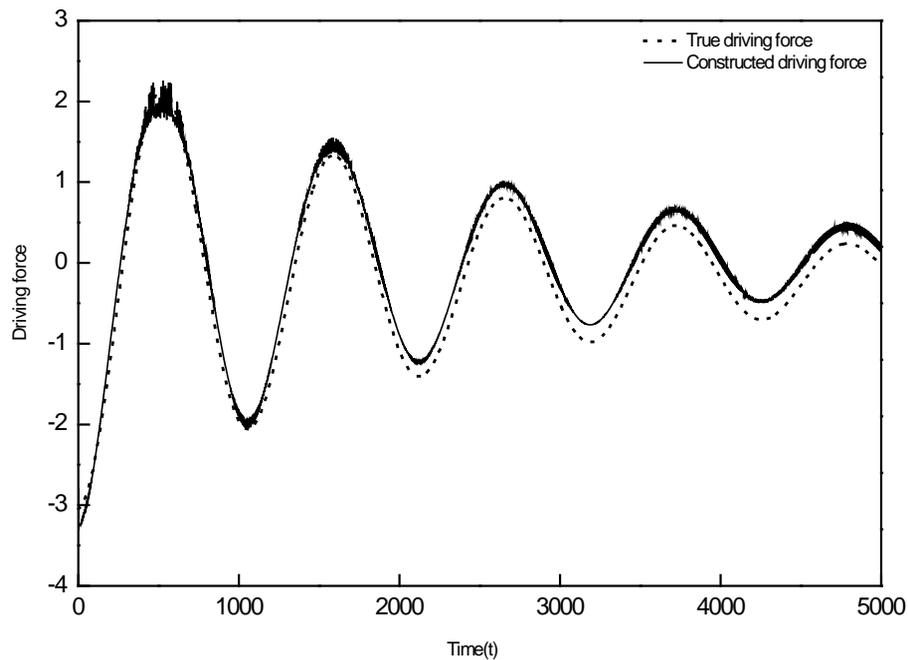
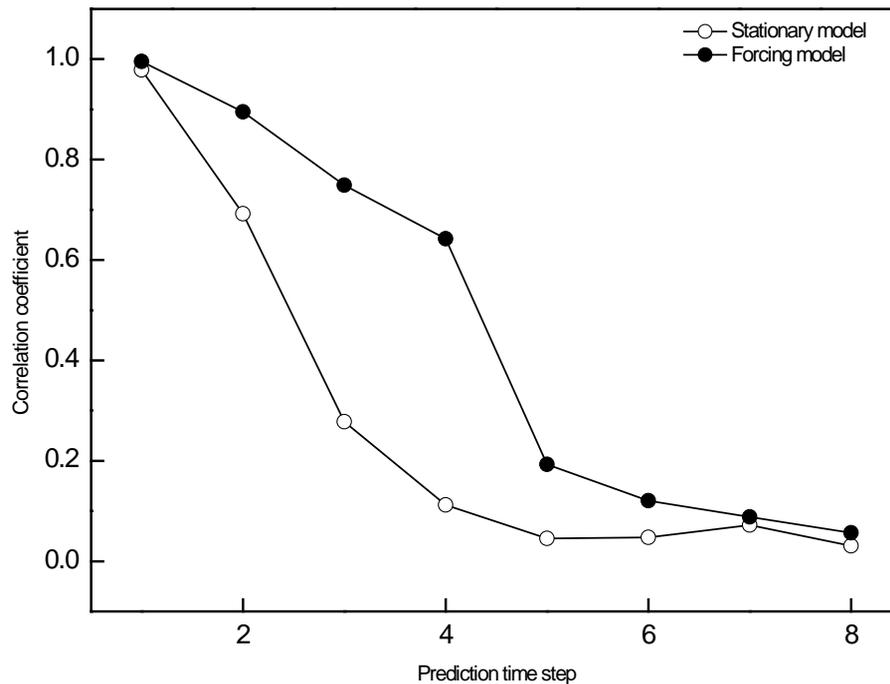


Figure 1. The true and reconstructed driving force.

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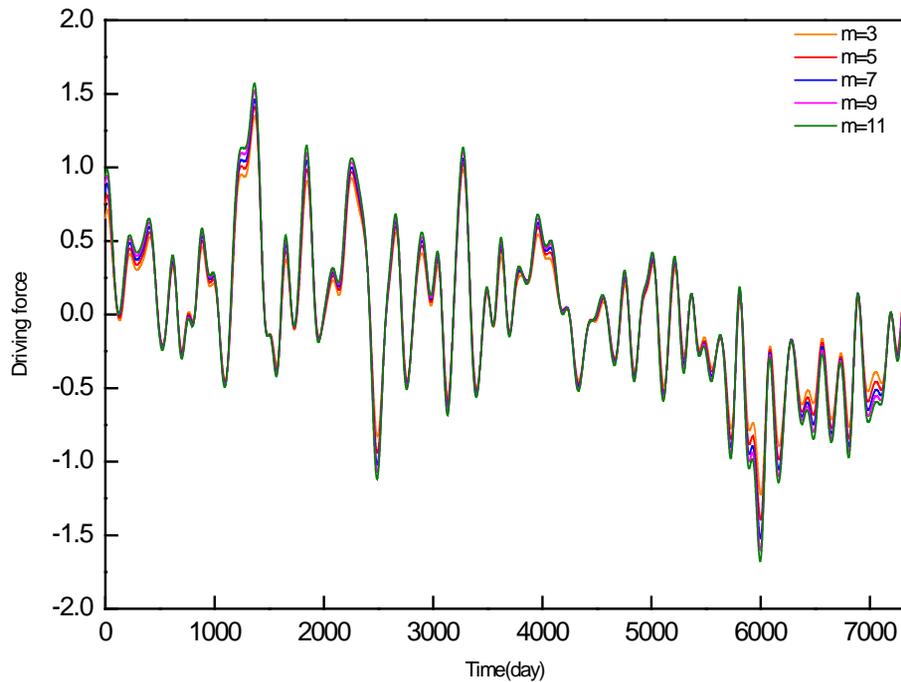
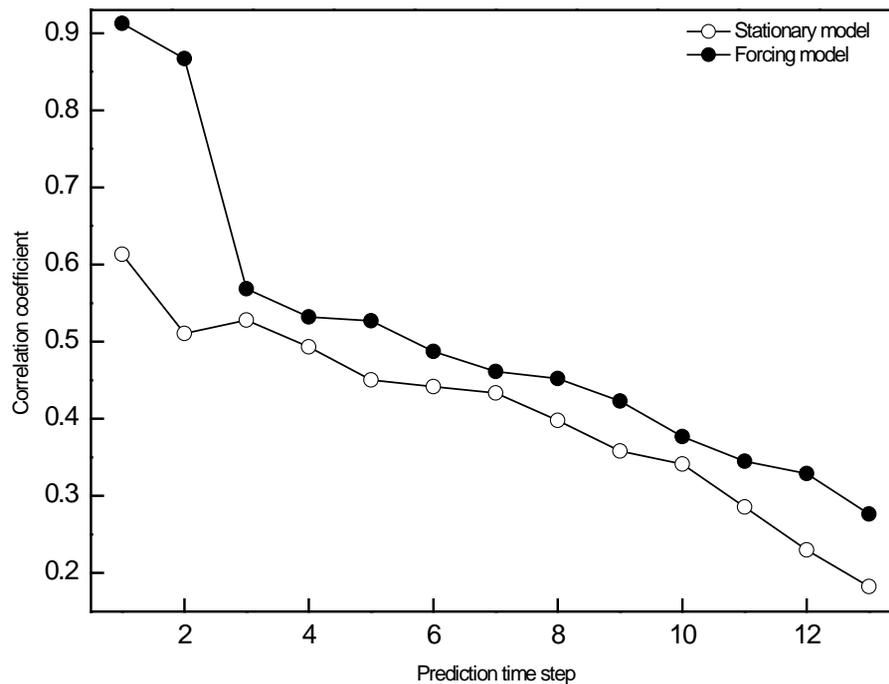


Figure 3. The slowest driving force with different embedding dimension for total ozone data.

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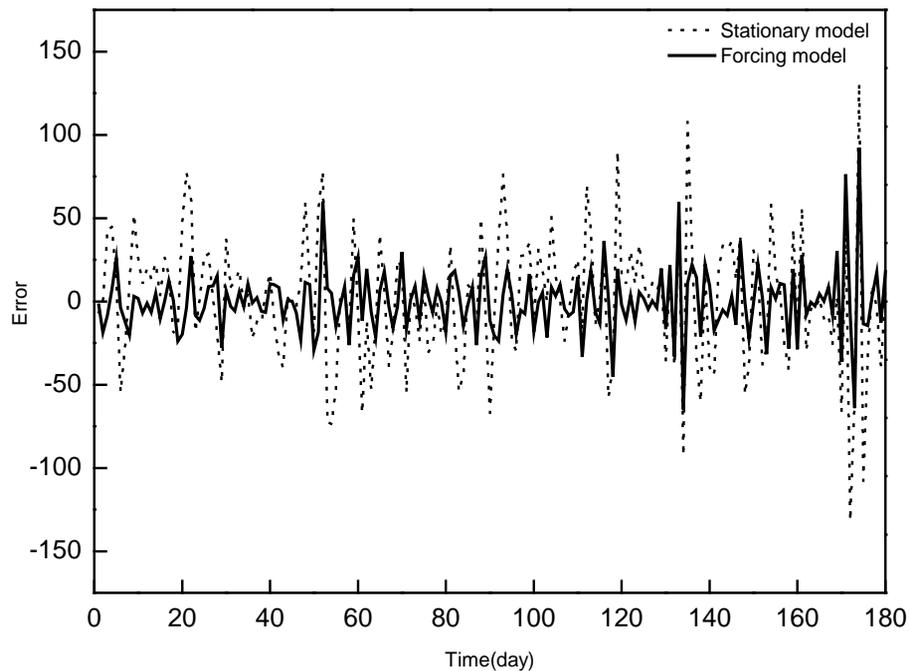


Figure 5. Error (Dobson Units) at prediction steps with or without forcing input.

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