



Sequential Bayesian approach for ice-core dating

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A sequential Bayesian approach for the estimation of the age–depth relationship of Dome Fuji ice core

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Abstract

A technique for estimating the age–depth relationship in an ice core and evaluating its uncertainty is presented. The age–depth relationship is mainly determined by the accumulation of snow at the site of the ice core and the thinning process due to the horizontal stretching and vertical compression of ice layers. However, since neither the accumulation process nor the thinning process are fully understood, it is essential to incorporate observational information into a model that describes the accumulation and thinning processes. In the proposed technique, the age as a function of depth is estimated from age markers and $\delta^{18}\text{O}$ data. The estimation is achieved using the particle Markov chain Monte Carlo (PMCMC) method, in which the sequential Monte Carlo (SMC) method is combined with the Markov chain Monte Carlo method. In this hybrid method, the posterior distributions for the parameters in the models for the accumulation and thinning processes are computed using the Metropolis method, in which the likelihood is obtained with the SMC method. Meanwhile, the posterior distribution for the age as a function of depth is obtained by collecting the samples generated by the SMC method with Metropolis iterations. The use of this PMCMC method enables us to estimate the age–depth relationship without assuming either linearity or Gaussianity. The performance of the proposed technique is demonstrated by applying it to ice core data from Dome Fuji in Antarctica.

1 Introduction

Ice cores provide vital information on the climatic and environmental changes over the past hundreds of thousands of years. It is thus crucial to obtain an accurate estimate of the relationship between age and depth in the ice cores. For determining the age–depth relationship, dating methods based on glaciological modeling are widely used. However, since the glaciological processes controlling this relationship are not fully known, it is essential to reduce uncertainty by incorporating various types of observa-

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tional information into the glaciological model. The Bayesian approach is a powerful way to combine observational information with a model, and it has been applied to the dating of ice cores in a number of studies. Parrenin et al. (2007) attributed the uncertainties in the estimates to those in the parameterization of the dating model and used a Bayesian approach for resolving these uncertainties. Klauenberg et al. (2011) took a Bayesian approach to estimate the accumulation and some parameters from $\delta^{18}\text{O}(z)$ data and to evaluate the uncertainty of the estimate.

In the dating of an ice core, age markers offer significant constraints on the age–depth relationship. In order to effectively make use of age markers, it is essential to ensure the consistency of the estimated age within the whole ice core, and it is thus necessary to simultaneously consider a large number of variables to represent the age–depth relationship for the entire ice core. Hence, the Bayesian estimation of the age–depth relationship becomes a high-dimensional problem. Some existing methods handle this high-dimensionality by assuming Gaussianity. Dreyfus et al. (2007) used age markers and a penalized least square method, which assumes Gaussianity, to estimate the age as a function of depth. Lemieux-Dudon et al. (2009) also started by assuming that the uncertainties are Gaussian and that the model is approximately linear. However, if any of the relationships among the variables are nonlinear, Gaussianity does not hold in general. In this paper, we propose a dating method to estimate the age for the entire ice core without assuming either linearity or Gaussianity. The proposed method formulates the age–depth relationship based on a sequential Bayesian approach. The estimation is then achieved using the particle Markov chain Monte Carlo (PMCMC) method (Andrieu et al., 2010), which is applicable to nonlinear non-Gaussian problems formulated as sequential Bayesian models. This method estimates the age by using the marginal distribution, in which the uncertainties of the parameters in the glaciological model are marginalized out. Hence, it evaluates the uncertainty of the estimated age after considering the effects of the uncertainties in the model parameters.

The remainder of the present paper is organized as follows. In Sect. 2, we describe the models of the accumulation and thinning processes that control the age–depth

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relationship. In Sect. 3, these models are formulated in a framework of the sequential Bayesian approach in order to estimate the age, accumulation rate, and the model parameters. The PMCMC algorithms are explained in Sect. 4. In Sect. 5, an application to the Dome Fuji ice core is demonstrated, and the performance of our method is evaluated. Finally, a summary and discussion are presented in Sect. 6.

2 Dating model

It is thought that the age–depth relationship is primarily determined by two processes: the accumulation of snow at the site of the ice core and thinning due to long-term deformations within the ice sheet (e.g. Parrenin et al., 2001, 2007). We can thus consider the following differential equation describing the relationship between age and depth:

$$dz = A(z)\Theta(z)d\xi \quad (1)$$

where z denotes the depth from the surface of the ice sheet, ξ is the age in year at the given z (past is positive), $A(z)$ is the annual rate of accumulation of snow, and $\Theta(z)$ represents the thinning factor. Equation (1) yields the age ξ in the following form:

$$\xi(z) = \int_0^z \frac{dz'}{A(z')\Theta(z')}. \quad (2)$$

This implies that the age ξ can be obtained by the integral from the surface at $z = 0$.

Assuming a steady state, the thinning factor $\Theta(z)$ in Eq. (2) can be written using the vertical velocity U :

$$\Theta(z) = U(z)/U(0). \quad (3)$$

Rescaling z and U as

$$\zeta = \frac{H-z}{H}, \quad u(\zeta) = -\frac{U(z)}{H}, \quad (4)$$

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Eq. (3) can be rewritten as:

$$\Theta(\zeta) = u(\zeta)/u(1). \quad (5)$$

In Eq. (4), H is the thickness of the ice. We write the rescaled vertical velocity $u(\zeta)$ in the following form (Parrenin et al., 2006):

$$u(\zeta) = u(0) + [u(1) - u(0)]\omega(\zeta), \quad (6)$$

where $\omega(\zeta)$ is a function satisfying $\omega(0) = 0$ and $\omega(1) = 1$. We assume the following form for $\omega(\zeta)$ (Lliboutry, 1979):

$$\omega(\zeta) = \zeta - \frac{1-s}{\rho+1}(1-\zeta) \left[1 - (1-\zeta)^{\rho+1} \right], \quad (7)$$

where s corresponds to the sliding ratio, which is the ratio of the basal horizontal velocity to the vertically averaged horizontal velocity. Denoting the accumulation at the surface by A_0 and the melting at the base of the ice sheet by m , A_0 and m correspond to the vertical velocity at $\zeta = 1$ and that at $\zeta = 0$, respectively, under a steady state. Equation (6) can thus be rewritten as:

$$u(\zeta) = -\frac{1}{H} [m + (A_0 - m)\omega(\zeta)]. \quad (8)$$

Using Eq. (8), Eq. (5) becomes

$$\Theta(z) = \frac{\omega(\zeta) + \mu}{1 + \mu}, \quad (9)$$

where μ is defined as $\mu = m/(A_0 - m)$. Using Eqs. (7) and (9), the thinning factor Θ can be determined if the parameters s , ρ , and μ are specified.

In order to obtain the age ξ using Eq. (2), it is also necessary to give the accumulation rate A . In this study, A is treated as an unknown variable to be estimated. However,

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since the accumulation rate A is related to the temperature in the Antarctica, it can be constrained by some proxy of the temperature. We used the $\delta^{18}\text{O}$ data taken at Dome Fuji as a proxy for the temperature to estimate A . Since the vertical profile of the age ξ is associated with the profile of A , the information from the $\delta^{18}\text{O}$ data is also effective to improve the estimate of the age ξ .

At several points, we can also use more reliable age information that was determined from the relationship between O_2/N_2 and the summer insolation (Kawamura et al., 2007). We used the age values deduced from O_2/N_2 as tie points when estimating the age–depth relationship. The age ξ was estimated considering both the $\delta^{18}\text{O}(z)$ data and the tie points.

3 Bayesian model

Discretizing the vertical coordinate z with an interval Δz , the integral in Eq. (2) for any discretized z can be calculated using the following recurrence relation:

$$\xi_{z+\Delta z} = \xi_z + \frac{\Delta z}{A_z \Theta_z} + \nu_z \Delta z \quad (z = 0, \Delta z, 2\Delta z, \dots), \quad (10)$$

where ξ_z denotes the age at z and Z denotes the depth at the bottom of the core. At the surface ($z = 0$), ξ_0 is defined as zero. The accumulation rate and the thinning factor in the interval from z to $z + \Delta z$ are denoted by A_z and Θ_z , respectively. The term ν_z represents unknown variations that are attributed to processes that are not taken into account in Eq. (2). The thinning factor Θ_z can be obtained according to Eq. (9). The accumulation rate A_z is treated as an unknown variable, and its transition from z to $z + \Delta z$ is described by the following recurrence relation:

$$\log A_{z+\Delta z} = \log A_z + \eta_z \Delta z \quad (z = 0, \Delta z, 2\Delta z, \dots). \quad (11)$$

Note that the transition of A_z is described using its logarithm in Eq. (11) in order to guarantee $A_z > 0$. The term η_z represents the (unknown) variation in the accumulation

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rate. We hereinafter assume $\Delta z = 1$ [m]. Equations (10) and (11) can thus be rewritten as follows:

$$\xi_{z+1} = \xi_z + \frac{1}{A_z \Theta_z} + \nu_z, \quad (12)$$

$$\log A_{z+1} = \log A_z + \eta_z, \quad (z = 0, \dots, Z - \Delta z). \quad (13)$$

In order to apply a Bayesian approach, we introduce conditional probability density functions based on Eqs. (12) and (13). We assume that ν_z and η_z obey the normal distributions $\mathcal{N}(0, \sigma_\nu^2)$ and $\mathcal{N}(0, \sigma_\eta^2)$, respectively, where we denote a normal distribution with mean μ and variance σ^2 by $\mathcal{N}(\mu, \sigma^2)$. Accordingly, the conditional distribution of ξ_{z+1} given ξ_z for each z becomes

$$p(\xi_{z+1} | \xi_z, \boldsymbol{\theta}) = \mathcal{N}(\xi_z, \sigma_\nu^2), \quad (14)$$

and the conditional distribution of A_{z+1} given A_z for each z becomes a log-normal distribution as follows:

$$p(A_{z+1} | A_z, \boldsymbol{\theta}) = \log \mathcal{N}(A_z, \sigma_\eta^2), \quad (15)$$

where $\boldsymbol{\theta}$ indicates a collection of unspecified parameters such as ρ and s in Eq. (7). The full definition of $\boldsymbol{\theta}$ will be provided later. Because $p(\xi_{z+1} | \xi_z, \boldsymbol{\theta})$ and $p(A_{z+1} | A_z, \boldsymbol{\theta})$ are given, the joint distribution $p(\xi_{z+1}, A_{z+1} | \xi_z, A_z, \boldsymbol{\theta})$ can also be defined. We hereinafter combine ξ_z and A_z into one vector \mathbf{x}_z . Thus,

$$p(\mathbf{x}_{z+1} | \mathbf{x}_z, \boldsymbol{\theta}) = p(\xi_{z+1}, A_{z+1} | \xi_z, A_z, \boldsymbol{\theta}). \quad (16)$$

Estimates of ξ_z and A_z for each z are obtained on the basis of their posterior distributions given the tie points and the $\delta^{18}\text{O}$ data. The tie points are available only at a limited number of depths. For the k th tie point τ_k at depth z_k , we assume the following relationship between τ_k and the modeled age ξ_{z_k} :

$$\tau_k = \xi_{z_k} + \varepsilon_k, \quad (17)$$

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where ε_k is the discrepancy between the age at the tie point and the modeled age. Assuming that ε_k obeys the normal distribution $\mathcal{N}(0, \sigma_\varepsilon^2)$, the conditional distribution of τ_k given ξ_{z_k} becomes

$$p(\tau_k | \xi_{z_k}) = \mathcal{N}(\xi_{z_k}, \sigma_\varepsilon^2). \quad (18)$$

5 The $\delta^{18}\text{O}$ data, which are associated with the accumulation rate, can be abundantly obtained from the ice core at Dome Fuji. Multiple data points for $\delta^{18}\text{O}$ are sometimes available within an interval of a single meter, and we used the mean $\delta^{18}\text{O}$ value for each such interval. It was assumed that A_z , the accumulation rate in the interval from z to $z + \Delta z$, is associated with $\delta^{18}\text{O}$ as follows:

$$10 \quad \delta^{18}\text{O}_z = a \log A_z + b + w_z. \quad (19)$$

Assuming that w_z obeys the normal distribution $\mathcal{N}(0, \sigma_w^2)$, the conditional distribution of $\delta^{18}\text{O}_z$ given A_z becomes

$$p(\delta^{18}\text{O}_z | A_z, \boldsymbol{\theta}) = \mathcal{N}(a \log A_z + b, \sigma_w^2). \quad (20)$$

We define the vector of the available data for each z as \mathbf{y}_z . It should be noted that a tie point τ_k is available at only a limited number of points and that it is missing for some values of z . If both the tie point τ_{k_z} and the $\delta^{18}\text{O}$ data $\delta^{18}\text{O}_z$ are available at z , then, $\mathbf{y}_z = (\tau_{k_z}, \delta^{18}\text{O}_z)^\top$. If the $\delta^{18}\text{O}$ data are available but a tie point is unavailable, we define $\mathbf{y}_z = \delta^{18}\text{O}_z$. In the case that neither a tie point nor $\delta^{18}\text{O}$ data are available, we define $\mathbf{y}_z = \emptyset$. Using \mathbf{y}_z , the conditional distributions in Eqs. (18) and (20) can then be combined into the conditional distribution $p(\mathbf{y}_z | \mathbf{x}_z, \boldsymbol{\theta})$ for any z , where we define $p(\mathbf{y}_z = \emptyset | \mathbf{x}_z, \boldsymbol{\theta}) = 1$.

Our aim is to estimate $\mathbf{x}_{0:Z} = \{\mathbf{x}_0, \dots, \mathbf{x}_Z\}$ based on the sequence of the data $\mathbf{y}_{1:Z} = \{\mathbf{y}_1, \dots, \mathbf{y}_Z\}$. If a set of the parameters $\boldsymbol{\theta}$ was given, we could obtain an estimate of

$x_{0:z}$ from the posterior distribution $p(x_{0:z}|y_{1:z}, \theta)$. However, since the value of θ is not specified, it is necessary to take into account the uncertainties of θ in estimating $x_{0:z}$. We obtain an estimate from the marginal posterior distribution given $y_{1:z}$, where θ is marginalized out:

$$p(x_{0:z}|y_{1:z}) = \int p(x_{0:z}|y_{1:z}, \theta) p(\theta|y_{1:z}) d\theta. \quad (21)$$

Since y_z is conditionally independent of $x_{z'}$ given x_z when $z' \neq z$, $p(x_{0:z}|y_{1:z}, \theta)$ satisfies the following recurrence equation:

$$\begin{aligned} p(x_{0:z}|y_{1:z}, \theta) &\propto p(y_z|x_z, \theta) p(x_{0:z}|y_{1:z-1}, \theta) \\ &= p(y_z|x_z, \theta) p(x_z|x_{z-1}, \theta) p(x_{0:z-1}|y_{1:z-1}, \theta). \end{aligned} \quad (22)$$

This equation expresses a sequential Bayesian model. By applying Eq. (22) recursively, we can obtain $p(x_{0:z}|y_{1:z}, \theta)$ for any z . Thus, sampling from $p(x_{0:z}|y_{1:z}, \theta)$ can be achieved using the sequential Monte Carlo (SMC) method (Doucet et al., 2001; Liu, 2001). If z is set at Z in Eq. (22), we obtain $p(x_{0:Z}|y_{1:Z}, \theta)$, which provides the estimate of the age given all the data for the entire ice core.

A Bayesian approach also enables us to estimate the parameter θ . The posterior distribution of θ given $y_{1:Z}$ in Eq. (21) is calculated using the following equation:

$$p(\theta|y_{1:Z}) \propto p(y_{1:Z}|\theta) p(\theta). \quad (23)$$

An approximation of $p(y_{1:Z}|\theta)$ can be calculated using the SMC method. Therefore, if the prior $p(\theta)$ is given, the posterior of θ can readily be obtained. The vector θ contains all of the unspecified parameters used above. The full definition of θ is as follows:

$$\theta = (A_0 \ a \ b \ \mu \ p \ s \ \sigma_v \ \sigma_n \ \sigma_w)^T. \quad (24)$$

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Since the accumulation at the surface A_0 is not specified in the above sequential model, A_0 is treated as one of unspecified parameters and is included in θ . The parameter vector θ also contains three hyper-parameters σ_v , σ_η , and σ_w , which represent the variabilities in the model. These hyper-parameters are estimated so as to well explain the variability observed in the data. For example, if σ_v is taken to be too small, the estimated age would not be well fit to the data. On the contrary, if σ_v is taken to be too large, the spread of the posterior density function would get too large, which means the uncertainty is estimated to be too large. The posterior given the data provides an appropriate value of σ_v , which is large enough to achieve a good fit but not too large. The uncertainty of the $\delta^{18}\text{O}$ data, σ_w , corresponds to the standard deviation of the deviation from the trend of the $\delta^{18}\text{O}$ variation. The posterior of σ_w thus provides the scale of diversity of the $\delta^{18}\text{O}$ data. We did not include σ_ε in θ , but σ_ε for each tie point was set at a fixed value which was determined according to Kawamura et al. (2007).

4 Estimation algorithm

In order to approximately obtain the conditional distributions $p(\mathbf{x}_{0:Z}|\mathbf{y}_{1:Z},\theta)$ and $p(\theta|\mathbf{y}_{1:Z})$, we employ a non-Gaussian algorithm called the PMCMC method (Andrieu et al., 2010), which is a hybrid method combining the SMC method and the Markov chain Monte Carlo (MCMC) method. In this hybrid method, the posterior distributions for the uncertain parameters in the model are computed using the standard MCMC with the exception that the likelihood of the parameters is estimated using the SMC method. Meanwhile, the age–depth relationship is estimated by performing many repetitions of the SMC procedure under iterations of the MCMC. In the following, we first present the SMC method on which the PMCMC method is based. We then describe the PMCMC method and explain how approximations of $p(\mathbf{x}_{0:Z}|\mathbf{y}_{1:Z},\theta)$ and $p(\theta|\mathbf{y}_{1:Z})$ can be obtained.

4.1 Sequential Monte Carlo method

The SMC method, which is sometimes referred to as the particle filter/smoothen in time-series analysis (Gordon et al., 1993; Kitagawa, 1996; Doucet et al., 2001), is used for sampling from the conditional distribution $p(\mathbf{x}_{0:z}|\mathbf{y}_{1:z}, \boldsymbol{\theta})$. The SMC method approximates the density function $p(\mathbf{x}_{0:z-1}|\mathbf{y}_{1:z-1})$ by a set of particles $\{\mathbf{x}_{0:z-1|z-1}^{(i)}\}$ as

$$p(\mathbf{x}_{0:z-1}|\mathbf{y}_{1:z-1}, \boldsymbol{\theta}) \approx \frac{1}{N} \sum_{i=1}^N \delta(\mathbf{x}_{0:z-1} - \mathbf{x}_{0:z-1|z-1}^{(i)}). \quad (25)$$

Drawing a particle $\mathbf{x}_{z|z-1}^{(i)}$ according to

$$\mathbf{x}_{z|z-1}^{(i)} \sim p(\mathbf{x}_z|\mathbf{x}_{z-1} = \mathbf{x}_{z-1|z-1}^{(i)}, \boldsymbol{\theta}), \quad (26)$$

the set of particles $\{\mathbf{x}_{0:z|z-1}^{(i)}\}$ provides an approximation of $p(\mathbf{x}_{0:z}|\mathbf{y}_{1:z-1}, \boldsymbol{\theta})$:

$$p(\mathbf{x}_{0:z}|\mathbf{y}_{1:z-1}, \boldsymbol{\theta}) \approx \frac{1}{N} \sum_{i=1}^N \delta(\mathbf{x}_{0:z} - \mathbf{x}_{0:z|z-1}^{(i)}). \quad (27)$$

An approximation of the distribution conditioned by the observation \mathbf{y}_z at z can be obtained using the importance sampling scheme (e.g. Liu, 2001; Robert and Casella, 2004):

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$$\begin{aligned} p(\mathbf{x}_{0:z}|\mathbf{y}_{1:z}, \boldsymbol{\theta}) &= \frac{p(\mathbf{y}_z|\mathbf{x}_z, \boldsymbol{\theta}) p(\mathbf{x}_{0:z}|\mathbf{y}_{1:z-1}, \boldsymbol{\theta})}{p(\mathbf{y}_z|\mathbf{y}_{1:z-1}, \boldsymbol{\theta})} \\ &\approx \sum_{i=1}^N \beta_z^{(i)} \delta(\mathbf{x}_{0:z} - \mathbf{x}_{0:z|z-1}^{(i)}). \end{aligned} \quad (28)$$

The weight $\beta_z^{(i)}$ for each i is defined as

$$\beta_z^{(i)} = \frac{p(\mathbf{y}_z|\mathbf{x}_{z|z-1}^{(i)}, \boldsymbol{\theta})}{\sum_{i=1}^N p(\mathbf{y}_z|\mathbf{x}_{z|z-1}^{(i)}, \boldsymbol{\theta})}, \quad (29)$$

5 where $p(\mathbf{y}_z|\mathbf{x}_{z|z-1}^{(i)}, \boldsymbol{\theta})$ is the likelihood of the particle $\mathbf{x}_{z|z-1}^{(i)}$ that indicates how well $\mathbf{x}_{z|z-1}^{(i)}$ explains the observation \mathbf{y}_z at z .

Equation (28) indicates that $p(\mathbf{x}_{0:z}|\mathbf{y}_{1:z}, \boldsymbol{\theta})$ can be approximated by weighting the particles $\{\mathbf{x}_{0:z|z-1}^{(i)}\}$. However, the weights are usually highly unbalanced and many of the particles have only negligible weights. Because particles with negligible weights no longer contribute to the estimation, this destroys the efficiency of the approximation. In order to resolve the imbalance in the weights, a new set of N particles $\{\mathbf{x}_{0:z}^{(i)}\}$ is obtained by resampling the original particles $\{\mathbf{x}_{0:z|z-1}^{(i)}\}$ such that each $\mathbf{x}_{0:z|z-1}^{(i)}$ is drawn with a probability of $\beta_z^{(i)}$. After resampling, the original particles in $\{\mathbf{x}_{0:z|z-1}^{(i)}\}$ that have low weights are removed, and those that have high weights are duplicated. The number of the duplicates of $\mathbf{x}_{0:z|z-1}^{(i)}$, $n_z^{(i)}$, becomes approximately equal to $N\beta_z^{(i)}$. The newly generated particles then provide an approximation of $p(\mathbf{x}_{0:z}|\mathbf{y}_{1:z}, \boldsymbol{\theta})$ as follows:

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$$\begin{aligned}
 p(\mathbf{x}_{0:z}|\mathbf{y}_{1:z}, \boldsymbol{\theta}) &\approx \sum_{i=1}^N \beta_z^{(i)} \delta(\mathbf{x}_{0:z} - \mathbf{x}_{0:z|z-1}^{(i)}) \\
 &\approx \sum_{i=1}^N \frac{n_z^{(i)}}{N} \delta(\mathbf{x}_{0:z} - \mathbf{x}_{0:z|z-1}^{(i)}) \\
 &= \frac{1}{N} \sum_{i=1}^N \delta(\mathbf{x}_{0:z} - \mathbf{x}_{0:z|z}^{(i)}). \tag{30}
 \end{aligned}$$

Applying the procedure from Eq. (25) to Eq. (30) recursively up to $z = Z$, we obtain samples from the conditional distribution $p(\mathbf{x}_{0:Z}|\mathbf{y}_{1:Z}, \boldsymbol{\theta})$. If only the marginal distribution $p(\mathbf{x}_z|\mathbf{y}_{1:z}, \boldsymbol{\theta})$, where $\mathbf{x}_{0:z-1}$ is marginalized out, is of interest, it is not necessary to keep the whole sequence of $\mathbf{x}_{0:z|z}^{(i)}$ for each particle; instead, at each iteration, it is sufficient to keep only the element $\mathbf{x}_{z|z}^{(i)}$ and discard the remaining $\mathbf{x}_{1:z-1}^{(i)}$.

4.2 Particle Markov chain Monte Carlo method

An approximation of the marginal likelihood $p(\mathbf{y}_{1:Z}|\boldsymbol{\theta})$ in Eq. (23) can be calculated using the SMC (Kitagawa, 1996). If we decompose $p(\mathbf{y}_{1:Z}|\boldsymbol{\theta})$ as

$$\begin{aligned}
 p(\mathbf{y}_{1:Z}|\boldsymbol{\theta}) &= p(\mathbf{y}_{1:Z-1}|\boldsymbol{\theta}) p(\mathbf{y}_Z|\mathbf{y}_{1:Z-1}, \boldsymbol{\theta}) \\
 &= p(\mathbf{y}_1|\boldsymbol{\theta}) \prod_{z=2}^Z p(\mathbf{y}_z|\mathbf{y}_{1:z-1}, \boldsymbol{\theta}), \tag{31}
 \end{aligned}$$

we can obtain $p(\mathbf{y}_z|\mathbf{y}_{1:z-1}, \boldsymbol{\theta})$ for each z , from the following equation:

$$\begin{aligned}
& \rho(\mathbf{y}_z | \mathbf{y}_{1:z-1}, \boldsymbol{\theta}) \\
&= \int \rho(\mathbf{y}_z | \mathbf{x}_z, \boldsymbol{\theta}) \rho(\mathbf{x}_z | \mathbf{y}_{1:z-1}, \boldsymbol{\theta}) d\mathbf{x}_z \\
&= \int \rho(\mathbf{y}_z | \mathbf{x}_z, \boldsymbol{\theta}) \rho(\mathbf{x}_z | \mathbf{x}_{z-1}, \boldsymbol{\theta}) \rho(\mathbf{x}_{0:z-1} | \mathbf{y}_{1:z-1}, \boldsymbol{\theta}) d\mathbf{x}_{0:z-1}.
\end{aligned} \tag{32}$$

Since samples from $\rho(\mathbf{x}_{0:z-1} | \mathbf{y}_{1:z-1}, \boldsymbol{\theta})$ can be obtained by the SMC, a Monte Carlo approximation of the integral in Eq. (32) can be obtained, and an approximation of the posterior $\rho(\boldsymbol{\theta} | \mathbf{y}_{1:z})$ in Eq. (23) can accordingly be obtained.

Using the Monte Carlo approximation of the marginal likelihood $\hat{\rho}(\mathbf{y}_{1:z} | \boldsymbol{\theta})$, we can obtain an approximation of the marginal posterior distribution of $\boldsymbol{\theta}$ using the MCMC (Andrieu et al., 2010), which sequentially produces samples that obey the target distribution. In order to obtain an approximation of $\rho(\boldsymbol{\theta} | \mathbf{y}_{1:z})$, we employ the Metropolis method. In this method, at the k th iteration, a proposal sample $\boldsymbol{\theta}^*$ is drawn from the proposal density $q(\boldsymbol{\theta} | \boldsymbol{\theta}^{(k)})$, which is conditioned by the sample $\boldsymbol{\theta}^{(k-1)}$, which was obtained at the previous iteration:

$$\boldsymbol{\theta}^* \sim q(\boldsymbol{\theta} | \boldsymbol{\theta}^{(k-1)}). \tag{33}$$

In this paper, we use a Gaussian distribution with a fixed variance for each element of $\boldsymbol{\theta}$ that satisfies

$$q(\boldsymbol{\theta} | \boldsymbol{\theta}') = q(\boldsymbol{\theta}' | \boldsymbol{\theta}) \tag{34}$$

for any $\boldsymbol{\theta}$ and $\boldsymbol{\theta}'$. The proposal sample $\boldsymbol{\theta}^*$ is accepted with the following probability:

$$\min \left(1, \frac{\hat{\rho}(\mathbf{y}_{1:z} | \boldsymbol{\theta}^*) \rho(\boldsymbol{\theta}^*)}{\hat{\rho}(\mathbf{y}_{1:z} | \boldsymbol{\theta}^{(k-1)}) \rho(\boldsymbol{\theta}^{(k-1)})} \right), \tag{35}$$

where $\hat{\rho}(\mathbf{y}_{1:z} | \boldsymbol{\theta})$ is an approximation of the marginal likelihood obtained by the SMC. If $\boldsymbol{\theta}^*$ is accepted, we set $\boldsymbol{\theta}^{(k)} = \boldsymbol{\theta}^*$; otherwise, we set $\boldsymbol{\theta}^{(k)} = \boldsymbol{\theta}^{(k-1)}$. Using $\boldsymbol{\theta}^{(k)}$, the

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proposal sample at the next iteration can be obtained according to Eq. (33). Iterating the above procedure generates a large number of samples that obey the posterior distribution $p(\boldsymbol{\theta}|\mathbf{y}_{1:Z})$.

In the above algorithm, an approximated value of the marginal likelihood $p(\mathbf{y}_{1:k}|\boldsymbol{\theta})$ is computed using the SMC method at each iteration of the Metropolis method. It should be noted that Eq. (32) can be modified as follows:

$$\begin{aligned} & p(\mathbf{y}_z|\mathbf{y}_{1:z-1}, \boldsymbol{\theta}) \\ &= \int p(\mathbf{y}_z|\mathbf{x}_z, \boldsymbol{\theta}) p(\mathbf{x}_z|\mathbf{x}_{z-1}, \boldsymbol{\theta}) p(\mathbf{x}_{z-1}|\mathbf{y}_{1:z-1}, \boldsymbol{\theta}) d\mathbf{x}_{z-1} d\mathbf{x}_z. \end{aligned} \quad (36)$$

Thus, in calculating $p(\mathbf{y}_{1:Z}|\boldsymbol{\theta})$ in Eq. (31), it is not necessary to consider the joint distribution of the sequence $\mathbf{x}_{0:Z}$; it is sufficient to consider the marginal distribution $p(\mathbf{x}_z|\mathbf{y}_{1:z}, \boldsymbol{\theta})$ for each z . As mentioned above, sampling from $p(\mathbf{x}_z|\mathbf{y}_{1:z}, \boldsymbol{\theta})$ can be achieved when discarding $\mathbf{x}_{1:z-1}$; this greatly reduces the computational cost. We then discard $\mathbf{x}_{1:z-1}$ in order to obtain an approximation of $p(\mathbf{y}_{1:k}|\boldsymbol{\theta})$ at each iteration of the Metropolis method.

As mentioned in Sect. 3, if we retain the samples for the whole sequence $\mathbf{x}_{0:Z}$ from a run of the SMC with a given $\boldsymbol{\theta}$, we obtain samples from $p(\mathbf{x}_{0:Z}|\mathbf{y}_{1:Z}, \boldsymbol{\theta})$. The Metropolis procedure sequentially generates a large number of samples that obey the marginal posterior distribution $p(\boldsymbol{\theta}|\mathbf{y}_{1:Z})$. By combining the SMC samples with various $\boldsymbol{\theta}$ values that obey $p(\boldsymbol{\theta}|\mathbf{y}_{1:Z})$, we can obtain the samples representing the marginal posterior distribution $p(\mathbf{x}_{0:Z}|\mathbf{y}_{1:Z})$ where $\boldsymbol{\theta}$ is marginalized out according to Eq. (21). If samples that obey $p(\boldsymbol{\theta}|\mathbf{y}_{1:Z})$ are obtained in advance, the sampling procedures from $p(\mathbf{x}_{0:Z}|\mathbf{y}_{1:Z}, \boldsymbol{\theta})$ for various $\boldsymbol{\theta}$ can be performed in parallel, and an approximation of the marginal posterior distribution $p(\mathbf{x}_{0:Z}|\mathbf{y}_{1:Z})$ can be obtained efficiently.

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5 Result

Following a burn-in period, we performed 250 000 iterations of the Metropolis sampling, and we retained a sample every fifth iteration. We thus drew 50 000 samples from the marginal posterior distribution of θ , $p(\theta|\mathbf{y}_1:z)$. For each run of the SMC, 5000 particles were used to obtain samples from $p(\mathbf{x}_{0:z}|\mathbf{y}_1:z, \theta^{(k)})$.

Figure 1 shows the marginal histograms for the estimated posterior distribution for each parameter. The posterior mean and standard deviation of the accumulation at the surface A_0 were 2.69 and 0.13, respectively. This result is in good agreement with the measurement by Kameda et al. (2008), who reported the surface mass balance at Dome Fuji to be $27.3 \pm 1.5 \text{ kg (m}^{-2} \text{ yr)}^{-1}$. The maxima of the posterior distributions for μ and s were estimated to be near zero. This result is similar to that obtained in a previous study that used the Metropolis–Hastings method (Parrenin et al., 2007), although this result estimated a smaller uncertainty for μ and a larger uncertainty for s . In the result by Parrenin et al. (2007), the posterior of p peaks around 3, and another peak was suggested around $p = 2$. In contrast, the results obtained in this study suggest that the posterior of p peaks around 4, and it is not clear whether there is another mode. It should be noted that these two results were based on different modeling for the accumulation rate, and thus it should not be expected that they would necessarily provide similar results.

Figure 2 shows the estimated age as a function of depth. The red solid line indicates the median of the posterior distribution and the blue dotted lines indicate the 10th and 90th percentiles of the posterior distribution. The black crosses in this figure indicate the tie points used for the estimation. In order to verify the convergence of the SMC sampling, we repeated sampling from the marginal posterior distribution $p(\mathbf{x}_{0:z}|\mathbf{y}_1:z)$ five times with different seeds. We confirmed that there were no apparent differences between the results of the five trials. Thus, the estimate shown in Fig. 2 is sufficiently converged. The SMC method often suffers from the degeneracy problem, especially when the number of steps is large. In the PMCMC method, this degeneracy problem

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is overcome by collecting a large number of the SMC samples that are obtained by iterations of the Metropolis method. Figure 3 shows the difference between the 10th and 90th percentiles of the posterior distribution of the age as a function of depth, which is shown in Fig. 2. The uncertainty of the age is minimized at each tie point where the age is known with high accuracy.

Figure 4 indicates the estimated thinning factor as a function of depth. Again, the red solid line indicates the median of the posterior distribution and the blue dotted lines indicate the 10th and 90th percentiles of the posterior distribution. Since $\Theta = 1$ at the surface by definition, the width of the posterior distribution is almost zero near the surface, and the uncertainty becomes larger in the deeper core. Figure 5 shows the estimated accumulation rate as a function of depth. As in Fig. 2, the red solid line indicates the median of the posterior distribution, and the blue dotted lines indicate the 10th and 90th percentiles of the posterior distribution. In this way, we can estimate the age and related variables, and we can also obtain information about the credibility of these estimates. The accumulation rate can also be considered as a function of age, as shown in Fig. 6.

In the SMC, if the number of particles N is large, each run of the sampling requires high computational cost. Thus, it is preferable that N should be as small as possible. We evaluated the performance with various smaller values of N . In Fig. 7, the cyan histograms shows the marginal posterior distribution that was estimated using 3000 particles in each run of the SMC method. For reference, the magenta histogram shows the estimate with $N = 5000$, which is the same as in Fig. 1. The two histograms are in good agreement, although the result with $N = 3000$ seems to have more noise. In Fig. 8, the cyan histogram shows the marginal posterior distribution estimated with $N = 1000$, and again the magenta histogram shows the estimate with $N = 5000$. The discrepancy between the two histograms increased. For example, the estimated variance for p was smaller when $N = 1000$. Indeed, the acceptance rate of the Metropolis transition was depressed when the number of particles used in the SMC sampling was reduced: the acceptance rate was 8.0% when $N = 5000$, 6.0% when $N = 3000$, and

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only 2.1% when $N = 1000$. When the number of particles was small, to some extent, the low acceptance rate can be compensated by increasing the number of iterations of the Metropolis method. Figure 9 shows the result when 1000 particles were used for the SMC run, but 1 250 000 iterations of the Metropolis sampling were performed in order to obtain 250 000 samples. The SMC method with $N = 1000$ requires only about 1/5 of the computational cost with $N = 5000$. Thus, 1 250 000 iterations with $N = 1000$ have approximately the same computational cost as 250 000 iterations with $N = 5000$. The estimate with $N = 1000$, as shown in the cyan histogram, is similar to the estimate with $N = 5000$. However, even though these two estimates have similar computational costs, the histogram of the estimate with $N = 1000$ appear to be noisier than that of the estimate with $N = 5000$.

6 Concluding remarks

We have developed a technique for the dating of an ice core by combining information obtained from age markers at various depths and with a model describing the accumulation of snow and glaciological dynamics. This technique provides estimates of unspecified parameters in the model from their posterior distributions as calculated with the PMCMC method. In the PMCMC method, the marginal posterior distributions of the parameters are obtained using the Metropolis method; this is similar to other existing techniques (Parrenin et al., 2007), but here the likelihood of the set of parameters is estimated with the SMC method. The age as a function of depth can also be estimated from the marginal posterior distributions where the parameters are marginalized out. The marginal posterior distribution of age at each depth is obtained by collecting the SMC samples produced by many iterations of the Metropolis method. We applied this PMCMC method to the data of the ice core at Dome Fuji. The estimates of the age–depth relationship and the parameters were successfully obtained.

The main advantage of the proposed technique is that it can be applied to general nonlinear non-Gaussian situations. Since the relationship between accumulation rate

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and a temperature proxy is typically nonlinear, it is not necessarily justified to assume linearity and Gaussianity in dating of an ice core using a temperature proxy. The PMCMC method allows us to use various kinds of data which are expected to have nonlinear relationship with model variables. Another advantage is that the PMCMC method estimates model parameters simultaneously with the age as a function of depth. The uncertainty of age is therefore evaluated after taking into account the uncertainties in the model parameters. However, the proposed technique requires high computational cost because the SMC sampling is performed at each iteration of the Metropolis method. At present, it takes about 52 h to complete 250 000 iterations of the Metropolis sampling with 5000 particles for the SMC on a workstation with two Intel Xeon processors (12 cores; 2.7 GHz). The efficiency could be improved by using a better proposal distribution (e.g. Doucet et al., 2001). This problem should be addressed in the future.

This study used the $\delta^{18}\text{O}$ data and tie points deduced from O_2/N_2 data to estimate the age–depth relationships. However, data from other sources could also be used to improve the accuracy of the estimates. For example, deuterium-excess data have been used to estimate temperatures (Uemura et al., 2012), and this could be used for improving the accuracy of the accumulation rate. Some recent studies have provided simultaneous estimates of the age as a function of depth at multiple sites (e.g. Lemieux-Dudon et al., 2010; Veres et al., 2013). The extension of the SMC approach such that information at multiple sites could be combined would be a useful area of future work.

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References

- Andrieu, C., Doucet, A., and Holenstein, R.: Particle Markov chain Monte Carlo methods, *J. Roy. Stat. Soc. B*, 72, 269–342, 2010. 941, 948, 952
- Doucet, A., de Freitas, N., and Gordon, N. (Eds.): *Sequential Monte Carlo Methods in Practice*, Springer-Verlag, New York, 2001. 947, 949, 957
- Dreyfus, G. B., Parrenin, F., Lemieux-Dudon, B., Durand, G., Masson-Delmotte, V., Jouzel, J., Barnola, J.-M., Panno, L., Spahni, R., Tisserand, A., Siegenthaler, U., and Leuenberger, M.: Anomalous flow below 2700 m in the EPICA Dome C ice core detected using $\delta^{18}\text{O}$ of atmospheric oxygen measurements, *Clim. Past*, 3, 341–353, doi:10.5194/cp-3-341-2007, 2007. 941
- Gordon, N. J., Salmond, D. J., and Smith, A. F. M.: Novel approach to nonlinear/non-Gaussian Bayesian state estimation, *IEE Proc.-F*, 140, 107–113, 1993. 949
- Kameda, T., Motoyama, H., Fujita, S., and Takahashi, S.: Temporal and spatial variability of surface mass balance at Dome Fuji, East Antarctica, by the stake method from 1995 to 2006, *J. Glaciol.*, 54, 107–116, 2008. 954
- Kawamura, K., Parrenin, F., Lisiecki, L., Uemura, R., Vimeux, F., Severinghaus, J. P., Hutterli, M. A., Nakazawa, T., Aoki, S., Jouzel, J., Raymo, M. E., Matsumoto, K., Nakata, H., Motoyama, H., Fujita, S., Goto-Azuma, K., Fujii, Y., and Watanabe, O.: Northern Hemisphere forcing of climatic cycles in Antarctica over the past 360,000 years, *Nature*, 448, 912–916, doi:10.1038/nature06015, 2007. 944, 948
- Kitagawa, G.: Monte Carlo filter and smoother for non-Gaussian nonlinear state space models, *J. Comput. Graph. Stat.*, 5, 1–25, 1996. 949, 951
- Klauenberg, K., Blackwell, P. G., Buck, C. E., Mulvaney, R., Röthlisberger, R., and Wolff, E. W.: Bayesian glaciological modelling to quantify uncertainties in ice core chronologies, *Quaternary Sci. Rev.*, 30, 2961–2975, 2011. 941
- Lemieux-Dudon, B., Parrenin, F., and Blayo, E.: A probabilistic method to construct an optimal ice core chronology for ice cores, in: *Proceedings of the 2nd International Workshop on Physics of Ice Core Records (PICR-2)*, edited by: Hondoh, T., Institute of Low Temperature Science, Hokkaido University, Hokkaido, 233–245, 2009. 941
- Lemieux-Dudon, B., Blayo, E., Petit, J. R., Waelbroeck, C., Svenson, A., Ritz, C., Barnola, J. M., Narcisi, B. M., and Parrenin, F.: Consistent dating for Antarctic and Greenland ice cores, *Quaternary Sci. Rev.*, 29, 8–20, 2010. 957

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Liu, J. S.: Monte Carlo Strategies in Scientific Computing, Springer-Verlag, New York, 2001. 947, 949

Lliboutry, L.: Local friction laws for glaciers: a critical review and new openings, *J. Glaciol.*, 23, 67–95, 1979. 943

5 Parrenin, F., Waelbroeck, J. J. C., Ritz, C., and Barnola, J.-M.: Dating the Vostok ice core by an inverse method, *J. Geophys. Res.*, 106, 31837–31851, 2001. 942

Parrenin, F., Hindmarsh, R. C. A., and Rémy, F.: Analytical solutions for the effect of topography, accumulation rate and lateral flow divergence on isochrone layer geometry, *J. Glaciol.*, 52, 191–202, 2006. 943

10 Parrenin, F., Dreyfus, G., Durand, G., Fujita, S., Gagliardini, O., Gillet, F., Jouzel, J., Kawamura, K., Lhomme, N., Masson-Delmotte, V., Ritz, C., Schwander, J., Shoji, H., Uemura, R., Watanabe, O., and Yoshida, N.: 1-D-ice flow modelling at EPICA Dome C and Dome Fuji, East Antarctica, *Clim. Past*, 3, 243–259, doi:10.5194/cp-3-243-2007, 2007. 941, 942, 954, 956

15 Robert, C. P. and Casella, G.: Monte Carlo Statistical Methods, 2nd Edn., Springer Science + Business Media Inc., New York, 2004. 949

Uemura, R., Masson-Delmotte, V., Jouzel, J., Landais, A., Motoyama, H., and Stenni, B.: Ranges of moisture-source temperature estimated from Antarctic ice cores stable isotope records over glacial–interglacial cycles, *Clim. Past*, 8, 1109–1125, doi:10.5194/cp-8-1109-2012, 2012. 957

20 Veres, D., Bazin, L., Landais, A., Toyé Mahamadou Kele, H., Lemieux-Dudon, B., Parrenin, F., Martinerie, P., Blayo, E., Blunier, T., Capron, E., Chappellaz, J., Rasmussen, S. O., Severi, M., Svensson, A., Vinther, B., and Wolff, E. W.: The Antarctic ice core chronology (AICC2012): an optimized multi-parameter and multi-site dating approach for the last 120 thousand years, *Clim. Past*, 9, 1733–1748, doi:10.5194/cp-9-1733-2013, 2013. 957

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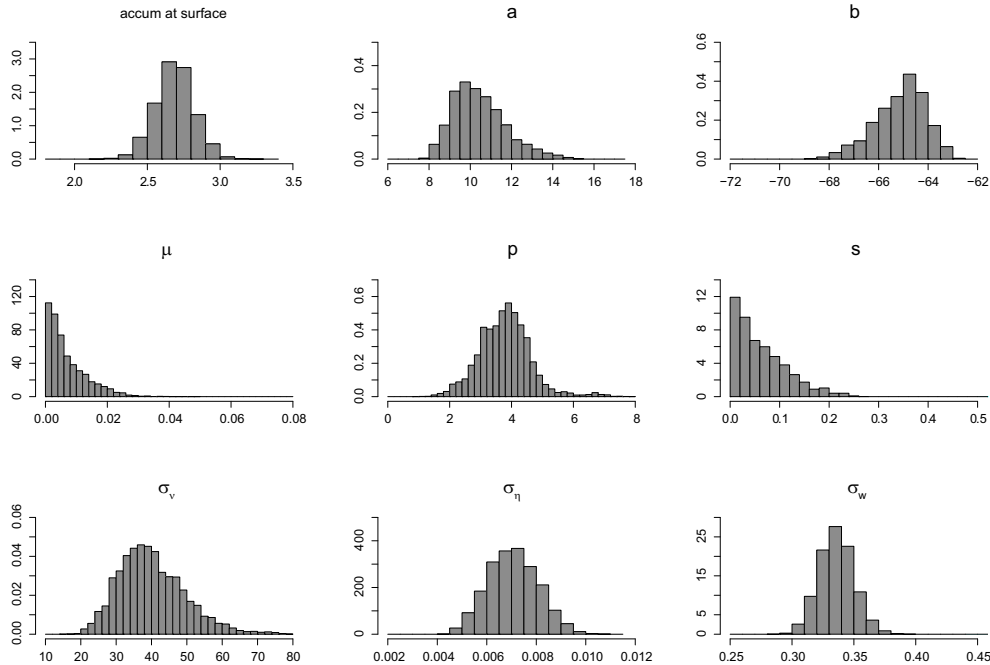


Figure 1. Estimated marginal distributions of the posterior distributions for the nine parameters: A_0 (upper left panel), a (upper center panel), b (upper right panel), μ (middle left panel), p (middle center panel), s (middle right panel), σ_v (lower left panel), σ_η (lower center panel), and σ_w (lower right panel).

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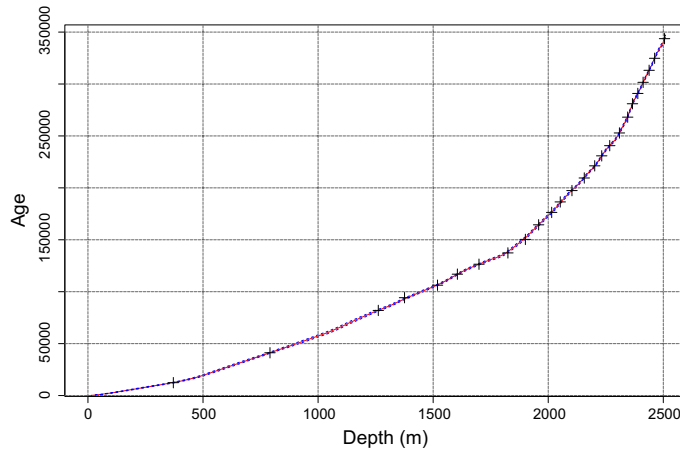


Figure 2. Estimated age as a function of depth. The solid line indicates the median of the posterior distribution. The 10th and 90th percentiles of the posterior are indicated by red dotted lines. The blue crosses indicates the age markers.

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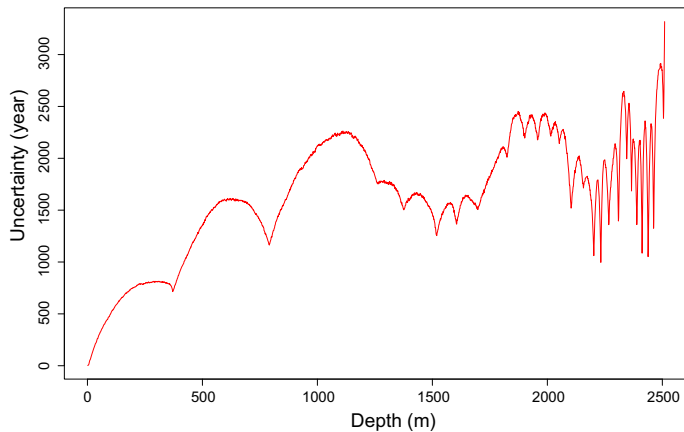


Figure 3. Difference between the 10th and 90th percentiles of the posterior of the age as a function of depth, which are shown in Fig. 2.

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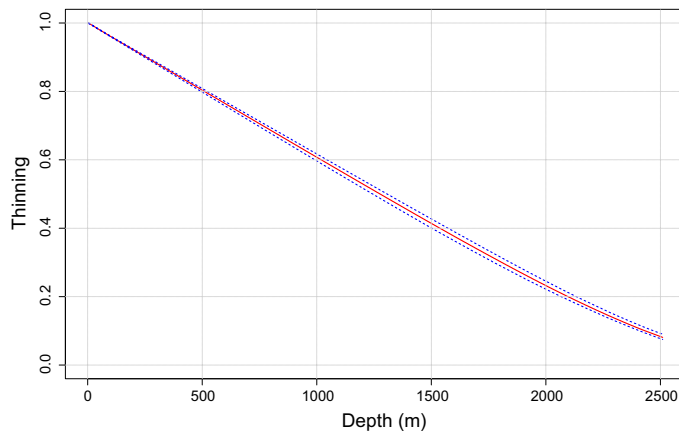


Figure 4. Estimated thinning factor Θ as a function of age. The median of the posterior is shown with a red solid line, and the 10th and 90th percentiles are shown with blue dotted lines.

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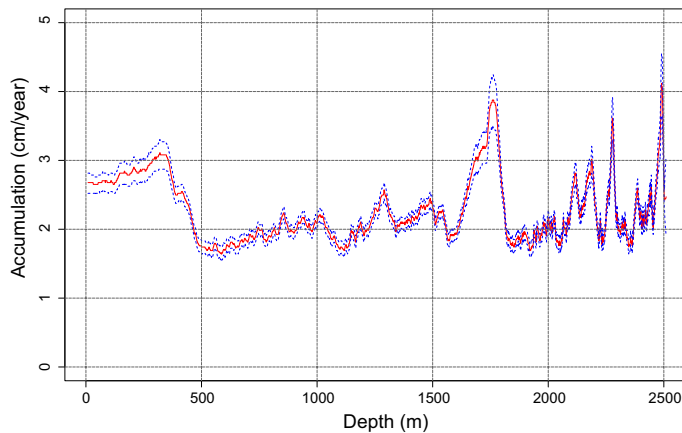


Figure 5. Estimated accumulation rate as a function of depth. The median of the posterior is shown with a red solid line, and the 10th and 90th percentiles are shown with blue dotted lines.

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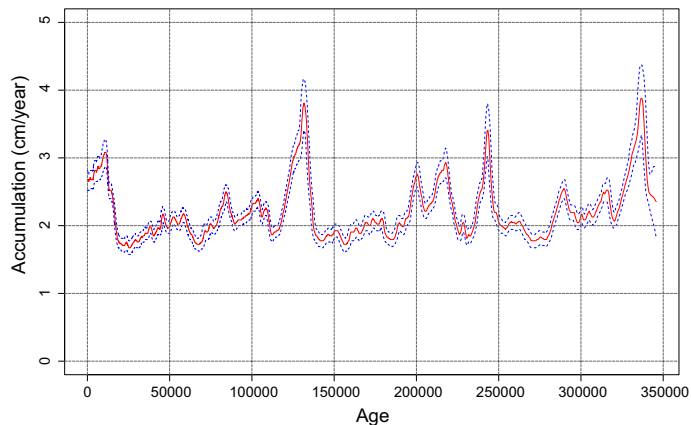


Figure 6. Estimated accumulation rate as a function of age. The median of the posterior is shown with a red solid line, and the 10th and 90th percentiles are shown with blue dotted lines.

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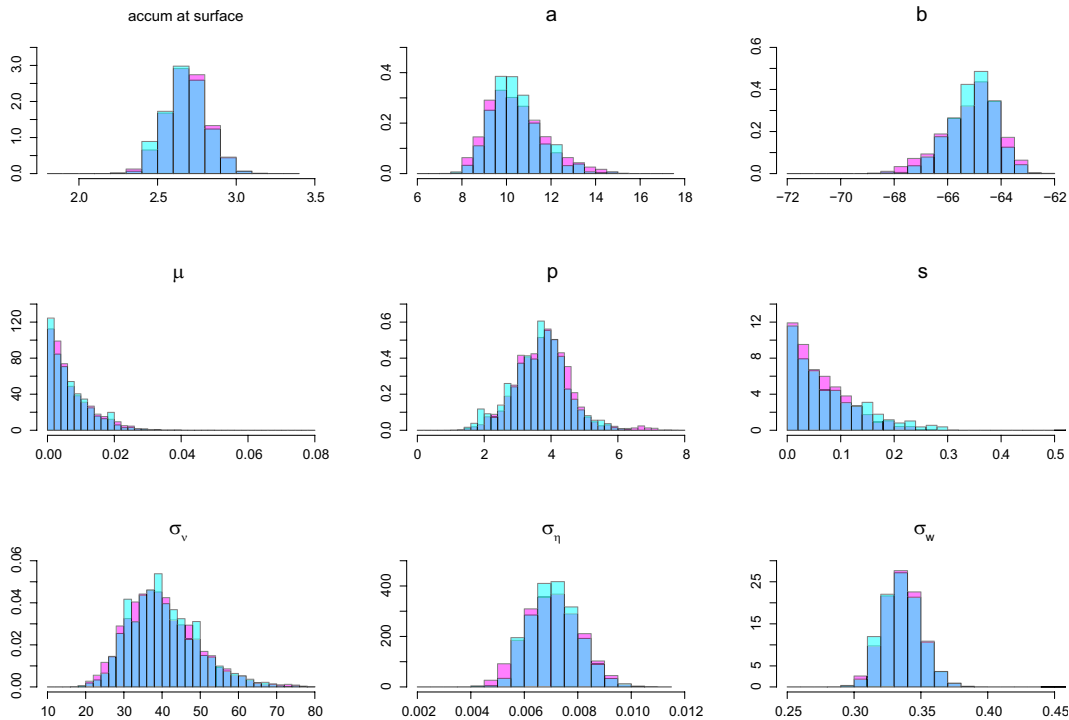


Figure 7. Comparison of the estimated marginal distributions between the result with 5000 particles (magenta) and the result with 3000 particles (cyan). The parameters for which the histograms are displayed are the same as in Fig. 1.

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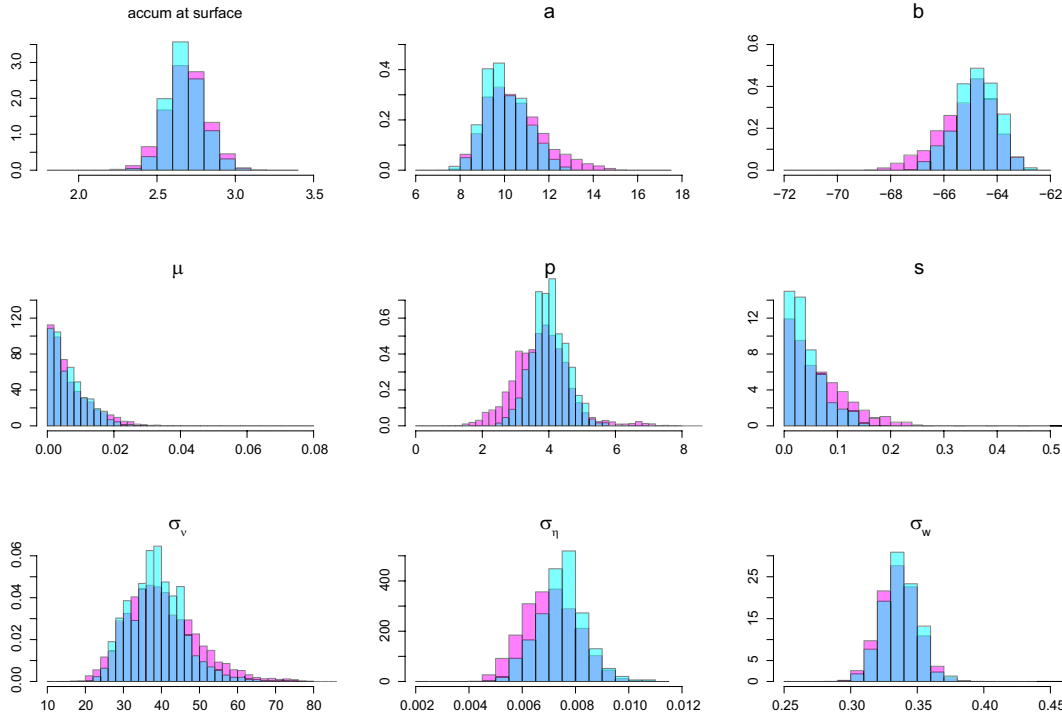


Figure 8. Comparison of the estimated marginal distributions between the result with 5000 particles (magenta) and the result with 1000 particles (cyan). The parameters for which the histograms are displayed are the same as in Fig. 1.

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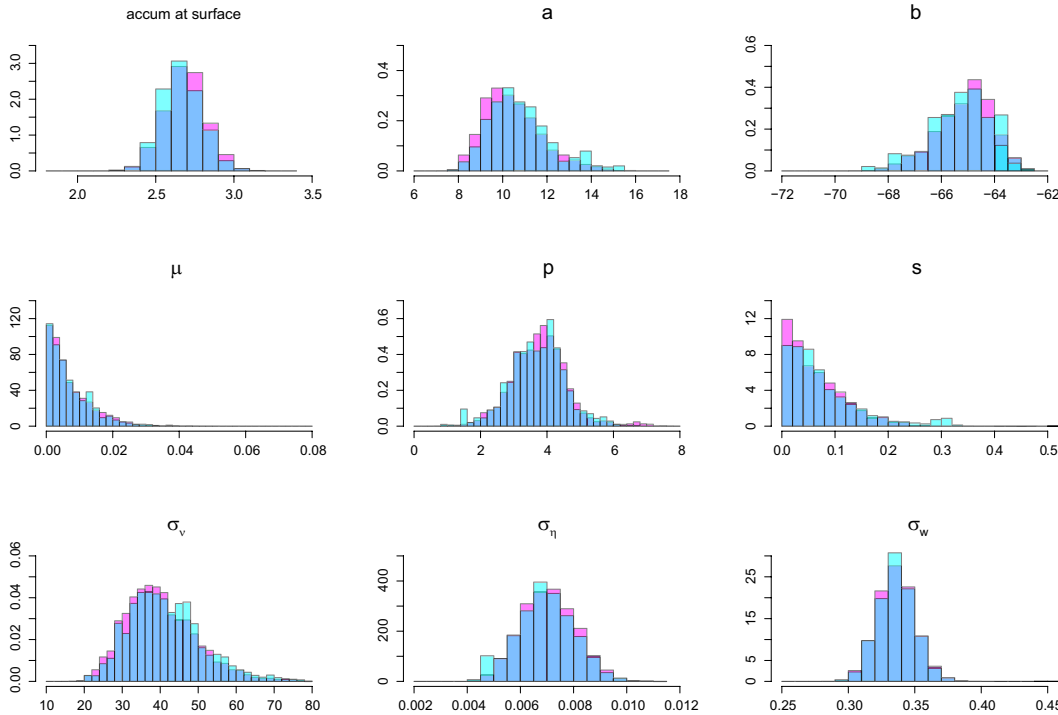


Figure 9. Comparison of the estimated marginal distributions between the result with 50 000 iterations and 5000 particles (magenta) and the result with 250 000 iterations and 1000 particles (cyan). The parameters for which the histograms are displayed are the same as in Fig. 1.

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