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Critical behavior in earthquake energy dissipation

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Abstract

We explore bursty multiscale energy dissipation from earthquakes flanked by latitudes 29 and 35.5° S, and longitudes 69.501 and 73.944° W (in the Chilean central zone). Our work compares the predictions of a theory of nonequilibrium phase transitions with nonstandard statistical signatures of earthquake complex scaling behaviors. For temporal scales less than than 84 h, time development of earthquake radiated energy activity follows an algebraic arrangement consistent with estimates from the theory of nonequilibrium phase transitions. There are no characteristic scales for probability distributions of sizes and lifetimes of the activity bursts in the scaling region. The power-law exponents describing the probability distributions suggest that the main energy dissipation takes place due to largest bursts of activity, such as major earthquakes, as opposed to smaller activations which contribute less significantly though they have greater relative occurrence. The results obtained provide statistical evidence that earthquake energy dissipation mechanisms are essentially "scale-free," displaying statistical and dynamical self-similarity. Our results provide some evidence that earthquake radiated energy and directed percolation belong to a similar universality class.

1 Introduction

Earthquakes represent an area of research with important theoretical and practical implications. In terms of basic human wellbeing, earthquakes are natural hazards with devastating consequences throughout history. As natural events, they possess qualities in common with other areas of scientific research and, as such, their universal qualities offer the tantalizing suggestion that with further research their shattering social and economic costs might be reduced.

A predominantly empirical approach is typically applied to energy dissipation and associated dynamics in earthquakes. For instance, well-known empirical laws include the Omori law giving measures of the temporal configuration of aftershocks and the

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Gutenberg–Richter law for the relation between earthquake frequency–magnitude of tremors. The standard view in the community is that seismicity is the product of multi-scale interactions between tectonic plates. In this case, relative movement of tectonic plates steadily leads to stress increases. Energy release in the form of seismic waves follows these increases, with dynamics exhibiting intermittent temporal patterns where periods of high activity are separated by periods of relative quiet. Rather than following a smooth path, earthquake dynamics follow an avalanche or punctuated type development reminding one of self-organized critical systems (Sornette and Sornette, 1989).

Over the past few decades research has begun to explore the possibility of our planet's seismicity being in a self-organized critical state (Pastén et al., 2011; de Carvalho and Prado, 2000; Chelidze and Matcharashvili, 2007; Ito and Matsuzaki, 1990; Bak and Tang, 1989; Sornette and Sornette, 1989). These studies show how the earthquake energy release distribution (seism energy or avalanche size) follows a power law distribution, and how spatial distributions (of avalanches or the seism magnitude) shows (multi)fractal structure.

In this paper we propose to study ensemble-averaged statistical properties of earthquakes in Chile. Since we do not have detailed knowledge about underlying dynamics, it is important to attempt to explain the ensemble averaged arrangement of correlations to extract information of the dynamical features of a time series. In general when a non-linear system has many sources of spatially dispersed instability it cannot be wholly characterized in deterministic terms. That is why a statistical analysis is necessary. Applicable statistical-physical methods of analysis allow one to extract a great deal of information about system dynamics found in multiscale correlations of non-Gaussian random variables (Chicheportiche and Chakraborti, 2014). Consequently, we frame the following working hypothesis: there exists a correlation, at least at the statistical level, between any two successive events. It does not matter how distant they are. In other words, one earthquake can be the trigger for the next one over 1000 km away (Steeples et al., 1996). This means the system correlation length can be divergently large, which in turn suggests a relationship or similarity to critical phenomena.

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Using this approach we seek to probe earthquake data towards answers of the following questions: Do seismic disturbances of various sizes exhibit a distinctive temporal signature? In terms of overall energy dissipation what are the relative contributions of large- and small-scale earthquakes? If there is a characteristic scale does this suggest the dominance of a specific dissipation mechanism?

Our work finds motivation in the above questions, which point us to a few interesting and novel results regarding the temporal dynamics of earthquake energy radiation and dissipation. Various studies of earthquakes frequently demonstrate robust power-law statistical relations. We will explore whether these are consistent with the dynamics of nonequilibrium systems undergoing transitions between several metastable states. Given that stress buildup is released within minutes, even seconds, we have an intuitive expectation that earthquakes, as a slowly driven threshold phenomenon, might exhibit such metastable, critical behavior. If such is the case it may suggest that the observed seismic activity bursts are not dominated by characteristic space, time, or energy scales; and so the energy dissipation mechanisms associated with earthquakes are essentially "scale-free".

We already know the Omori and Gutenburg–Richter scalings hold for both energy and timing data. However, not all power law behavior is necessarily an effect of dynamical self-organization into a critical stationary state (Watkins, 2002; Watkins et al., 2009). In this paper we wish to go beyond the well-known scaling behaviors and explore the question of whether earthquakes fit the hypothesis of an avalanching critical system. We propose to do this by defining and finding a set of new power law exponents, and compare these explicitly to predictions for critical avalanching systems.

2 Statistical theory

When equilibrium systems display critical behaviour the most common characterization is via long-range correlations propagating through the system. This is usually realized by the fine-tuning of a control parameter (Stanley, 1999). What this demonstrates is

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that many systems, near their critical points, tend to produce long-range and scale-free correlations having universal statistical properties. The system in thermodynamic equilibrium, and system phase transitions, is the most familiar setting for discussion of critical system reconfigurations.

However, nonequilibrium situations such as the onset of fluid convection, also exhibit critical reconfigurations. Similarly, models of dynamics in markets (Lammoglia et al., 2008), epidemics (Cardy and Grassberger, 1985; Janssen, 1985), space weather (Klimas et al., 2000; Sitnov et al., 2000; Valdivia et al., 2006, 2013; Dominguez et al., 2014), and city traffic (Toledo et al., 2004; Villalobos et al., 2010; Toledo et al., 2013) are other respectable examples, and references above suggest the earthquakes may be party to a similar universality.

The nascent theory of nonequilibrium critical systems offers an explanation for scale-invariant dynamics in systems both driven and distorted. Critical behaviour is, as we mentioned above, often associated with the fine tuning of one or more system parameters. Systems in a so-called self-organized critical (SOC) state exhibit scale-free correlations, correlations usually accompanying criticality, giving the appearance of arising spontaneously. That is why it can be a frustrating exercise to search for a specific trigger that initiates the explosive system reconfiguration – there is often simply no unambiguously and unique trigger (Wanliss, 2005). For example, if a system is in a SOC state it will be barely stable and far from equilibrium. Yet it will reliably return to the critical state again and again, thus responding resiliently to driving.

We note that a mere demonstration of scale-invariance at a point in parameter space is insufficient to elucidate the behaviour of nonequilibrium natural systems. Furthermore, it does not directly offer any explanation of how to maintain the system at, or close to, the critical point. In the language of earthquake dynamics, the existence of power law behavior (e.g. Omori or Gutenberg–Richter) is not sufficient to guarantee that the dynamics are consistent with the statistics of a non-equilibrium critical avalanching system.

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To explore such issues we will use recently developed frameworks (Dickman, 1996; Muñoz et al., 2001) to describe dynamics of critical reconfigurations – the so-called avalanches (Bak et al., 1988) – taking place in a general class of nonlinear dynamical systems with coupled degrees of freedom. We will define $N(\tau)$ as the average number of avalanche sites which are active. This is the same as the average excitation area in the continuum limit. Here we define the parameter τ as the temporal interval between each initiation of an avalanche. The nonequilibrium models mentioned above, for delay times less than the time-scale introduced by finite-size effects, predict that close to the critical point, the number of avalanche sites follows a power law scaling as follows: $N \propto \tau^{\eta}$. In like manner, if we define $P_s(\tau)$ as the probability of an avalanche surviving by this time, then theory also predicts a power law scaling: $P_s \propto \tau^{-\delta}$. Two power law exponents η and δ are introduced, and these are termed spreading-exponents (Muñoz et al., 2001). The avalanche size S is the total number of active sites participating in the system rearrangement. The avalanche lifetime is represented by T. For SOC, which is a nonequilibrium critical state, there is a strong connection between the avalanche lifetime and size. Following the definition of δ and η, one can demonstrate how there must exist a scaling relationship for the average number of active sites in surviving runs which participated in producing avalanches with $T > \tau$. The scaling is on the order of $\tau^{\eta+\delta}$. In addition, upon creating the time integral of this quantity to compute the characteristic size S of the event, we realize $S \sim T^{1+\eta+\delta}$ (details are found in Muñoz et al., 2001). Such scaling relations comprise a central role in the statistical analysis of nonequilibrium critical systems (Paczuski, et al., 1996; Marro and Dickman, 1999; Dickman et al., 2000).

In this paper we wish to examine the extent to which the avalanches of nonequilibrium statistical physics, mentioned above, can be related to bursts of seismic activity in the crust and mantle of the earth. In pursuit of this objective, we here demonstrate the time-series based version of the spreading exponent study (Wanliss and Uritsky, 2010). We treat bursts of seismic activity as physical indicators of spatiotemporal spreading dynamics.

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3 Data and analysis method

The data used for this paper are produced by the *Chilean Servicio Sismologico Nacional* (National Seismologic Service). They were recorded from October 2000 to January 2007 and comprise in excess of 17 000 seismic events of magnitude above 1.6. All data were recorded between longitudes 69.501 and 73.944° W and latitudes between 29 and 35.5° S. The data fall within a volume having dimensions $L_z = 700$ km in depth, $L_{\mathrm{NS}} = 730$ km long in the North–South direction, and $L_{\mathrm{EW}} = 500$ km in the East–West direction.

From these data we have the hypocenter (the 3-D point of the seismic event), the time of the seismicity, and the Richter or local magnitude M_{L} (Richter, 1958), which is directly related to the energy release and to the amplitude of the seismic event:

$$M_{\mathrm{L}} = \log_{10}(A) - \log_{10}(A_0)\delta_0$$

In this equation δ_0 is the distance from observation location to the epicenter, A is the amplitude of the S waves measured 600 km from the epicenter, and A_0 is a standard value, depending on the temporal interval between P and S wave observation at the recording station.

Since we wish to examine energy dissipation, the earthquake magnitudes must be convolved to seismic radiated energy, according to the following formula:

$$E = 10^{4.8+1.5M_{\mathrm{L}}}$$

where E is the radiated energy in Joules (Båth, 1966). Radiated energy is a dynamic measure of earthquake size that depends on the details of the rupture process. For instance, when there is slow slip on a fault, perhaps only a small amount of energy is radiated, yet this may have the same Richter magnitude in a felt earthquake which actually radiates far more energy.

Figure 1 shows the raw, unsmoothed radiated energy data. Note that these data, covering over seven orders of magnitude in energy, show evidence of strong intermittency, and have a multifractal nature (Pastén and Comte, 2014). Given that earthquake

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magnitudes, from which the radiated energy curve is calculated, are often like point processes in a given geographic region, there may be periods where radiated energy is below the measurement threshold so that there are only apparent intermittent spikes in energy above a zero threshold. Since the original data set considers seismic magnitudes only above 1.6, any lower magnitude tremors contribute zero in the energy budget as they are effectively ignored. In addition, each earthquake magnitude is given at a particular instant, whereas in reality the radiated energy profile is spread out, rather than a delta function. Since this is unrealistic, and since the effect is to have long gaps where there is no radiated energy reflected in our raw data, we convolve the time series of earthquake magnitudes to create a modified radiated energy profile.

First we produce a time series of radiated energy on a uniform grid. This mitigates the effects of long gaps with no seismic activity. Next we smooth the radiated energy data using a running average filter. Smoothing the radiated energy data using a running average filter avoids the problem of the most energetic seismic events destroying information contained in lower energy events found in an averaging box. Specifically we use a second degree polynomial model to perform a local regression with weighted linear least-squares. This reduces the impulsive nature of the raw data by flattening the raw energy profile. Without such a convolution it is impossible to perform critical scaling analysis on our data.

We create the final cumulative radiated energy time series by summing the energy within a box of length 1 week, then moving the box through the data with a step size of 1 min.

We selected this temporal cadence and box size because the minimum gap between recorded seismic events is on the order of a minute and the maximum time interval between seismic activity above the minimum threshold is 2.2 days. We experimented to see the results of our analysis with different box sizes, and found little difference within the experimental uncertainty, and thus restrict our discussion to the box size of 1 week.

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The smoothing reduces the dominance of local seismic events that are orders above the average, and spread the energy of these events. This can be seen in Fig. 2, which shows energy data. These data are less intermittent than the raw data of Fig. 1, reflecting the smoothing effect of the box size of one week. To further demonstrate the difference between this energy profile and the point process nature of the raw data, we now plot a four month interval of these data. Figure 3 shows a comparison of the raw radiated energy profile, for the interval from March through June 2002, which is highly intermittent compared to the modified radiated energy computed in the box of length one week.

Note from this figure how the modified radiated energy profile smooths the data but retains the continued effect of the dominant seismic event within a given box which, though still evident, no longer makes irrelevant the other seismic events within any given box. For instance, see around 1 April 2002, 1 May 2002, and just prior to 1 June 2002.

Now that we possess a reasonable measure of cumulative energy radiation, we are able to assess the extent of the seismic behavior that reflects critical behavior. If the energy perturbations are near a critical state, it should possible to recover some, perhaps all, power-law relations introduced in the previous section.

To do so we follow the methodology of Wanliss and Uritsky (2010) and decompose the radiated energy data into series of activity bursts. We define an activity burst (AB) to be a transient increase of the seismic radiated energy, from the time series shown in Figs. 2 and 3. Such an AB is flagged if the data exceed a specified constant threshold. We define the waiting-time as the time interval between two such bursts of activity. If a system is SOC-like there can be a broad range of thresholds where gradients of the power-law *portion* of the burst lifetime distribution are constant (Wanliss and Weygand, 2007).

We define as the lifetime T for each AB the temporal interval during which the data exceeds the threshold. In addition we compute the AB size, $S = \int_{\{T\}} (E_{\text{rad}}(t) - L)\mathrm{d}t$, where L is the selected threshold. E_{rad} is the cumulative radiated energy data, reflecting the dissipation of stress within the earth's crust.

We follow definitions of S and T that match energy-based estimates of sizes and lifetimes of critical avalanches in "running, continuously driven " SOC models (Hwa and Kardar, 1992). Since in some sandpile models the number N of active elements is commonly proportional to the energy dissipated, we use an analogous reasoning for the cumulative radiated seismic energy to define an N^* scaling in much the same manner as N; $N^* = \langle E_{rad}(t_1 + \tau) - L \rangle$. As mentioned above the prospect an AB lasts beyond a certain time interval is termed the survival probability: $P_s(\tau) = n(\tau)/m$, where n is the number of ABs with $T > \tau$ and m is the total number of ABs.

4 Results and discussion

Figure 4 shows plots of $N^*(\tau)$ for cumulative radiated seismic energy. Figure 5 shows the associated plots for the survival probabilities $P_s(\tau)$. In these figures we show results for the threshold of $L = 1.39 \times 10^{12} J$ which corresponds to the 50th percentile for our data signal; we find simultaneous power-law fits over a broad range of thresholds L for cumulative radiated seismic energy, up to the 90th percentile. For both cases there is some divergence for the power law fits for the smallest lifetimes. Note that there is a strong break in the average number of activity bursts occurring past periods of 3 days. We therefore attempt to fit a power law over the interval $\tau < 3.5$ days. Fitting of power laws is difficult and an extensive literature demonstrates a preference for maximum-likelihood methods (Corral et al., 2014), which we follow as well. We use the robust Levenberg–Marquardt algorithm for nonlinear least squares (Seber and Wild, 2005), to estimate best-fit critical scaling exponents, viz. $\eta = 1.11 \pm 0.01$ and $\delta = 0.079 \pm 0.001$.

In the event they exist it is possible to relate theoretically, to some extent, the dynamic critical exponents with avalanche scaling exponents. It is via probability distributions of avalanche sizes and lifetimes that the latter are estimated. Here we approximate avalanche and lifetime distributions by power laws with exponential cutoffs,

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namely $P(S) \propto S^{-t_s} \exp(-S/S_c)$ and $P(T) \propto T^{-t_T} \exp(-T/T_c)$. The latter exponential cut-offs account for deviances from self-similar statistics at the largest scales which are the product of the limited number of massive seismic events possessing these largest timescales. In Fig. 6 we plot these distributions for the threshold at the 50th percentile.

To demonstrate the robustness of the power law exponents, irrespective of L-threshold used, Table 1 presents the fitting parameters calculated from the probability distributions for all studied thresholds. There is reasonable agreement except for at and below the 25th percentile for t_T. This deviation may exist because there are simply not enough avalanches at this threshold to measure reasonable statistics.

Consider now the overall earthquake radiated energy dissipation. Our results present the opportunity to contrast and compare the relative importance of small and large ABs. Given the scaling ansatz for $p(S)$ previously introduced the rate of total energy release by the ensemble of bursts scales like

$$E_{\mathrm{tot}} \sim \int_{S_0}^{S_c} S\, p(S)\, \mathrm{d}S \sim \left. \frac{S^{2-t_S}}{2-t_S} \right|_{S_0}^{S_c}, \quad t_S \neq 2$$

where $S_0 \ll S_c$ is the tiniest burst size in the data. One notices that large bursts dictate the release of energy if $t_S < 2$. On the other hand small ABs dominate the energetics when $t_S > 2$. We find t_S much below 2, which suggests that large ABs are most important in the dissipation of earthquake total energy.

Let us now attempt to relate the avalanche exponents t_S and t_T. First note that conservation of probability gives $p(S)\mathrm{d}S \sim p(T)\mathrm{d}T$ and then apply the scaling law $S \sim T^{1+\eta+\delta}$ which was derived previously. Then, over the power-law regions, we find $t_T = (1+\eta+\delta)(t_S-1)+1$. Because δ is the survival probability exponent, it should relate directly to the cumulative avalanche lifetime probability distribution; it equals the power-law slope $t_T - 1$. This produces $t_T = (\eta[t_S-1]+1)/(2-t_S)$. The scaling relation we find matches rather well with the analysis of spreading and avalanche exponents first introduced by Muñoz et al. (2001). They found that $t_S = (\eta+2\delta+1)/(\eta+\delta+1)$. This expression shrinks to our formula following the substitution $\delta = t_T - 1$. The theoretical value then, calculated from $N^*(\tau)$ is $t_T = 1.08 \pm 0.01$ which compares favourably with the values calculated from $p(T)$, viz. $t_T = 1.02 \pm 0.09$.

The relation $t_S = (\eta + 2\delta + 1)/(\eta + \delta + 1)$ works fairly well in the case of the earthquake radiated energy for the area of Chile we have examined: the predicted value is $t_S = 1.03 \pm 0.01$, which is close to the value measured directly via $p(S)$, viz. $t_S = 1.00 \pm 0.05$ (the two estimates are statistically indistinguishable at the confidence test level $p = 0.95$).

In Fig. 7 we show the size of avalanches vs. lifetimes. The effect of the largest earthquake seisms is evident with the broadening of the range of sizes for the largest lifetimes. The best-fit critical exponent calculated from the local slope was 2.04 ± 0.16. In critical avalanching systems, the lifetimes and sizes are related by $S \propto T^{1+\eta+\delta}$. Our results show the strong connection between the experimental theoretical value between avalanche lifetime and size ($1 + \eta + \delta = 2.19 \pm 0.02$).

In Table 2 we summarize the results, giving scaling exponents measured directly from power laws produced by the seismic processes, and comparing these to theoretical predictions calculated from η and δ values. These latter values we derive from $N^*(\tau)$, the average number of active avalanche sites, and the survival probabilities $P_s(\tau)$.

5 Conclusions

We have find two different echelons of results. The first is observational evidence of a broad-range scaling in the earthquake radiated energy dynamics between October 2000 and January 2007, for latitudes 29 to 35.5° S, and between longitudes 69.501 and 73.944° W.

Our analysis adds a helpful piece of information to the existing portrait of the earthquake energy dissipation by uncovering its multiscale nonlinearity. We have shown the absence of any time scales where the earthquake radiated energy response is linear.

Our results go beyond standard power-law behaviors like the Omori and Gutenberg–Richter laws, and may serve as an additional validation tool for models of earthquake energy dissipation in terms of their ability to represent correctly effects of cross-scale coupling (Bhattacharya et al., 2011).

The second level of results, so far less concrete, shows how earthquake energy dissipation, and its multiscale dynamics, results from cooperative behaviour ruled by a specific statistical principle. We associate such dynamical behaviour with nonlinear interactions of spatially extended degrees of freedom (e.g. multiscale fracture and stress regimes in rock) that maintain the system in the region of a global critical point. Such an interpretation, while supported by the achieved statistics, is far from final. On the basis of simulation results for nonlinear critical models (Muñoz et al., 1999) our evidence points to the idea that earthquake radiated energy comes from a geophysical mechanism that connects a one-dimensional mass and/or energy transport realized in a medium of two-dimensions. This matches expectations for the fracture process, or even dynamics of large-scale fault slippage (Ben-Zion and Rice, 1997; Xu et al., 2014).

In summary, we have found that on average temporal development of bursts of radiated earthquake energy follows statistical forms matching predictions from theories of nonequilibrium phase transitions. Our results provide direct evidence for dynamical and statistical self-similarity in earthquake energy dissipation and signal its possible critical behavior.

The existence of a dynamic critical power-law exponent of burst size distribution suggests that the main energy dissipation is associated with large ABs such as major quakes. Smaller activations contribute less despite their larger relative occurrence. This is consistent with the well-known energy dissipation laws. Based on the t_s value obtained, the rate of total seismic energy radiated scales as $S_c^{-t_s+2} \sim S_c^{0.9}$ meaning it is governed by the upper cutoff scale, which diverges in the limit $S_c \to \infty$. However, we find that we can describe the $P(S)$ distribution by a single power-law exponent. This suggests that both small- and large-scale ABs can be a indicator of emergent behavior allied with intensive cross-scale coupling of multiple intrinsic dissipation mechanisms.

These results begin to paint a coherent portrait of critical fluctuations in earthquakes. Of course, the mere existence of uncomplicated algebraic relations between multiple kinds of scaling exponents is not a definitive indication of critical systems. Scaling relations can be expected for any fractal stochastic process. But the form of the scaling can vary quite a bit depending on the underlying physics. For example, it is well known in fractional Brownian motion (fBm), multifractional Brownian motion (mfBm) (Wanliss et al., 2014), or other processes (Anh et al., 2005). Peng et al. (1994) used a time-varying threshold in fBm to determine that $t_S = 2/(1-H)$ and $t_T = 2-H$, and so $t_T = 3-(2/t_S)$. This implies that t_T can be defined uniquely via the value of the exponent t_S irrespective of the Hurst exponent H which describes the time series correlation structure. Therefore, fBm scaling is quite different from that found in some critical avalanching systems, and in the dynamics of earthquake radiated energy.

One can imagine a situation where some probabilistic model in which H is controlled by multiple exponents – for instance, linear fractional stable motion dependent on memory d and a stability exponent *alpha* – reproduces scaling relations without needing to rely on scaling arguments of the type we have invoked. We advocate the search for suitably rich, yet still stochastic, models as complex "null models" by which to compare with SOC (Wanliss et al., 2005; Yu et al., 2010). The scaling relations of Carbone and Stanley (2004) were subsequently perceived to exist via fixed thresholds, for fBm, by Rypdal and Rypdal (2008). In a future study we will explore the possible (multi)fractal structure of earthquake cumulative radiated energy.

We do not wish to overstate the consistency of the earthquake energy dissipation dynamics with the scaling relations predicted for a specific class of critical models with absorbing configurations (see e.g. Dickman, 1996; Muñoz et al., 1999, 2001). In Hinrichsen's (2006) review, there is noted the existence of the absorbing states. He calls these, *"a highly idealized requirement that is difficult to realize experimentally"*. But we note that it has recently been achieved (Kazumasa et al., 2009; Hinrichsen, 2009). It remains open whether it is possible to treat earthquake dynamical states as

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absorbing configurations. Our analysis of seismic dynamic critical scaling exponents supports the confirmatory reply.

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Table 1. Avalanche scaling exponents from the lifetime and avalanche size probability distributions for various AB thresholds.

Threshold	t_S	t_T
90 (6.93×10^{12} J)	1.00 ± 0.04	0.81 ± 0.08
75 (3.03×10^{12} J)	0.92 ± 0.02	0.82 ± 0.06
50 (1.39×10^{12} J)	0.96 ± 0.02	0.81 ± 0.05
25 (0.78×10^{12} J)	1.00 ± 0.03	0.94 ± 0.05
10 (0.45×10^{12} J)	0.99 ± 0.03	0.93 ± 0.04

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Table 2. Compilation of the theoretical values for scaling exponents, calculated from η and δ, and the equivalent experimental measurements from actual distributions.

Exponent	Measurement	Theory
η	1.11 ± 0.01	
δ	0.079 ± 0.001	
t_S	1.00 ± 0.05	1.03 ± 0.01
t_T	1.02 ± 0.09	1.08 ± 0.01
$\eta + \delta + 1$	2.04 ± 0.16	2.19 ± 0.02

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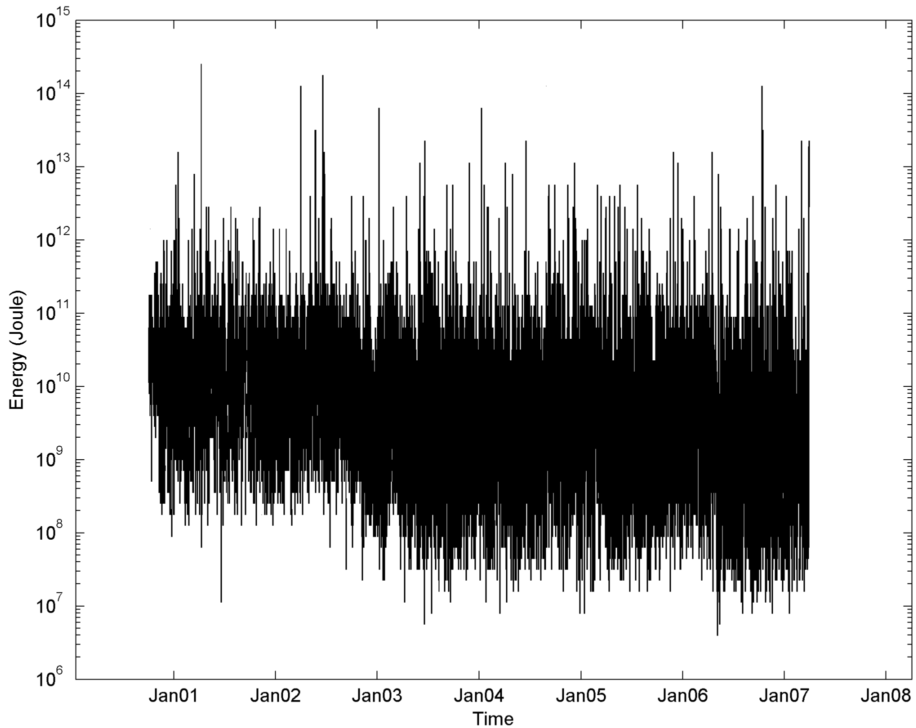

Figure 1. Radiated energy for 2001 through 2007.

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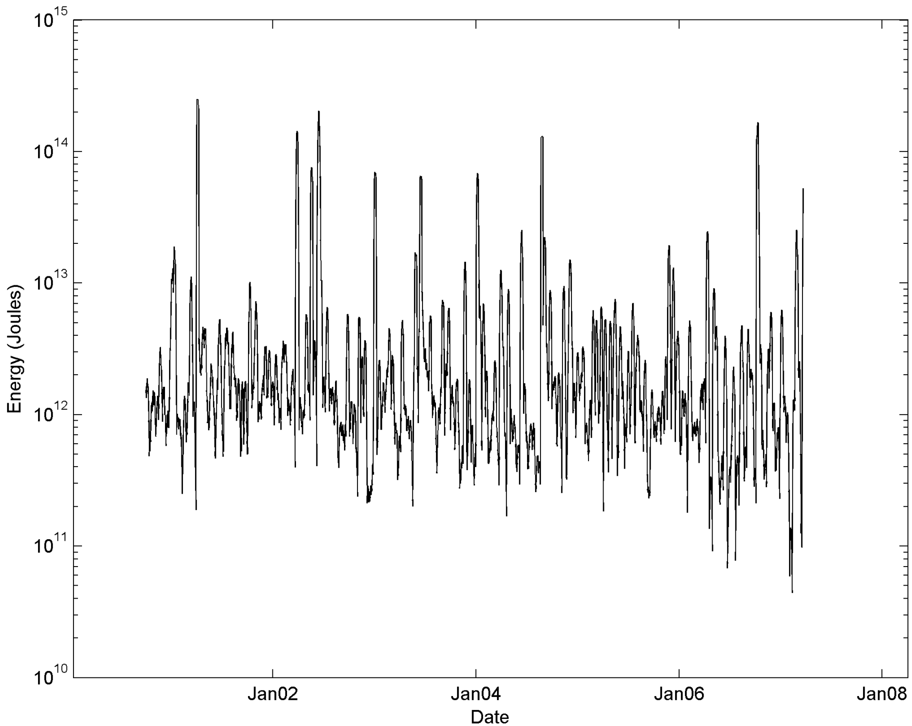

Figure 2. Radiated energy profile computed in windows of size 1 week.

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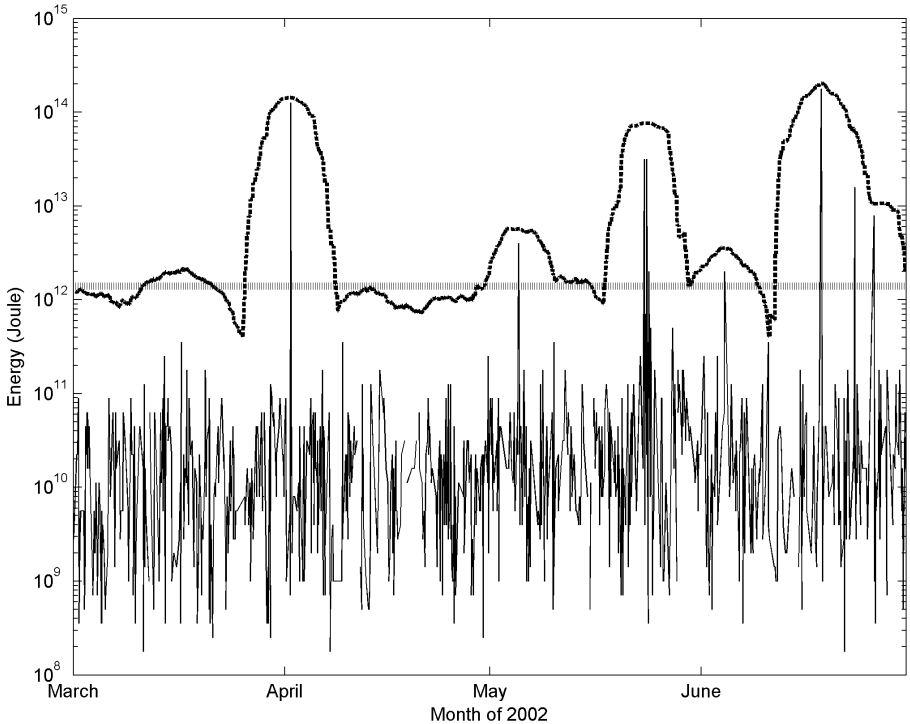

Figure 3. Comparison of the raw point process radiated energy profile for March through June 2002, showing large intermittency, and the smoothed total radiated energy computed in boxes of 1 week. The horizontal line shows the 50th percentile for the full data set shown in Fig. 2.

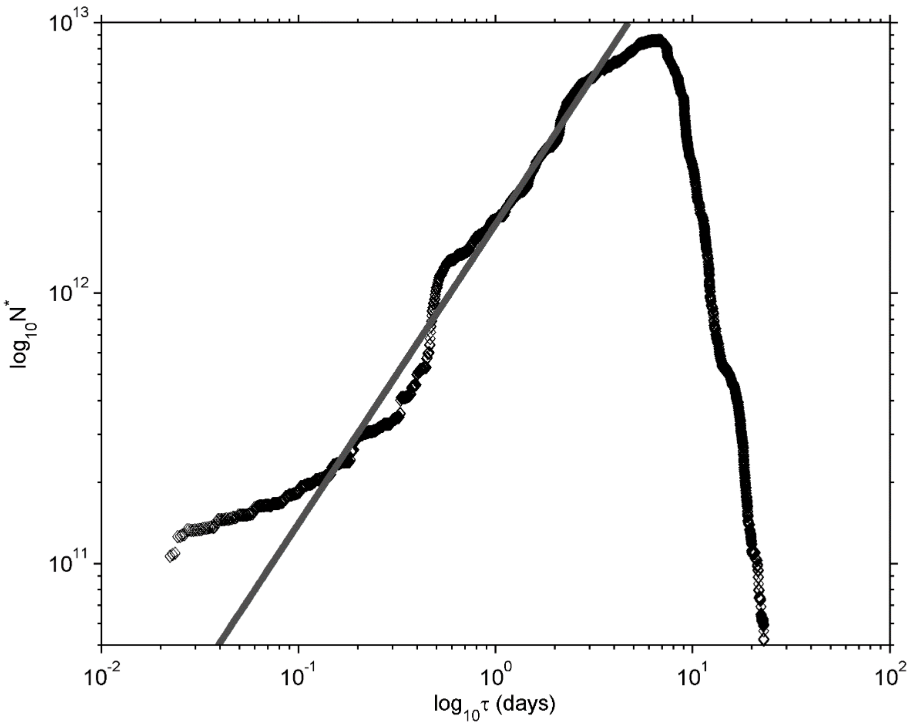

Figure 4. N^* as a function of lifetime τ.

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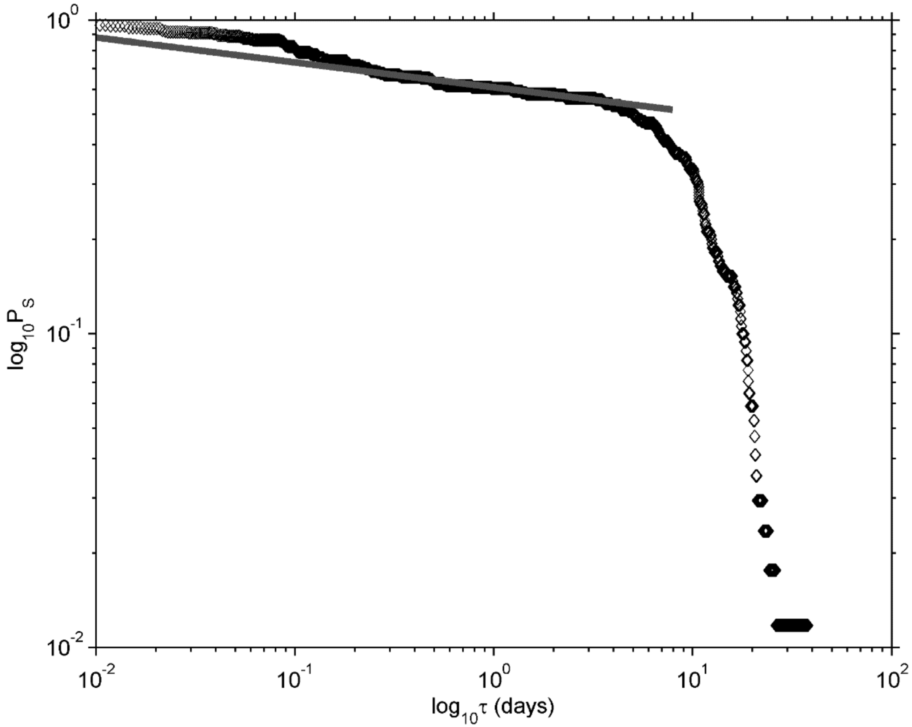

Figure 5. Survival probability P_S as a function of lifetime τ.

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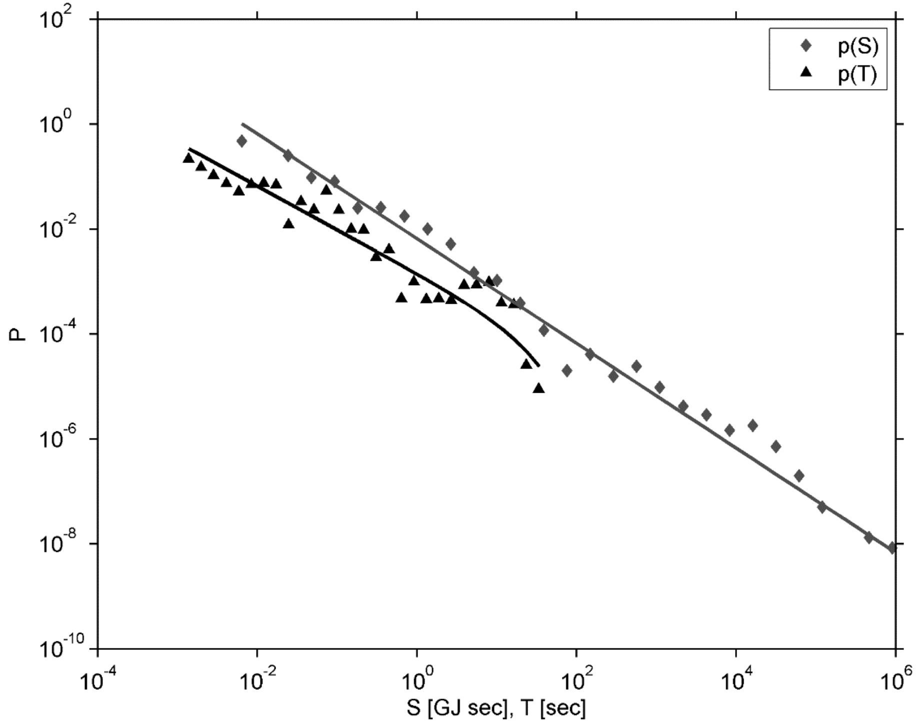

Figure 6. Avalanche probability distributions $P(S)$ (grey diamonds) and $P(T)$ (black triangles) for the threshold of $L = 1.39 \times 10^{12}$ J.

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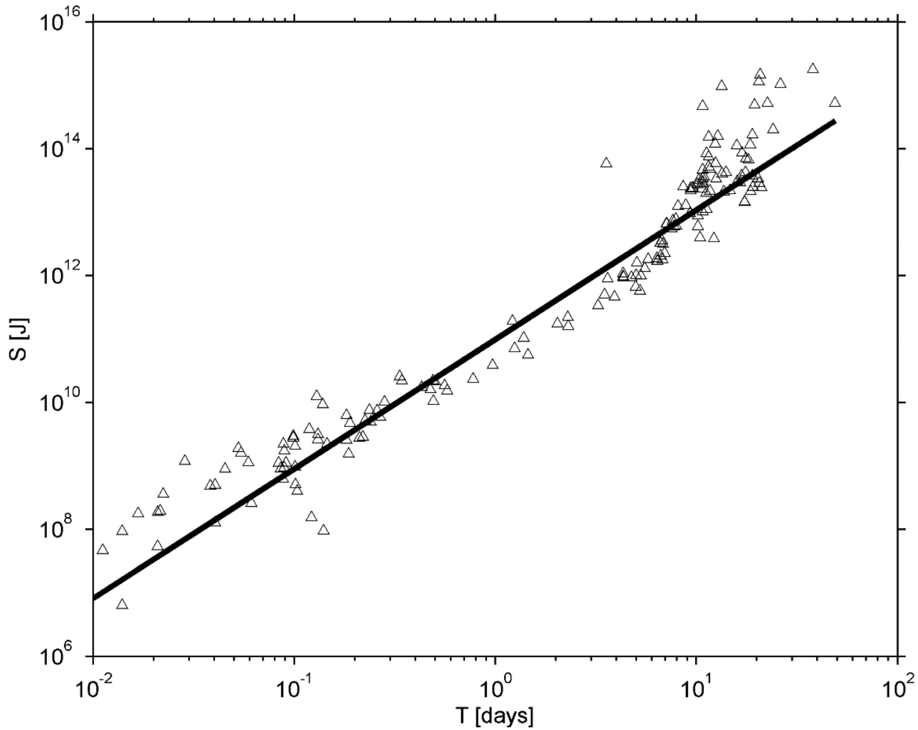

Figure 7. Burst size S as a function of lifetime T.

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