Contents

Part 1 Comments and replies

Comments of Referee #1

Received and published: 3 February 2015

The paper applies a new method to detect abrupt changes in an observable. It is based on the famous logistic equation studied by Robert May in the mid-70s. The method is straight forward and the authors have done a good job in presenting it. They apply it first to an ideal model, thereby demonstrating the principle, and then they apply it to a time series representing the PDO. They conclude that the abrupt changes found in the PDO relate to global warming. I have the following comments:

1) In figure 5b why state 4 does not have the same frequency as state 2? I would haveexpected (due to the symmetry of the ideal model) to be the same.

2) That the PDO is involved in global temperature variability has been demonstrated in other previous studies (for example, Tsonis et al: A new dynamical mechanism for major climate shifts, Geophys. Res. Lett., 34, L13705, 2007).

3) I would like the authors to discuss if their method can be used to predict the next abrupt change. It's one thing to detect past abrupt changes and another to predict the next one. Can future values of parameters μ and ν be predicted from a series of past values?

Reply to Referee#1

Received and published: 6 February 2015

1) Referee. In figure 5b why state 4 does not have the same frequency as state 2? I would have expected (due to the symmetry of the ideal model) to be the same.

Authors. In figure 5b, only the "start states" is counted. For a system which is similar with the ideal model, the method is applied to detect the start states and end states. However, sometimes the length of sub-sequence is too short to cover the entire abrupt change process because the truly length of the process is unknown. Several condition could be happened during the detection as follows:

Condition 1. When the sub-sequence is moved to the moments before 150, such as 30(as shown as box 1 in Fig. 1a), the "start states" are always counted as state 2. And this is how we get the frequency of state 2.

Condition 2. When the sub-sequence is moved to the moments after 150, such as 170-250(as shown as box 2 in Fig. 1b), the "start states" could be counted as state 2.5, 3.0 et al. And this is why the states between 2~4 have values in figure 5b of the paper.

Condition 3. When the sub-sequence is moved to the moments after 350, such as 360(as shown as box 3 in Fig. 1c), the "start states" are always counted as state 4. And this is how we get the frequency of state 4.

Condition 4. There is no condition 4. Because in box 4(Fig 1d), the location of sub-sequence is break away from the sequence of the model.

Thus, the frequency of state 4 is less than the frequency of state 2.

On the contrary, when the end states" is counted only, the frequency of state 2 is less than the frequency of state 4, as shown in Fig. 2.
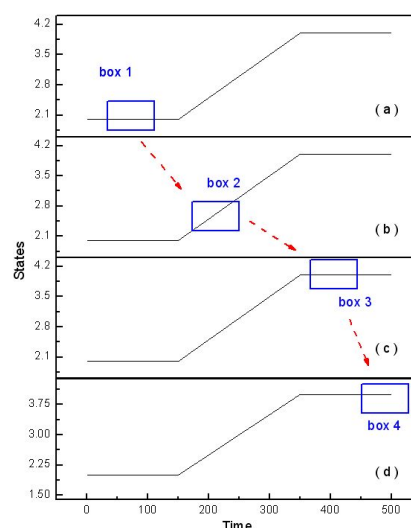

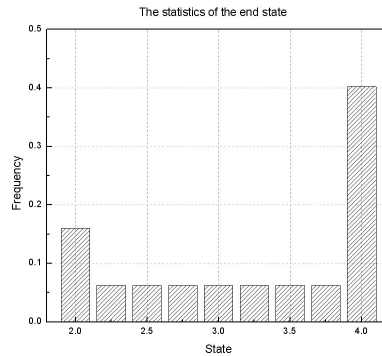Fig 1. Moving process of sub-sequence on the entire time series

Fig 2. The statistics of the end state

Changes: Page 5 Line 21

We added one sentence to give an explanation about this issue

It should be noticed that the reason of the frequency of state 2 higher than state 4   is only the start moment , just like state 2 ,is counted here. In contrast, the frequency of state 4 will   higher than state 2 if the end moment, just like state 4 ,is counted only.

2) Referee. That the PDO is involved in global temperature variability has been demonstrated in other previous studies (for example, Tsonis et al: A new dynamical mechanism for major climate shifts, Geophys. Res. Lett., 34, L13705, 2007).

Authors. It's true that the PDO index captures the essence of climate variability as saying in the Referee #1. And for the same reason, we use this index to detect the process of abrupt change. Our result shows that the process is exist as same as what we get from an ideal model, and the persist time of the process is related to global warming.

Changes: Page 7 Line 9

We added the article mentioned by referee#1

3) Referee. I would like the authors to discuss if their method can be used to predict the next abrupt change. It's one thing to detect past abrupt changes and another to predict the next one. Can future values of parameters μ and ν be predicted from a series of past values?

Authors. The answer is "Yes". We are going to use this method to predict the end time of an abrupt change process. According to the result in this paper, especially Eq.8, the quantitative relationship shows us a possible way to solve the prediction problem. But Prior to this,there are another worksneed to be done firstly, including understand the character of parameter k which might be relate to some meteorological elements, and the parameters h and μ which could be calculated if the abrupt change has begun. In addition, parameter v is constant based on Eq.8, which means the

persist time of abrupt change process is almost fixed.

Reply to reply to Referee#1

Received and published: 6 February 2015

The authors have answered my questions satisfactorily.

Comments of Referee #2

Received and published: 4 February 2015

In this paper the authors consider a toy model to understand abrupt changes in a observables and posterior they applied the method to the PDO series. the idea is not new, other authors have used simple models to explain changes in complex systems like climate system. However it is interesting. In the complex global model are very difficult to follow the all interactions and to understand singular process like for example an abrupt climate change. the authors explain the model in clear way, however, I consider that the case of PDO is not sufficiently explained or maybe it is not clear. What real variables are the responsible of the change?. When you use a toy model you must give a role to the variables of model because this toy model can be applied to several fields, (ecology, climate sciences,...)With the title of the paper the reader wait a better explanation of the abrupt changes in the series of PDO. Can we predict changes in the PDO with this model?. It would be really interesting. Therefore I consider that the paper could be published in the NPG journal after a revision and better explanation of the section 3 of the paper. The authors must explain the role of each variable of the toy model in the particular analyzed case.

Reply to Referee#2

Received and published: 11 February 2015

1) Referee. The authors explain the model in clear way, however, I consider that the case of PDO is not sufficiently explained or maybe it is not clear. What real variables are the responsible of the change?

Authors. This paper focuses on proposing a novel method based on Logistic model to detect the process of an abrupt climate change.

In order to investigate that the real variables are the responsible of the change, there are so many

works to be done, and the method to detect the process of an abrupt climate change may be a new powerful tool. Although we just apply this method to the PDO index now, but we are going to work on more variables of climate system in the future and try to get the most key factors.

2) Referee. When you use a toy model you must give a role to the variables of model because this toy model can be applied to several fields, (ecology, climate sciences,...)

Authors. In section 2, the parameters of Logistic model are discussed in detail. Based on this discussion, the characteristics of the process of abrupt change of the ideal time series are shown, then we can detect the transition process of the PDO via research its characteristics. According to the results of the ideal time series,we suggest that this method can be used for other fields.

Changes: Page 7 Line 11

We added and rewrote the first sentence of the paragraph "By analyzing the PDO index over the past 100 years, the statistical distribution······ "

3) Referee. With the title of the paper the reader wait a better explanation of the abrupt changes in the series of PDO. Can we predict changes in the PDO with this model?. It would be really interesting. Therefore I consider that the paper could be published in the NPG journal after a revision and better explanation of the section 3 of the paper. The authors must explain the role of each variable of the toy model in the particular analyzed case.

Authors. As what we have replied to referee #1(question 3), the answer to the question about prediction work is "Yes". We are going to apply this method to predict the end time of an abrupt change. According to the Eq.8, the quantitative relationship shows us a potential way to predict the end time.

4) Referee. Therefore I consider that the paper could be published in the NPG journal after a revision and better explanation of the section 3 of the paper. The authors must explain the role of each variable of the toy model in the particular analyzed case.

Authors. Based on the Logistic model, an interesting relationship between these parameters is shown. And according to Eq 2 & 8, we can calculate the parameters v, u, k of Logistic model, where parameters v and u correspond to the states before and after the abrupt change respectively, k is a instability parameter(page 46~49). During the application to the real time series, parameters v and u correspond to the states before and after the abrupt change of PDO, and k is a parameter indicated the instability of PDO. In section 3, we will add more explanation about the meaning of these variables.

Changes: Page 7 Line 11

As what we replied to issue 2 of referee #2, we added and rewrote the first sentence of the paragraph "By analyzing the PDO index over the past 100 years, the statistical distribution······ "

In addition, we also did some changes:
1, Page 10, line 18
A word (system) was added.
2, Page 10, line 27
A word(investigated) was corrected.
3, Page 14, line1
An reference was added.

Part 2 List of changes

Changes: Page 5 Line 21
We added one sentence to give an explanation about this issue
It should be noticed that the reason of the frequency of state 2 higher than state 4   is only the start moment , just like state 2 ,is counted here. In contrast, the frequency of state 4 will   higher than state 2 if the end moment, just like state 4 ,is counted only.

Changes: Page 7 Line 9
We added the article mentioned by referee#1

Changes: Page 7 Line 11
We added and rewrote the first sentence of the paragraph "By analyzing the PDO index over the past 100 years, the statistical distribution…… "

Changes: Page 10 line 18
A word (system) was added.

Changes: Page 10 line 27
A word(investigated) was corrected.

Changes: Page 14 line1
A reference was added.

A novel method for analyzing the process of abrupt climate change

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Abstract

A climate system which is transitioning from one state to another is known as an abrupt climate change. Most of the recent studies regarding abrupt climate change have focused on the changes occurring before and after the abrupt change point, while little attention has been given to the "transition process" which occurs when the system breaks away from the original state to a new state. In this study, a novel method for analyzing the process of abrupt climate change was presented. By using the mathematical model based on the Logistic model, the process of the abrupt change could be analyzed and divided into different phases which include start moment, end moment, stable state, and unstable transition state. Meanwhile, the method was confirmed to be effective by testing in a study of Pacific decadal oscillation(PDO) time sequence, and the results of this study specify that this abrupt change process of PDO has a relationship with global warming.

1 Introduction

The climate system is complex and chaotic(Shi, 2009). The abrupt climate change is described as the system transitioning from one stable state to another(Thom, 1972; Tong et al 2014), i.e. the system swings between different states(Lorenz, 1976; Charney et al 1979), and it also has been verified in the climate system(Dai et al, 2012;Bake, 1990; Wang et al, 2012; Alley et al, 2003;Xiao et al, 2012). Detection methods(Wan et al, 2008; Fu et al, 1992;

1 Yamamoto, 1985;) have been greatly developed. Since the abrupt change theory was

2 developed, increasing numbers of research(Goossens et al, 1986; Feng et al, 2011; Feng et al,

3 2008; Stefan, 2002) studies have been launched. Most of the current detection methods judge

4 the abrupt change as the changing of the statistics, such as the mean, variance and trend in

5 different moments. In order to clarify, such methods detect the abrupt change as one "point",

6 and also the time sequence change occurring abruptly before and after the "abrupt change

7 point"(He et al, 2012; He et al, 2012).

8 Neither the external force nor the feedback mechanism can guarantee that the abrupt change

9 occurs without a transition process, but the abrupt change must be obtained through a

10 transition process(Li et al, 1996). Unfortunately, it seems that the transition process has not

11 been mentioned in previous researches. In this study, in order to determine when the onset,

12 development, and extinction of the abrupt climate change events occur, a new novel detection

13 method has been proposed. Through the definition of the transitional process, the abrupt

14 climate change events were investigated as more than just a considered point, which may

15 prove helpful in eliminating missed and false detection.

16 An "analytical method of abrupt change process (AM-ACP)" introduced by Yan et al. (2013,

17 2014) was proposed to objectively deal with the transition process. By means of giving the

18 parameters actual physical meaning, the Logistic model could be used to detect the transition

19 process. This method is able to recognize the abrupt change process, which is different from

20 the method detecting the "abrupt change point". In this study, the AM-ACP has been used to

21 analyze the PDO(http://jisao.washington.edu/pdo/) index. It was able to point out that the time

22 sequence had several abrupt changes, and the processes had a relationship with global

23 warming. Also, the PDO index started from 1900.

24

2   Theory on Transition Process of Abrupt Change

26 The AM-ACP is based on the modified biologic model——Logistic model(May, 1976) as

27 shown in Eq. (1). This model showed the switching of the system among the different stable

28 states, as shown in Fig. 1 (black line), and was able show the different degrees of abrupt

29 change when the parameters of the model is changed, as shown in Fig. 1(gray dash line).

30 $\dot{x} = \kappa(x - \mu)(v - x), v < \mu$.   (1)

31

1

2   In Eq. (1), the system remains in the stable state: $x = \mu, or, x = v$; when the rate of the system

3   variable is zero, $\dot{x} = 0$.

4   By using a numerical method, the relationship between the solution of the model, and the

5   parameter κ is shown in Fig. 2. In Fig. 2a, the system reached the same state $x = \mu$, during

6   the evolution by change of the initial variable in threshold ($x_0 \in (v, \mu)$), when parameter

7   ($\kappa = 0.1$) was positive (black lines). The dash lines were the condition that the system reached

8   to the state $x = v$, when parameter ($\kappa < 0$) was negative. In Fig. 2b, the parameter $\kappa(=105)$

9   was larger than before, and the system became bifurcated. If the parameter $\kappa(=135)$ is much

10   larger, then the system becomes chaotic as shown in Fig. 2c. Therefore, the parameter κ is a

11   stability parameter, and the parameters μ, v are the start and end states before and after the

12   abrupt change(Yan and Hou et al, 2012, Yan and Yan 2013).

13

14

15   Fig. 3 describes an abrupt change process when the system transforms from one stable state to

16   another. In order to simplify the model, it was divided into 3 sections as shown in Fig. 3, and

17   the length of each section is marked as n_1, n_2, n_3 , respectively.

18

19

20   1) In section 1, during the period before the abrupt change, the system remained in the fixed

21   stable state $x = v$.

22   2) In section 2, the transition process whereby the system transformed from a stable state

23   ($x = v$), to another state ($x = \mu$), was considered to be linear. This means that the process

24   could be fitted by the method of the least squares. The fitting equation is: $x = h \cdot i + e$; where

25   h is the slope, and reflects the rate of the amplitude over time during the transition process.

26   3) In section 3, the system transformed to the stable state ($x = \mu$) after the abrupt change,

27   which is also represented as the equal of the sequence.

28   These parameters have been computed as a piecewise function：

1

$$\begin{cases} v = \sum_{i=1}^{n_1} x_i / n_1 \\ h = \sum_{i=n_1+1}^{n_1+n_2} \bar{i} \cdot \bar{x}_i / \sum_{i=n_1+1}^{n_1+n_2} \bar{i}^2, e = h \cdot \bar{x}_i - \bar{i} . \\ \mu = \sum_{i=n_1+n_2+1}^{n_1+n_2+n_3} x_i / n_3 \end{cases}$$ (2)

2   In section 2, the linear transition process has been marked as a black line. It is fixed by two

3   points, A and B, expressed as: (x_α, t_α) and (x_β, t_β), respectively. Then, the slope of the linear

4   equation is expressed as:

5     $$h = \frac{x_\alpha - x_\beta}{t_\alpha - t_\beta} .$$ (3)

6   The location parameters α, β have been defined to describe the location of these two points

7   as follows:

8     $$\begin{cases} x_\alpha = \alpha(\mu - v) + v \\ x_\beta = \beta(\mu - v) + v \end{cases}.$$ (4)

9   The parameter $\alpha(or\,\beta)$ is the ratio of the distance that the point deviates from the start state

10   and the amplitude of the abrupt change, then $\alpha \in [0,1], \beta \in [0,1]$.

11   Integral Eq. (1),

$$\frac{1}{\mu - v} \int_{x_0}^{x} \left(\frac{1}{x - v} - \frac{1}{x - \mu} \right) dx = \kappa \int_{t_0}^{t} dt$$

12     $$\Rightarrow \ln\left(\frac{x - v}{x - \mu} \cdot \frac{x_0 - \mu}{x_0 - v} \right) = \kappa(\mu - v)(t - t_0).$$ (5)

$$\Rightarrow \frac{x - v}{x - \mu} = \frac{x_0 - v}{x_0 - \mu} \cdot e^{\kappa(\mu - v)(t - t_0)}$$

13   Using the intermediate variable $\xi = \frac{x_0 - v}{x_0 - \mu} \cdot e^{\kappa(\mu - v)(t - t_0)} = \frac{x - v}{x - \mu}$, then:

14     $$t = t_0 + \frac{1}{\kappa(\mu - v)} \ln\left(\frac{x_0 - \mu}{x_0 - v} \cdot \xi \right).$$ (6)

15   The Eq. (3) is expressed by:

$$h = \frac{\alpha(\mu-\nu) - \beta(\mu-\nu)}{\frac{1}{\kappa(\mu-\nu)}\left(\ln\left(\frac{x_0-\mu}{x_0-\nu}\cdot\xi_\alpha\right) - \ln\left(\frac{x_0-\mu}{x_0-\nu}\cdot\xi_\beta\right)\right)}$$

1

$$= \kappa(\mu-\nu)^2 \frac{\alpha-\beta}{\ln(\xi_\alpha/\xi_\beta)} \qquad\qquad (7)$$

$$= \kappa(\mu-\nu)^2 \frac{\alpha-\beta}{\ln\left(\frac{\alpha}{\beta}\cdot\frac{1-\beta}{1-\alpha}\right)}$$

2   A new parameter is defined as $\chi = \dfrac{\alpha-\beta}{\ln\left(\dfrac{\alpha}{\beta}\cdot\dfrac{1-\beta}{1-\alpha}\right)}$, which has a relationship with the location

3   parameters α, β. The relationship is shown in Fig. 4, and the parameter χ is almost constant

4   when the values of α, β are within a certain range. In this paper, $\alpha = 0.2, \beta = 0.8$ and

5    $\chi = 0.2164$.

6

7

8   The Eq. (7) is rewrite as follows:

9    $h = \kappa(\mu-\nu)^2 \chi$.   (8)

10   As it is known that $\mu-\nu$ is the amplitude during the abrupt change, Eq. (8) illustrates the

11   quantitative relationship among the rates, the amplitude, and the stability parameter of the

12   abrupt change process.

13   Therefore, by changing the lengths n_1, n_2, n_3, the piecewise Eq. (2) could then be used to

14   optimally fit an abrupt change process with a group of parameters $\phi(h, \mu, \nu, n_1, n_2, n_3)$ from a

15   sequence which occurred during a similar abrupt change.

16   By testing a sub-sequence obtained from an entire time sequence, a group of parameters

17    $\phi(h, \mu, \nu, n_1, n_2, n_3)^1$ could then be calculated. Also, by moving the sub-sequence onto the

18   entire time sequence (as shown in Fig. 5a), a series of groups of parameters

19   (ϕ^1, ϕ^2, ϕ^3,···, ϕ^m) could then be calculated. The process of the abrupt change could then

20   be studied by analyzing the parameters. In Fig. 5b, in order to count the start moment ν, the

21   two states ($\nu = 2.0$ and $\nu = 4.0$) are detected with higher frequency. It should be noticed that

the reason of the frequency of state 2 higher than state 4   is only the start moment , just like state 2 ,is counted here. In contrast, the frequency of state 4 will higher than state 2 if the end moment, just like state 4 ,is counted only. To clarify, this means that two stable states exist in the time sequence, i.e. the system presents a double-peak structure. In regards to the relationship between the detecting moment and the start moment as shown in Fig. 5c (blue line), the same start moment has always been detected over a long period, with the detecting moment changing from point 100 to point 500. That start moment is calculated (red line) in Fig. 5c. At point 150, the frequency is much higher than in the others. The reason being that the point is the start moment of the abrupt change process of the ideal model. By utilizing the frequency of the start moment to judge the abrupt change, this method would improve the situations of missing and the false detection, which were happening by using the traditional method. Fig. 5d is a phase diagram of the "start-end" states, and each point in the figure represents a detection result. The x-axis is the start state, and the y-axis is the end state.

In the vertical line (the red line), the points present a process that the start state stays constant, and the end state increases. This indicates that the sub-sequence has passed by the start moment during the moving process. Therefore, the vertical line represents a process in which the system deviates from a stable state.

In the horizontal line (the blue line), the points represent a process that the start state is increasing, and the end start state remains constant. This indicates that the sub-sequence has passed by the end moment during the moving process, which indicates that the horizontal line represents a process that the system is approaching a stable state.

The green line, which runs parallel to the diagonal, shows that the sub-sequence is in the process of an abrupt change, since both the start state and the end state are increasing, and the difference between them is constant.

It should be noted that the line running parallel to the diagonal would vanish if the length of the sub-sequence is larger than the length of the abrupt change process. Also, the vertical line and the horizontal would still exist

Therefore, based on the analysis of the result of the detection, the AM-ACP can be used to effectively detect the process of the abrupt change. In future studies, this method will be used

to detect a real time sequence, and the statistical characters would then be shown in order to study the abrupt change process.

3   Characteristics of abrupt change process of Pacific decadal oscillation

In this study, based on the theory of the AM-ACP, the PDO index has been analyzed to study the abrupt change process. The PDO has been considered as an important indicator of the decadal variability of the Pacific Ocean(Meehl et al, 1993), which has a great relationship with China climate(Wang et al, 2010). It has been shown in many previous studies, that this index has experienced several abrupt changes over the past 100 years(Mantua, 1997, Tsonis et al, 2007), and that all of these abrupt changes correspond to the global climate changes.

During the application of AM-ACP on the PDO index, parameters μ, ν represent the states before and after the abrupt change process respectively. Parameter κ still represents the stability of the PDO index, a larger κ means more stability and vice versa(Yan and Hou et al, 2012, Yan and Yan 2013). And then, many families of parameters could be obtained by moving the sub-sequence whose length is set as 10 years like what have been done to the ideal model in section 2. By analyzing many families of parameters, the statistical distribution of the start state was determined and is illustrated in Fig. 6. During the calculation, the length of the sub-sequence was set to be: 10 years (Fig. a); 20 years (Fig. b); 30 years (Fig. c); and 40 years (Fig. d). All of these 4 frequency distributions had two peaks, which means the system mainly distributed on two stable states, as illustrated in Fig. 5b. Furthermore, under certain conditions, the system may have crossed over from one state to another. Such a structure has been verified several times in the climate system(Goldblatt et al, 2006; Alexander et al, 2012; Zerkle et al, 2012).

In Fig. 7, it is shown that the different start moments of the abrupt changes in the PDO index time sequence, have been detected at different detecting moments. The results showed that the frequency of the abrupt change start in some years was much higher than in other years. Also, when the length of sub-sequence was set as 40a, as shown in Fig. 7d, then the start moments of the abrupt changes started in 1940 and 1976 were detectable for a rather long time period (detecting moment). These results are precisely when two transitions occurred. These

transitions were when the PDO index time sequence transferred from a positive phase to a negative phase in 1940, and from a negative phase to a positive phase in 1976(Francis, 2007). The abrupt change in 1955, was detected by an abnormally smaller value of the PDO index time sequence in that year. In addition, the frequency of detected abrupt changes in 1934, and 1988 was higher, which indicated that the system did deviate, but with only a small amplitude, from a stable state.

Fig. 8 shows the "start-end states" phase diagram of the PDO system during the period from 1900 to 2010, and the five clear abrupt changes are marked with five different colors in the diagram. As shown in Fig. 5d, the vertical and horizontal lines represent different sections of the abrupt change process. In regards to the different lengths of the sub-sequence, all five abrupt change processes have been detected. By taking a 30a length as the set time length for example, the result was analyzed as follows.

The first abrupt change process started in 1934 (marked with blue dots), and the vertical section line which is consisting of blue dots in the phase diagram, is located on the left hand side of the diagonal line. By comparing this with Fig. 5d, we concluded that this was a continuously increasing process, and had increased to a new stable state.

The second abrupt change process started in 1940 (marked with light blue dots), and it is the vertical line located on the right hand side of the diagonal line. By comparing this with Fig. 5d, we concluded that this indicated a decreasing process from a stable state.

The third abrupt change process started in 1955(marked with green dots), and it is located in a neighboring area of the diagonal line. By comparing this with Fig. 5d, we concluded that this was a non-abrupt change process, for example, abnormally small values had been detected in the original time sequence.

The fourth abrupt change process started in 1976 (marked with yellow dots), and the red dots consist of a vertical line located on the left hand side of the diagonal line. This indicates an increasing process to a stable state, which is similar to the abrupt change that started in 1934 marked with a blue line.

The fifth abrupt change process started in 1988 (marked with red dots), which has been divided into two sections of an approximate vertical and horizontal lines, where the vertical

line indicates a decreasing process from an stable state, and the horizontal line indicates a decreasing process to an stable state.

In conclusion, the dotted line in the "start-end states" diagram indicates different abrupt change processes which may be included in certain stages of an abrupt change. For all of the abrupt changes detected, two changes which started in 1940 and 1976 have longer vertical lines, indicating these two abrupt changes were more severe. This is the reason why these two abrupt changes also had been detected by some other detection methods.

In Fig. 9, some further research is shown concerning the relationship between the persist time, and the start moment of the abrupt change processes. The results indicated that the abrupt changes starting from different moments have clear differences in persist times. The abrupt changes which started in 1940 and 1976, continued for 136 and 144 months, respectively. This is consistent with the observations of the time sequence of the PDO index. The abrupt change which started in 1988, continued for 120 months. The abrupt change started in 1934, continued for 70 months, and the abrupt change started in 1955, only continued for 38 months. In addition, by sliding the sub-sequence onto the entire time sequence, the persist time of each abrupt change may be detected more than once, the abrupt changes started in 1940 and 1976. The reason for this multiple detection is that such abrupt changes belong to different abrupt change processes.

To further inspect the relationship between the start moment of the abrupt changes and their persist time as shown in Fig. 10, the distribution of the relationship is shown with the length of sub-sequence set as 10a, 20a, 30a, and 40a. The number of abrupt changes following the 1960s, significantly exceeded that before the 1960s, which reflects the fact that the climate system might have become progressively unstable. In regards to the several abrupt changes occurring before the 1960s, the later the abrupt change started, the shorter the persist time of the change became. As for the abrupt changes after the 1960s, these abrupt changes can be divided into two groups, which are those lasting for more than 100 months, and those lasting for less than 50 months. In regards to the persist times, in the former group it was found to be shorter, while in the later group the persist time was extended, and the two groups were in a symmetrical distribution for approximately 70 to 80 months. In addition, the global temperature continued to increase prior to 1940, and reached its peak value during the 1940s

decade, then it remained at a lower level for the following 10 years, before beginning to slowly rise at the start of the 1950s(IPCC, 2007). This is consistent with the abrupt change in the PDO index time sequence. These facts indicated that, under the context of global warming, the oceanic system showed an evidently unstable state, and that the persist time of the abrupt changes which began after the 1950s presented a trend of becoming closer to the range of 70 to 80 months.

4 Conclusions

As the evidence of a climate system swing among different states has undoubtedly given us an opportunity to learn more about abrupt climate change events, a new method to analysis the process of abrupt climate change, which named AM-ACP, was proposed in this study, and the method has also provided a new perspective to research abrupt climate change.

By using an ideal mean change time sequence to simulate the process of the abrupt change , there are three characteristics can be shown: 1) multistable states exist in the abrupt change events; 2) the start moment of the abrupt change is detectable in different times; 3) the "start-end states" phase diagram could be used to express the duration of the abrupt change.

Moreover, These characteristics also could be obtained as the AM-ACP is applied to the PDO index. The results shown that the ocean system is bi-stability, and the transition among different stable states has occurred 5 times over the past 100 years in 1934, 1940, 1955, 1970 and 1976 respectively, and the "start-end states" phase diagrams gives the transformation processes clearly. In addition, the persist time of each abrupt change is almost constant, and the persist time become shorter with the global warming. After 1960s, the persist time is getting longer when it is shorter than 100 months, On the contrary, the persist time is getting shorter when it is longer than 100 months. As regards the process of abrupt climate change, further work is needed to research this subject. ,and more evidence concerning abrupt climate change is needed to be identified following more efficacious method is needed to be investigated also. Now we may unable to accurately forecast exactly when an abrupt climate change will occur, but at least we hope that in the future we can devise a methodology which can be used to determine whether we are in the process of an abrupt climate change, it is of great importance to human society.

Acknowledgements

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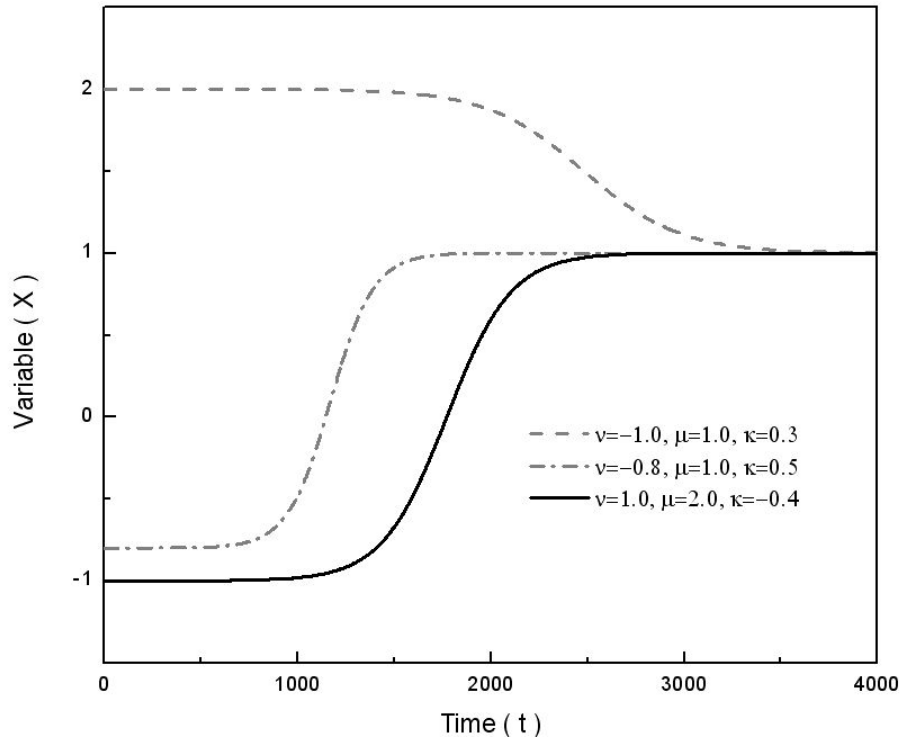
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3 Figure 1 The evolution of the system over time, computed by rewritten Eq. (1) as its

4 difference scheme: $x_{(t+1)} = \tau\kappa(x_t - \mu)(v - x_t) + x_t$; where $\tau = 0.01$ is the time step.
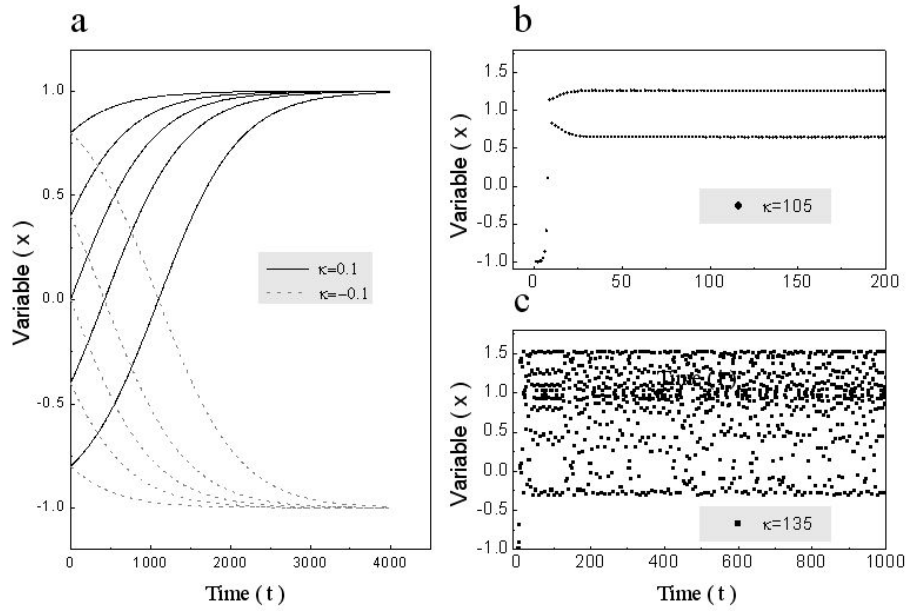
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3   Figure 2 The evolution of the system over time, with different stability parameters: a. the

4   system reaches to the stable states with a different initial variable when parameter $\kappa = \pm 0.01$;

5   b. the system becomes bifurcated when the parameter $\kappa = 105$; c. the system becomes chaotic

6   when the parameter $\kappa = 135$.

7

The figure shows a plot with x-axis labeled "Time (t)" ranging from 0 to 40, and y-axis labeled "Variable (x)" ranging from -1.5 to 1.5.

Parameters shown in the plot:
$\nu = -1.0$ $\mu = 1.0$
$\kappa = 0.3$ $\tau = 0.1$
$x_0 = 0.9995$

Points labeled: $A(x_\alpha, t_\alpha)$ and $B(x_\beta, t_\beta)$

Sections labeled: n_1, n_2, n_3

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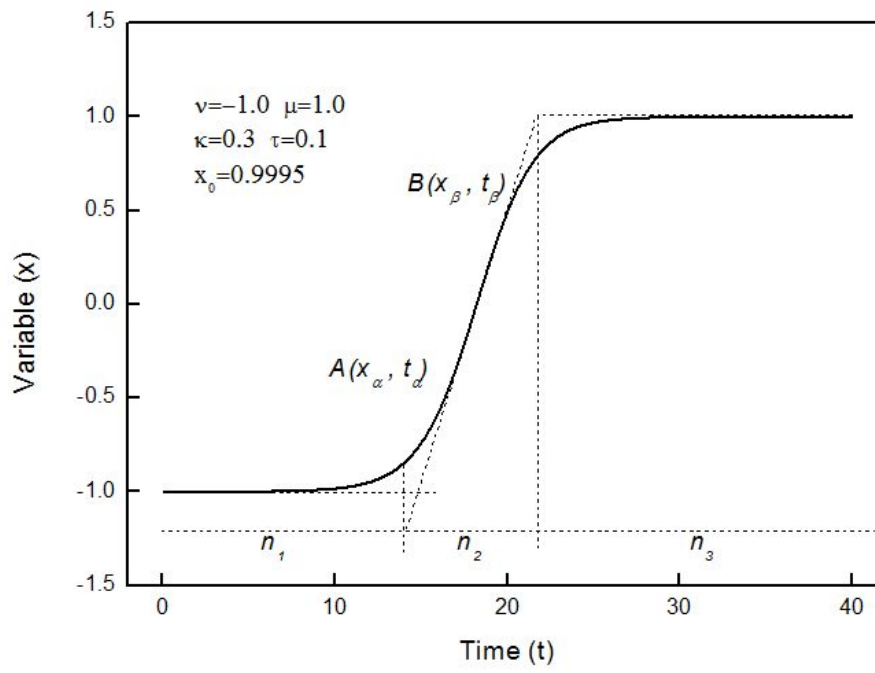
3   Figure 3 The abrupt change process and its sections. The three sections are period before the

4   abrupt change; period of abrupt change process; and period after abrupt change, respectively.

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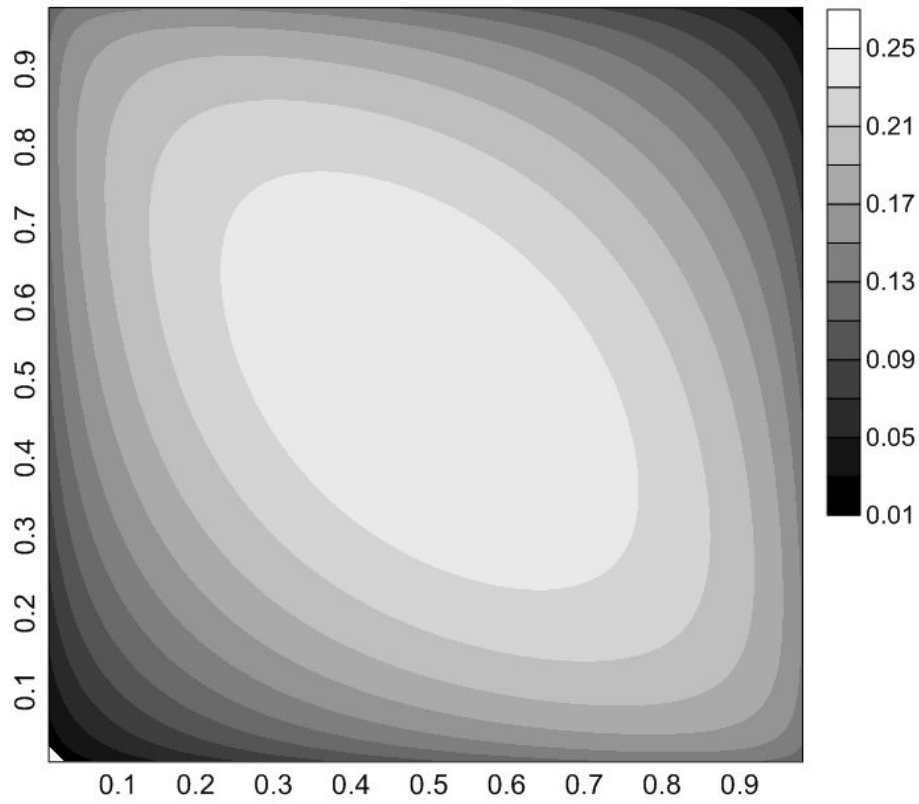
3 Figure 4 The relationship parameter χ and parameters α, β, the x-axis is parameter α, the y-

4 axis is parameter β and the contour is parameter χ.

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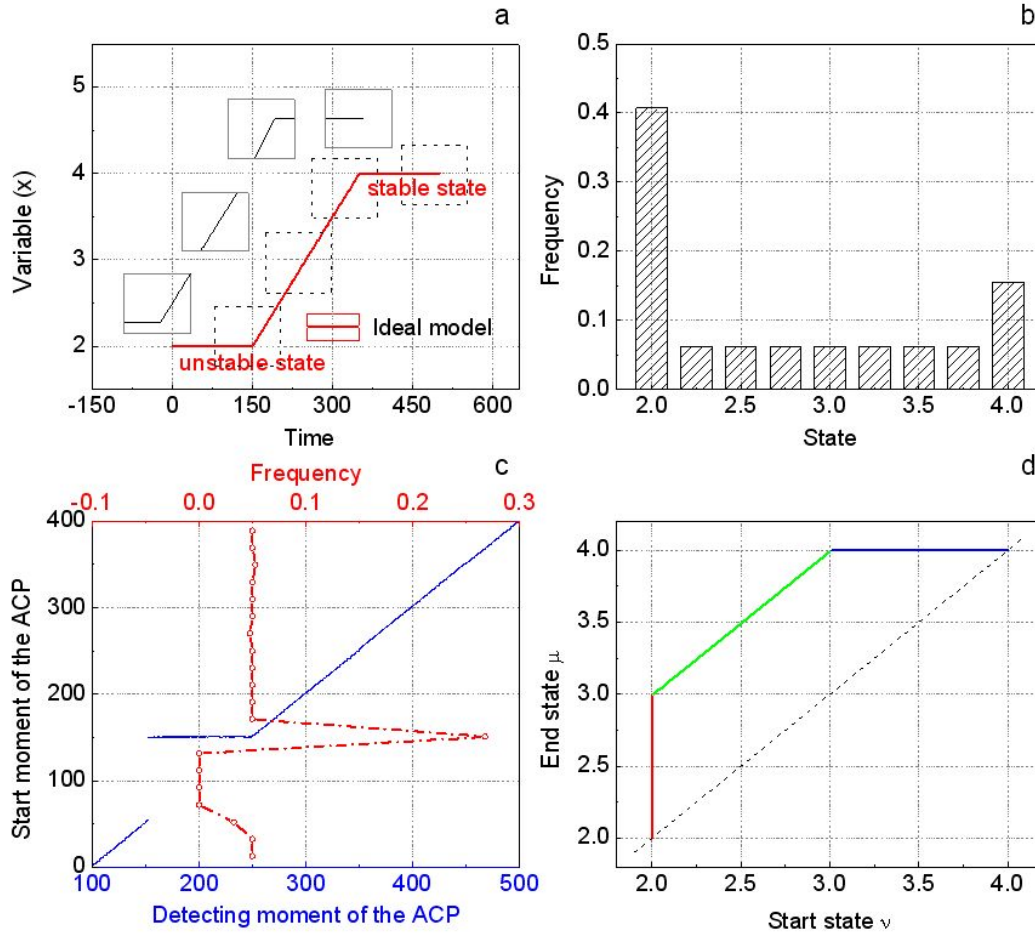
Figure 5 The detection of the abrupt change process of the ideal model and the analysis of the results. Fig. 5a: the ideal model (red line) and its detection of the abrupt change process in different observed windows (black lines) with sub-sequences; Fig. 5b: the statistics of the start states; Fig. 5c: the relationship between the detecting moment and the start moment (blue line), and the frequency of the star moment (red line); Fig. 5d: the phase diagram of "start-end" states.

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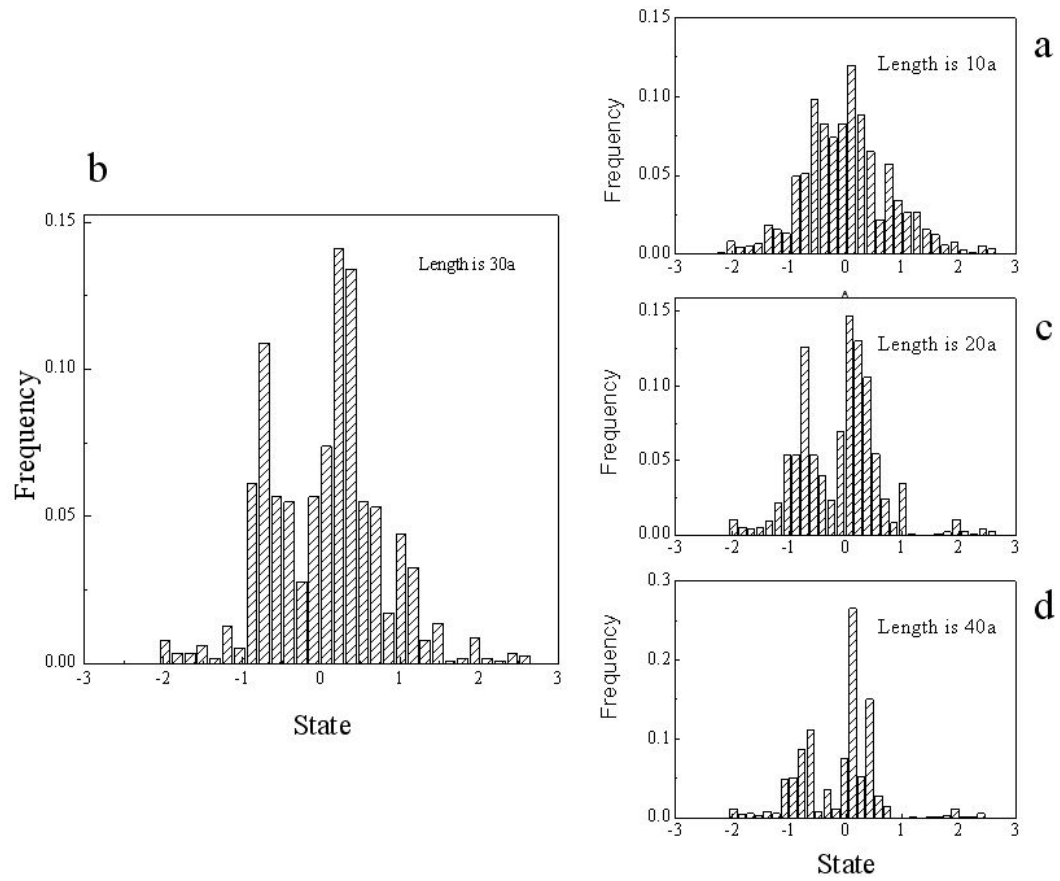
3 Figure 6 Histogram of statistical distribution of the start state. The x-axis refers to the

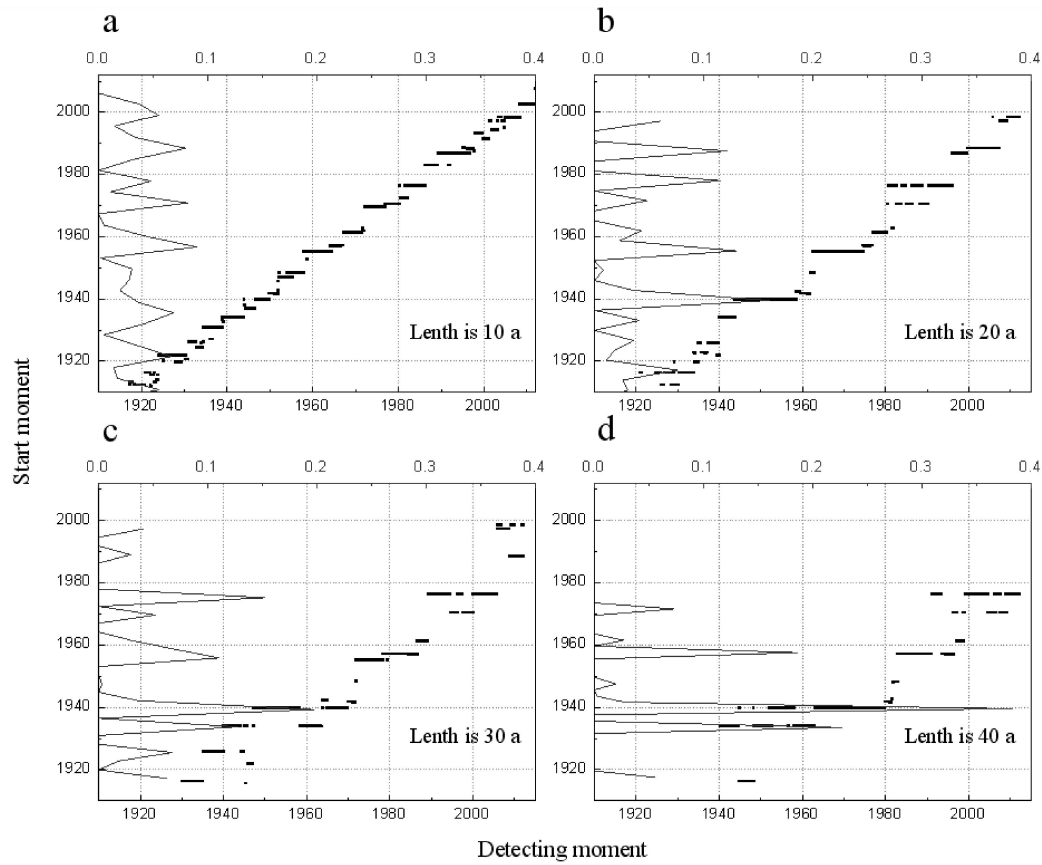4 statistical interval for the PDO index, and the y-axis refers to the frequency.

5

Figure 7 Start moment of the abrupt change in PDO index series with different detecting moments. The x-axis refers to the detecting moment of each sub-sequence detected, or the moment for detecting the time sequence, while the y-axis refers to the start moment. The black short line indicates the start moment of the abrupt change, which is detectable in a rather long time period; and the red line refers to the frequency of the start moment. Figs. a-d are the different conditions that the lengths of sub-sequence are set as 10a, 20a, 30a, and 40a, respectively.
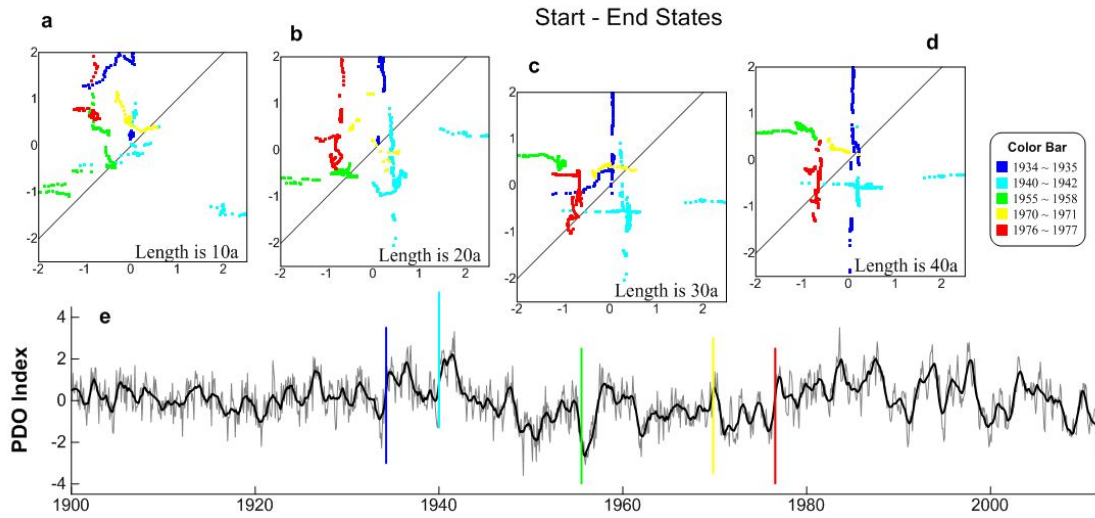
Figure 8 The PDO index and its "start-end states" phase diagram. Fig. 8a-d are the phase diagrams; the x-axis refers to start state of the abrupt change detected through each sub-sequence; the y-axis refers to the end state; and each point in the figure refers to one detection; the different colors refer to different processes with different start moments of abrupt change; Fig e is the PDO index; the x-axis refers to time; and the y-axis refers to the index.
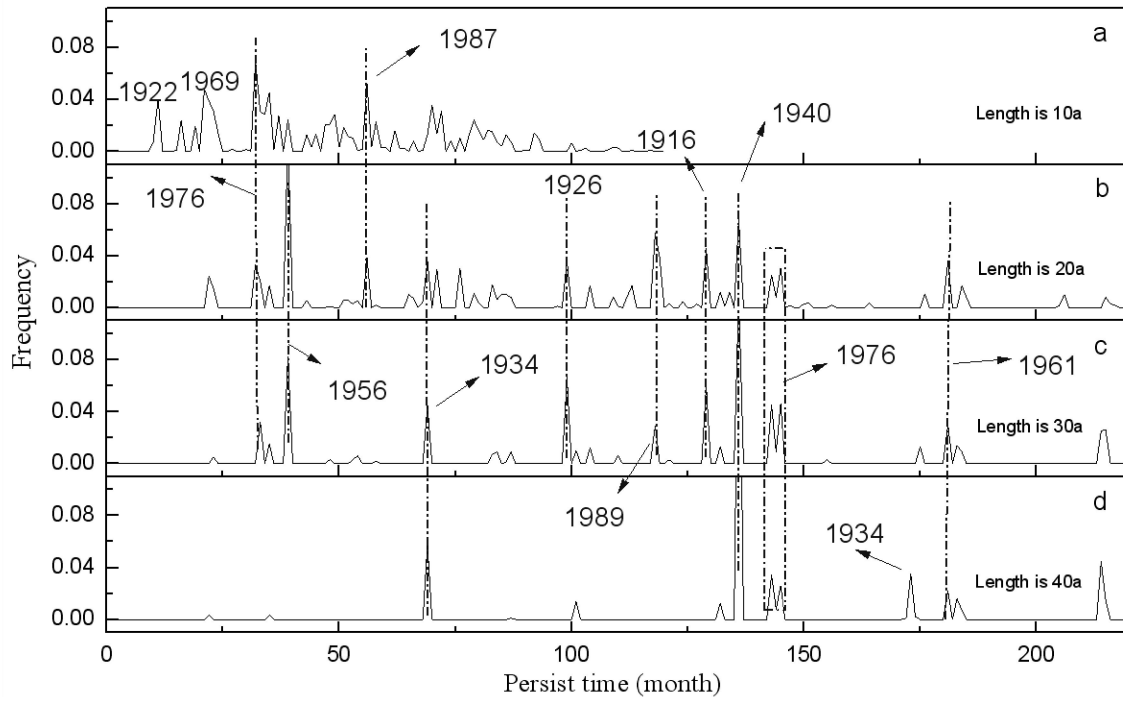
Figure 9 Distribution of the abrupt change persist time with the start moment. Fig. 9a-d show the different conditions in which the length of the sub-sequence are set, which are 10a; 20a; 30a; and 40a, respectively. The x-axis refers to the persist time of the abrupt change; the y-axis refers to the frequency of the abrupt changes that satisfies the conditions; and the characters mark the start moments of each abrupt change process.
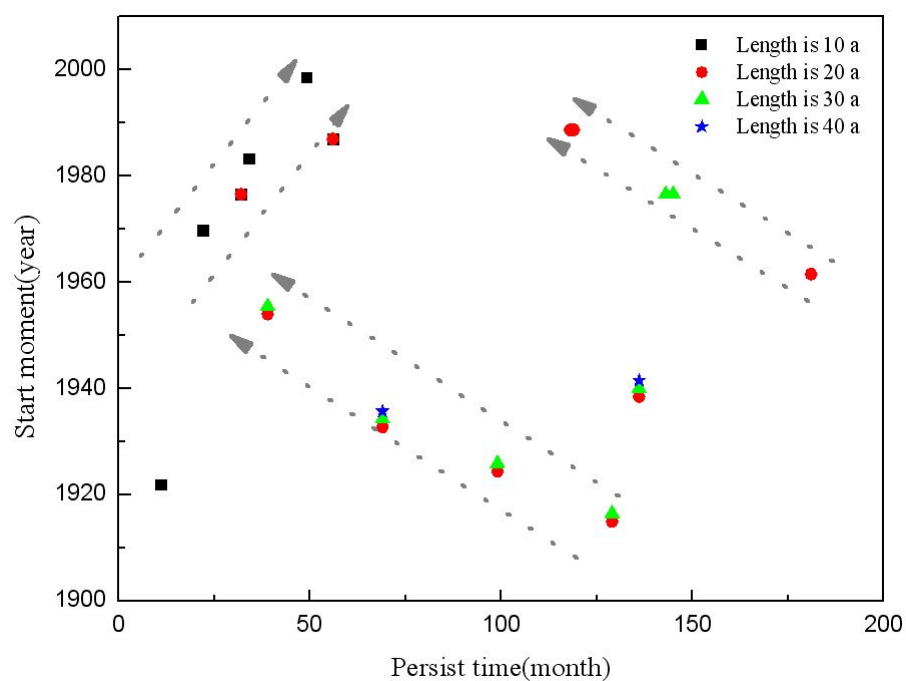
Figure 10 Abrupt changes start at different moments than their persist times. The x-axis refers to the persist time of each abrupt change process, and the y-axis refers to the start moment.