Manuscript prepared for Nonlin. Processes Geophys.
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Date: 31 August 2015

Earthquake source parameters which display first digit phenomenon

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Abstract. We study main parameters of earthquakes from the perspective of the first digit phenomenon: the nonuniform probability of the lower first digit different from zero compared to the higher ones. We found that source parameters like coseismic slip distributions at the fault and coseismic inland displacements show first digit anomaly. We also found the tsunami runups measured after the earthquake to display the phenomenon. Other parameters found to obey first digit anomaly are related to the aftershocks: we show that seismic moment liberation and seismic waiting times also display an anomaly. We explain this finding by invoking a self-organized criticality frame. We show that critically organized automata show the first digit signature and we interpret this as a possible explanation of the behavior of the studied parameters of the Tohoku earthquake.

1 Introduction

With the advent of modern seismological and geodetical instrumentation, the study of the earthquake process has experienced great advances, much of it punctuated by the occurrence of giant earthquakes. Since 2004, three of these spectacular events occurred, each producing large tsunamis, followed by human and material losses. These are the 2004 Northern Sumatra (Lay et al., 2005), the 2010 Central Chile (Vigny et al., 2011) and the 2011 Tohoku Japan (Simons et al., 2011) earthquakes, each of them representing an opportunity to advance in the comprehension of geophysical phenomena. Three key elements in the understanding of these events are: a) the source process, a highly nonlinear and heterogeneous phenomenon regarding the initiation, growth and stopping of the earthquake itself, b) the postseismic relaxation effects, which comprise the later perturbations at the crust and fault itself after the stop of the slip phase, and c) the water wave produced by the sudden uplift of the ocean floor, its propagation through the ocean and the, often destructive, arrival inland.

Of the aforementioned events, the Tohoku earthquake is by far the best recorded, at the local and global level. The Japanese and worldwide effort, lead by universities and public institutions, gathered a great bulk of information regarding this event, most of it public. We use data and models of this event, to assess an establish a regularity of the source and later events known as the first digit anomaly (Benford, 1938).

This phenomenon consist in the nonuniform statistical distribution of the first digit different from zero present in a —usually large— population of data $d = \{d_1, \ldots, d_n\}$ coming from natural systems. The law states that the probability of finding the number one as the first digit different from zero in d is higher than the probability of finding the number two and so on according to the formula:

$$P_D = \log_{10}(1 + 1/D), \quad D = 1, 2, ..., 9, \tag{1}$$

first proposed in the 19 century (Newcomb, 1881) by noticing the wear accumulated in the first pages of logarithm tables relative to the last ones.

The first digit phenomenon has received considerable attention (Hill, 1998) and is known to be found in diverse areas like: physics (Berger et al., 2005; Tolle et al., 2000), mathematics (Cohen and Katz, 1984), computation (Knuth, 1981), economy (Nigrini, 1996) and recently it has found some application in geophysics (Sambridge et al., 2010; Nigrini and Miller, 2007; Geyer and Martí, 2012). From the angle of statistics there are known theorems, some of them regarding scale invariant properties (Hill, 1995a) and others related to random sampling (Hill, 1995b) and. Moreover it has been found that datasets possesing multidecadal range should display the anomaly (Fewster, 2009). A depth review of these properties is outside of the scope of this work, and we recommend the paper of Sambridge et al. (2010) for more information. Despite efforts, little is known concerning why some numbers collected from natural data follow this regularity (Berger and Hill, 2011). However it is known since the beginning of 20 century that natural processes following geometric laws are equidistributed over the circle as long as an irrational base is considered, known as the ergodic property of geometric maps (Arnol'd, 2014). It has been argued by Tarantola (2006) that the effect appears related to the so called Jeffreys' pairs: physical variables endowed with the property of being as meaningful as their inverses. For instance: period and frequency, conductivity and resistivity (hydraulic, electric or thermal) or compliance and stiffness. We must note that Jeffreys' pairs usually display large dynamical ranges, a fact evident because of the regular use of logarithm scales when working with these variables. From now on, we will work under the hypothesis that an underlying physical process connects these elements (random sampling, scale invariance, broad dynamical range) and could explain the ubiquitous presence of the first digit phenomenon.

2 The Tohoku earthquake from the point of view of the first digit

As we pointed out, the first digit phenomenon has recently found some applications in geophysics. Sambridge et al. (2010) demonstrated that the initial signal of an earthquake, as viewed in a typical seismogram, displays Benford's anomaly. This effect was proposed as an event trigger, because the noise registered in the seismogram, displays a kind of gaussian behavior very different from first digit phenomenon. Our insight is that this property of the earthquake arrival is directly related to the seismic source and related processes. To go further into this track, we revisited some published data from the Tohoku earthquake in terms of the first digit distribution.

Leaving aside seismograms, we studied physical parameters closely related to the coseismic and postseismic processes. We choose to review data from the Tohoku earthquake mainly because of data quality. The data used comes from direct measurements of earthquake effects and indirect estimations as well. Care was taken in regard to the statistical significance of the samples selected, as we left out interesting data with few samples.

In Table 1 we present first digit statistics of various parameters closely related to the seismic source process in the case of the Tohoku earthquake. For each parameter, a χ^2 goodness of fit test with 9 degrees of freedom is presented. Also, each test is accompanied with the 5% significance level p, values of this last parameter close to one indicate statistical agreement with the hypothesis of the empirical data following the first digit anomaly. First, we show the finite fault model, as regularly published by the U.S. Geological Survey (Hayes, 2011a). This set is showed in Fig. 1, the data corresponds to an inversion of P wave, SH wave and long period surface waves of the source from globally located stations (Ji et al., 2002; Hayes, 2011b). We collected the 240 slips (Δu) and seismic moments (M_0) which give form to the finite fault model of the earthquake. We found the slips to follow the first digit anomaly. Seismic moments present the anomaly as well, and this is expected because seismic moment is a affine scaling of slips at the fault. From Fig. 1 the high dynamical range of the data can be clearly evidenced, and as it was mentioned this is one of the known characteristics of parameters showing first digit anomaly. Second, we used a GPS inversion of the coseismic inland deformation. This inversion uses data from the GPS Earth Observation Network (GEONET Ozawa et al. (2011)) and it represents an ensemble of 357 points inverted, showed in Fig. 2 are the total displacement magnitudes, which describe the effect of slip distribution on the fault and the effects over the Earth's surface from geodetic data, observe the high dynamical range of displacements. The absolute value of the deformation $|u_c|$ shows a clear first digit anomaly as shown in Table 1. Third, from the same dataset, we studied the first digit distribution of the postseismic relaxation process $|u_p|$ proposed by the authors. The data shown in Fig. 3 present the expected dynamical range for data which shows agreement with the expected probabilities. Fourth, Sambridge et al. (2011) showed that the waiting times between earthquakes presented first digit anomaly. Also in Table 1 we show selected events of the aftershock series as recorded by the Global Centroid Moment Tensor (GCMT Ekström et al. (2012)) of the Tohoku earthquake. We collected data from 11 March

2011 until 31 January 2012, considering a restricted geographic location of the earthquake, to avoid sophisticated filtering of events. The aftershock series is composed of 172 events, located between 12 and 80 kilometres depth and ranging from moment magnitude 4.9 to 9.1 a representation of the aftershock series can be seen in Fig. 4, the color of the circles clearly show the high dynamical range reached by waiting times. From this set, the first digit distribution of the seismic moment released $M_0(t)$ is remarkable and the waiting times $\Delta\tau$ between aftershocks was found to obey a weak statistical significant first digit anomaly at the 5% level. Fifth, regarding the tsunami phenomenon, we analyzed runups (r) data measured by Mori et al. (2011). This data set comprises 5260 points, each of them representing the maximum height inland reached by the water wave generated by the dislocation in the ocean floor. An image is presented in Fig. 5, where the different scale colors present in tsunami data can be appreciated. This data set also presents first digit anomaly.

As a summary, parameters closely related to the source process display Benford's effect and those parameters include: slip and moment distribution on the fault inverted from seismic data, surface deformation inverted from geodetic data, tsunami heights (possibly related to the source itself) surveyed directly and the GCMT aftershock series' moment release and waiting times.

3 A possible explanation of the ubiquity of Benford's law

As has been shown, the first digit anomaly appears in various variables regarding the process of seismic rupture. The earthquake, now viewed not just as the slip phase, contains this signature and it seems natural to search for a unique mechanism, which could explain the anomaly. Indeed a model capable of accounting for global features of earthquake has already been proposed and it is known as self-organized criticality, SOC (Bak and Tang, 1989; Ito and Matsuzaki, 1990; Sornette and Sornette, 1989). We will not try to demonstrate that SOC is the mechanism behind earthquakes, as there is a considerable debate about the relation between SOC and earthquakes (Ramos, 2010) but we will show that the paradigm of SOC, the two-dimensional sand pile cellular automaton (Bak et al., 1988) shows a remarkable first digit anomaly.

A SOC state is a special equilibrium reached by extended systems which are governed by non-linear rules generally under dissipative conditions. This regimen is characterized by power laws and fractal geometries. The existence of various laws of this type in seismology: Gutenberg-Richer, Omori, Båth and lately aftershock density distance decay (Felzer and Brodsky, 2006) are the strongest evidence of some critical mechanism at work, although the exact conditions are still unknown. For a recent view of current research see Pruessner (2012) and for a throughout exposition of the subject see Christensen and Moloney (2005) and Jensen (1998).

We tested two cellular automata, known to present very different behaviors: the one-dimensional sand pile (Bak et al., 1988) and the two-dimensional Bak-Tang-Wiesenfeld (BTW) automaton (Bak et al., 1987, 1988). The one-dimensional pile consists of an array of L integers subjected to random forcing. When a threshold is reached, the forced cell yield, transferring its burden to the next neighbour. Those rules are played asynchronously for a period of time T until meaningful statistics revealing the special equilibrium reached can be collected. This automaton does not present the properties of SOC since the correlation between cells is weak so that the global energy distribution of the pile (the number of consecutive transfers or avalanche) presents exponential properties, the interactions decay fast. On the other hand, the BTW automaton is formed by a bidimensional grid of $L \times L$ points. Again the cells are submitted to random forcing, a threshold is set, and when a cell yields, it transfers its burden to four neighbours. On both automata the borders of the grid are the dissipative points ($1/4$ of the burden is lost in the $2D$ case). After asynchronously playing of the rules, the BTW automaton reaches a state of dynamic equilibrium characterized by avalanches of all sizes. These simple rules give rise to a highly correlated state in time and space as well. The global energy distribution of the automaton is a self-similar power law.

In Table 2, we show the results of the one-dimensional sand pile. We present statistics of automata of different sizes, ranging from the small 11 points grid automaton to the bigger 301 one. It is shown that waiting times between avalanches show Benford's anomaly while the energy E release does not. The explanation to this is quite simple: as the pile does not reach whole size avalanches, because of the weak correlation, most of the events represent avalanches of size one or two, giving to the digits 1 and 2 such high frequencies. On the other hand, the automaton was run for a very long time relative to its size, this long time allows it to reach a diverse population of times between avalanches with first digit anomaly.

In Table 3 we show the BTW sand pile. Again the statistics are shown for automata of different sizes. The range of size is wider because of the higher dimension of the automata. The waiting times and the energetics of the automaton show a remarkable Benford's effect. It should be noted that the lower size automaton exposes weak correlation effect, likewise the $1D$ sand pile. This is related to the finite size of the grid (Bak et al., 1987); the higher the automaton size, the better the first digit anomaly.

There are other models that are more akin to model seismicity. One of the more severe criticisms to the BTW model, is the lack of aftershocks, a common and well established property of earthquakes. However Ito and Matsuzaki (1990) showed an automaton with minor changes in relation to the BTW model that display aftershocks and is capable of reproducing Omori's law. There are even models with no stochastic mechanisms, like the Carlson-Langer model (Carlson and Langer, 1989) in the tradition of the well known Burridge-Knopoff model (Burridge and Knopoff, 1967). Moreover there are automata with nonconservative rules like the Olami-Feder-Christensen (OFC) (Olami et al., 1992), all of them are believed to present self-organized critical equilibria. If they show Benford's effect, then they will be the subject of future studies. But we believe that the first digit anomaly is a symptom of these systems.

Recent studies on these systems (Sarlis et al., 2011a) revealed a striking similarity of the fluctuations of the order parameter, e.g. in the OFC model (Sarlis et al., 2011b), with seismicity. Focusing on these fluctuations before the Tohoku earthquake, it has been found that they exhibit an unprecendented minimum almost two months before in the beginning of January 2011 (Sarlis et al., 2013, 2015; Varotsos et al., 2012, 2014) these fluctuations could be mapped to the exponent in the Gutenberg-Richter law (see figure 6 and the discusion) therefore the first digit anomaly may be used as a general proxy regarding the dynamics of these phenomena.

4 Discussion

Concerning the actual relationship between earthquakes and SOC, we are bringing new information to light. What we have learnt is that if SOC is the underlying mechanism behind the complexity of earthquakes, revealed in power laws, then its first digit imprint is translated into the main observables of seismicity like the energy, tsunami runups and waiting times. That's the case of the earthquake source parameters presented. The aftershocks are an interesting matter as it is believed that the heterogeneous stress drop at the fault generates barriers, which at the end generate the complex patterns found in aftershock series, with Omori's law as one of the main characteristics (Aki, 1979). That the first digit phenomenon encounters stable parameters, like the released seismic moment and waiting times is a strong indication that SOC is at work not only on the generation process, but also on the later liberation at the fault itself.

How far this mechanism could be pushed? Actually the spectral analysis at the core of the criticality (the so called pink noise fingerprint) offers a very general explanation. In figure 6 we present a theoretical variable K which describes some parameter of a natural phenomena at hand, it maybe dissipated energy, waiting times or some other observable. The controlling parameter is the power law behavior with respect to the variable k, modeled as $K \sim k^{-\zeta}$, with the exponent a real number. If we observe the first decade only, one may find the geometrical roots of the first digit anomaly, because the space between 1 and 2 (populated with numbers all starting with 1) is 30.1% of the total decade, the space between 2 and 3 is 17.6% and so on. Therefore the uniform sampling of the process with respect to k implies the first digit anomaly in K, as long as the power law scaling is valid. More important is the repetitive nature of this process: what happens with the first decade, happens all over the available range in K, or in mathematical terms we may map a process ranging various orders of magnitude to the behavior at the first decade, this implies a map from the real line into the circle, generally know as periodicity. The conditions imposed over this supposed system are very general, consequently we expect this behavior to be common in nature, in concordance with the reported analysis of Sambridge et al. (2010). Is it possible to recover a specific SOC model from first digit anomaly alone? At this stage we can not distinguish between them. As discussed, the scaling structure of a critical model is mapped into a periodic space where first digit statistics are calculated, so just with the anomaly it is not possible to retrieve the original SOC model. With respect to the studied parameters. We are considering two kinds of data. Observed and recorded. As long as the models or the instrumentation do not filter out the scaling of the phenomena, turning the power law into something else, we expect the first digit anomaly to be clearly recognized. How many features of the studied phenomena do we need to stablish criticality? It is not clear to us if there is a specific number data to collect or a fixed number of models to ran, but we expect the spectral content to be the key, i.e. we need to preserve the power law scaling.

5 Conclusions

The first digit phenomenon has been taken for a simple mathematical property, but it has proven to be hard to elucidate the true origins of it (Berger and Hill, 2011). We have demonstrated that the phenomenon is not only present in the seismic source process, but it is also present in one of the most remarkable explanations of the earthquake phenomena. We claim that an imprint of the SOC mechanism could be traced back by way of Benford's effect, by the study of time and space observables be those indirectly derived or measured in situ.

The main properties seems to be: 1) the stochastic nature of the earthquake phenomena in study, 2) a scale independent mechanism, ranging in various orders of magnitude, from short period GPS source inversions to long period seismic wave imaging and 3) nonlinear laws of interaction powering the long range correlations.

Acknowledgements. We are specially grateful to professor Armando Cisternas for the critical review of the first drafts. Special thanks go to Lily Seidman who made a throughout review of the english writing. PT and SR were partially supported by postgraduate Conicyt fellowships. JC was partially supported by Fondecyt grant 1130636.

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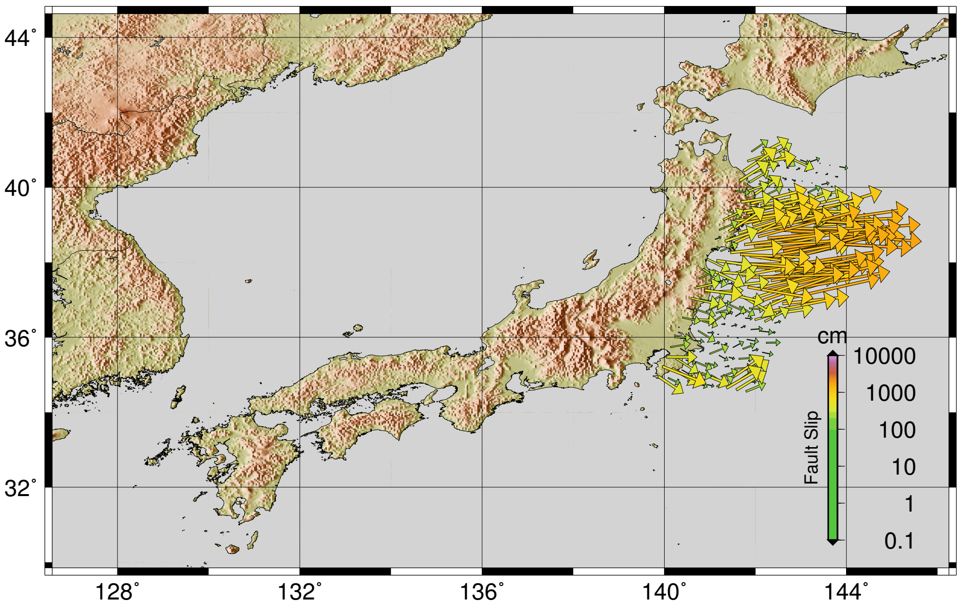

Fig. 1. Finite fault model from NEIC (Hayes, 2011a). Colorbar slip magnitude in centimeters. Size of arrows proportional to slip, rake represented as direction of arrows. From the size of arrows it is clear the existence of displacements in a broad dynamical range, covering at least six orders of magnitude.

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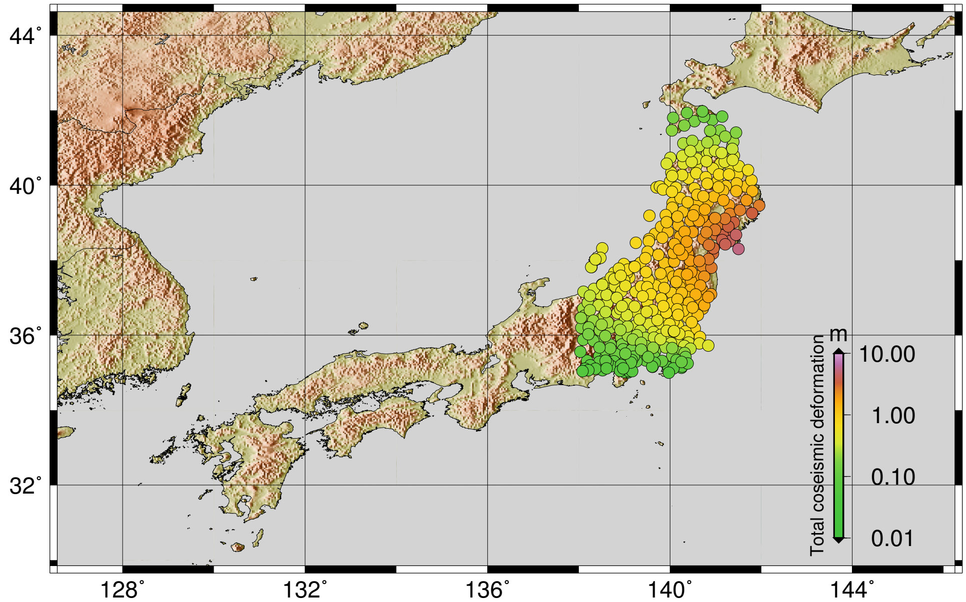

Fig. 2. Coseismic slip distribution (Ozawa et al., 2011). Displacement in meters. Note the dynamical range of data spanning four orders of magnitude.

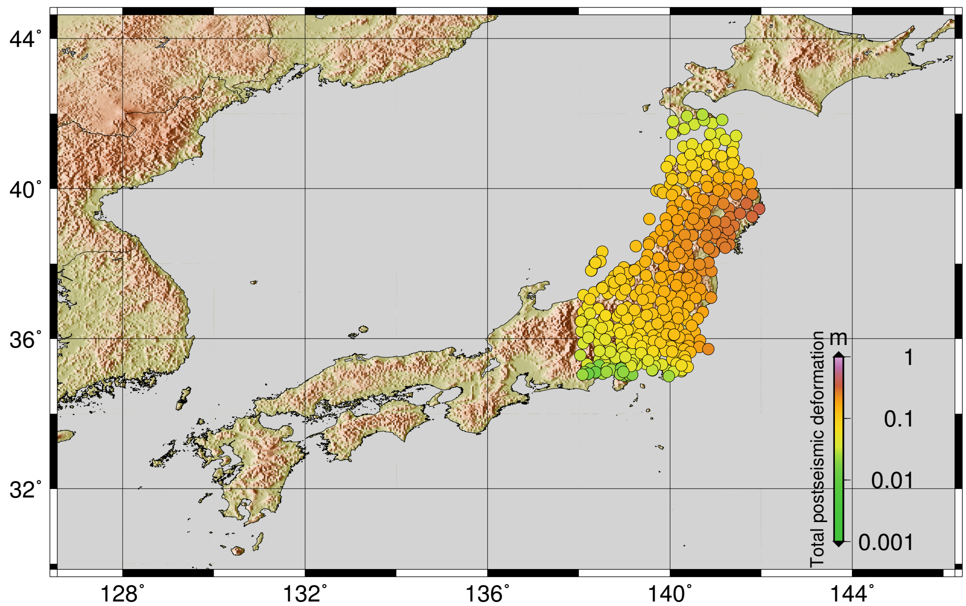

Fig. 3. Postseismic slip distribution (Ozawa et al., 2011). Displacement in meters. Dynamical range of data clearly evidenced from color distribution of circles running at least four orders of magnitude.

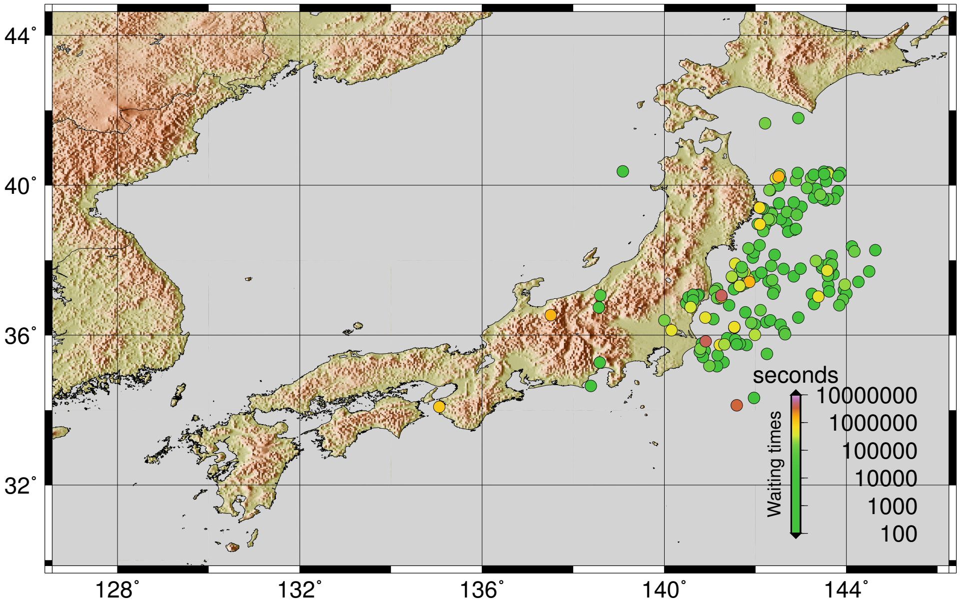

Fig. 4. Selected events of the aftershock series, from 11 March 2012 until 31 January 2012, from GCMT database (Ekström et al., 2012). Dynamical range of waiting times is at least four orders of magnitude.

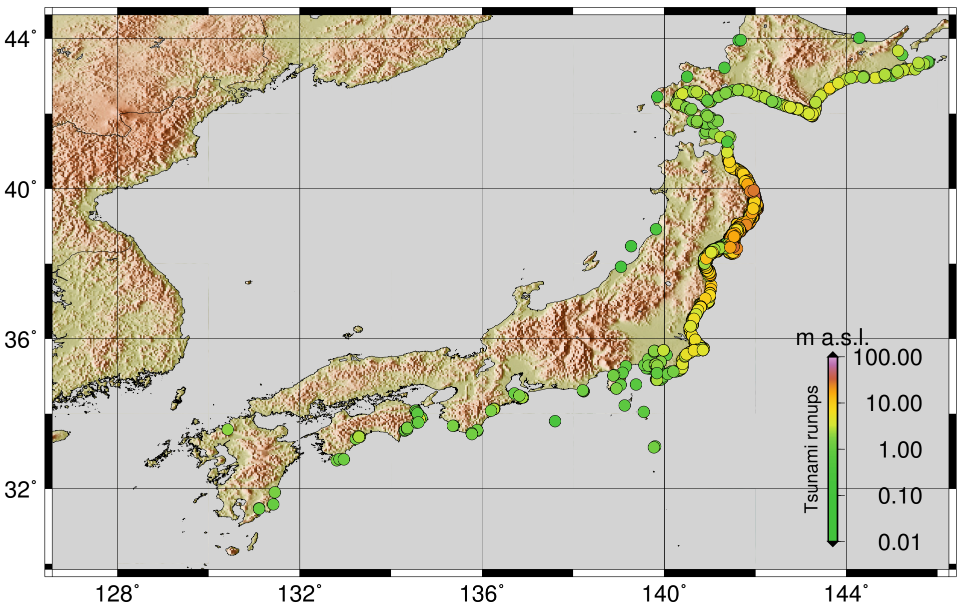

Fig. 5. Runup data measured by Mori et al. (2011). Colorbar in meters. Different scales present in runup data clearly evident from populations present in figure spanning at least 4 orders of magnitude.

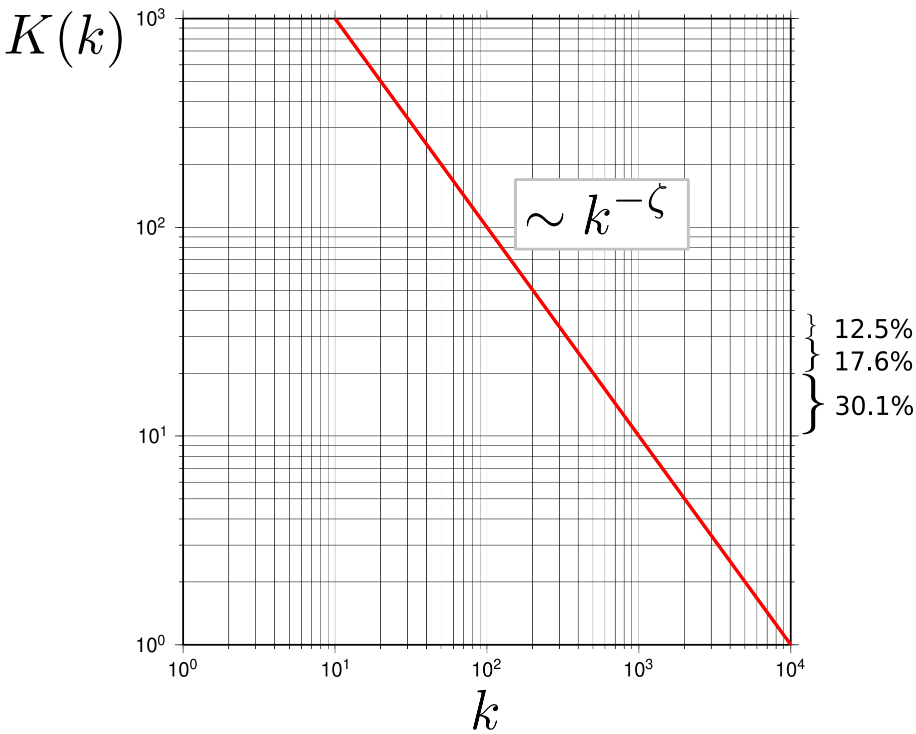

Fig. 6. Theoretical power law behavior of a natural system.

Table 1. Benford's law probabilities P_D in conjunction with the first digit distribution of various parameters related to the source of the Tohoku earthquake and related phenomena.

D	P_D	Finite Fault[a]		Surface GPS[b]		GCMT Aftershock[c]		Runups[d]
		Δu	M_0	$\|u_c\|$	$\|u_p\|$	$M_0(t)$	$\Delta\tau$	r
1	0.301	0.346	0.325	0.340	0.387	0.314	0.327	0.360
2	0.176	0.163	0.154	0.154	0.190	0.169	0.123	0.173
3	0.125	0.113	0.142	0.101	0.078	0.140	0.070	0.104
4	0.097	0.096	0.104	0.101	0.064	0.093	0.076	0.080
5	0.079	0.067	0.104	0.078	0.059	0.070	0.053	0.062
6	0.067	0.058	0.050	0.064	0.056	0.087	0.070	0.062
7	0.058	0.046	0.042	0.064	0.062	0.041	0.058	0.057
8	0.051	0.046	0.025	0.045	0.053	0.052	0.152	0.054
9	0.046	0.067	0.054	0.053	0.050	0.035	0.070	0.049
χ^2_{9-1}		0.340	0.583	0.0123	0.451	0.271	4.628	0.161
p		1.000	1.000	1.000	1.000	1.000	0.797	1.000
n		240	240	377	357	172	172	5260

[a]Slips and Seismic Moment from Hayes (2011b).

[b]Coseismic and Postseismic displacements from Ozawa et al. (2011).

[c]Moment and waiting times aftershock data from GCMT (Ekström et al. (2012)). From 11 March 2011 to 31 January 2012.

[d]Runups data from Mori et al. (2011).

Table 2. Sand pile $1D$ Cellular automaton (Bak et al., 1988). First digit statistics for various $1D$ cellular automata of different sizes.

L		11		21		31		101		201		301	
D	P_D	$\Delta\tau$	E	$\Delta\tau$	E	$\Delta\tau$	E	$\Delta\tau$	E	$\Delta\tau$	E	$\Delta\tau$	E
1	0.301	0.335	0.786	0.328	0.733	0.326	0.796	0.322	0.780	0.315	0.802	0.318	0.812
2	0.176	0.217	0.181	0.204	0.201	0.191	0.163	0.186	0.168	0.189	0.160	0.184	0.154
3	0.125	0.173	0.030	0.137	0.053	0.140	0.034	0.140	0.041	0.139	0.030	0.136	0.028
4	0.097	0.066	0.004	0.093	0.010	0.103	0.006	0.107	0.008	0.103	0.006	0.104	0.005
5	0.079	0.077	0.000	0.083	0.003	0.080	0.001	0.079	0.002	0.080	0.001	0.081	0.001
6	0.067	0.055	0.000	0.058	0.000	0.056	0.001	0.061	0.001	0.063	0.000	0.062	0.000
7	0.058	0.026	0.000	0.044	0.000	0.042	0.000	0.045	0.000	0.048	0.000	0.049	0.000
8	0.051	0.022	0.000	0.026	0.000	0.037	0.000	0.035	0.000	0.035	0.000	0.037	0.000
9	0.046	0.029	0.000	0.027	0.000	0.023	0.000	0.025	0.000	0.028	0.000	0.029	0.000
χ^2_{9-1}		1.110	9.100	0.550	8.130	0.450	9.080	0.410	8.760	0.310	9.200	0.260	9.380
p		1.000	0.330	1.000	0.420	1.000	0.340	1.000	0.360	1.000	0.330	1.000	0.310
T		1210		4410		9610		102010		404010		906010	

Table 3. BTW $2D$ Cellular automaton (Bak et al., 1987, 1988). First digit statistics for various $2D$ cellular automata of different sizes.

$L \times L$		11×11		21×21		31×31		101×101		201×201		301×301	
D	P_D	$\Delta\tau$	E	$\Delta\tau$	E	$\Delta\tau$	E	$\Delta\tau$	E	$\Delta\tau$	E	$\Delta\tau$	E
1	0.301	0.333	0.512	0.438	0.360	0.384	0.327	0.387	0.360	0.368	0.337	0.364	0.318
2	0.176	0.231	0.242	0.185	0.230	0.219	0.165	0.206	0.183	0.209	0.184	0.207	0.183
3	0.125	0.190	0.130	0.148	0.153	0.130	0.133	0.135	0.115	0.138	0.126	0.136	0.131
4	0.097	0.095	0.072	0.105	0.105	0.100	0.107	0.088	0.087	0.091	0.094	0.095	0.099
5	0.079	0.061	0.025	0.038	0.060	0.076	0.084	0.063	0.066	0.065	0.073	0.063	0.078
6	0.067	0.041	0.012	0.043	0.042	0.023	0.064	0.043	0.059	0.048	0.058	0.051	0.061
7	0.058	0.027	0.004	0.016	0.024	0.026	0.048	0.037	0.050	0.035	0.049	0.036	0.051
8	0.051	0.007	0.003	0.019	0.017	0.023	0.039	0.026	0.043	0.024	0.042	0.027	0.042
9	0.046	0.014	0.000	0.008	0.009	0.021	0.033	0.015	0.038	0.021	0.036	0.021	0.036
χ^2_{9-1}		2.120	4.590	2.250	1.810	1.500	0.200	1.120	0.180	0.930	0.140	0.830	0.110
p		0.980	0.800	0.970	0.990	0.990	1.000	1.000	1.000	1.000	1.000	1.000	1.000
T		1210		4410		9610		102010		404010		906010	