

Report #1  
Submitted on 29 Jun 2015  
Anonymous Referee #1

**Anonymous during peer-review: Yes No**  
**Anonymous in acknowledgements of published article: Yes No**

**Recommendation to the Editor**

Does the paper contain new and significant results?	<b>Yes No</b>
Is the paper of an international standard?	<b>Yes No</b>
Is the presentation clear and concise?	<b>Yes No</b>
Does the paper put the obtained results into context, with relevant references?	<b>Yes No</b>
Is the length of the paper appropriate?	<b>Yes No</b>
Is the text fluent and precise?	<b>Yes No</b>
Are the title and the abstract pertinent and understandable to a wide audience?	<b>Yes No</b>
Are all figures necessary, and of appropriate quality?	<b>Yes No</b>

For final publication, the manuscript should be

2 **accepted subject to minor revisions.**

**Suggestions for revision or reasons for rejection (will be published if the paper is accepted for final publication)**

A) The authors have addressed all my issues, except my comment 6). I know that the authors, in the response to my first review, recognized that the point was not still deeply understood and that more investigation should be done. However, even if this comment raises any limit of the proposed method, this should be highlighted in the revised paper.

**We agree with the comments of Referee #1. We have highlighted this point in our revised version. We agree with the referee that more investigations need to be done in this area.**

B) The figures 6a) and 6b) should be redrawn. The problem is in the upper timescale tau. As far as I can guess, the upper timescale in both figures is  $10^9$  sec. Since they analysed the catalogue spanning from 1982/01/01 to 2008/03/31, the whole observation period T is about  $10^9$  sec. The reliable estimate of the FF and AF curves should be done up to at most T/10, which means up to  $10^8$  sec. In fact, if you look at the last boxes in both plots between  $10^8$  to  $10^9$  sec, you will see a very erratic behavior of the curves

with a very high value of the exponents (2.88 and 3.33). These value of the scaling exponents are not reliable and do not inform on any high time-clustering. These values and the erratic behaviors of the FF and AF curves at these timescales are simply effect of the very poor statistics at these timescales. That's why, the upper timescale is at most  $T/10$ . So, the plots should be re-drawn, considering only timescales up to about  $10^8$  sec.

In response to the referee's comments, we have redrawn Figures 6a) and 6b) with times agreeing with the calendar times if the catalogue and with the upper timescale to at most  $T/10$  or  $10^8$  sec.

Submitted on 30 Jul 2015

Referee #2: Dr. Reik Donner, reik.donner@pik-potsdam.de

**Anonymous during peer-review: Yes No**

**Anonymous in acknowledgements of published article: Yes No**

**Recommendation to the Editor**

Does the paper contain new and significant results?	<b>Yes No</b>
Is the paper of an international standard?	<b>Yes No</b>
Is the presentation clear and concise?	<b>Yes No</b>
Does the paper put the obtained results into context, with relevant references?	<b>Yes No</b>
Is the length of the paper appropriate?	<b>Yes No</b>
Is the text fluent and precise?	<b>Yes No</b>
Are the title and the abstract pertinent and understandable to a wide audience?	<b>Yes No</b>
Are all figures necessary, and of appropriate quality?	<b>Yes No</b>

For final publication, the manuscript should be  
**accepted subject to minor revisions.**

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**Suggestions for revision or reasons for rejection (will be published if the paper is accepted for final publication)**

(M) The authors have made a great step forward by addressing much of the concerns raised by the reviewers of the original discussion paper. The manuscript is much clearer now in terms of its methodological approach and results. However, before final publication in NPG, there are some further minor aspects to be addressed and/or clarified:

(M) We have taken steps to attend to all of the remarks by Referee # 2. Where appropriate, we have modified/edited the text.

(1) p.5, l.2: "...for one Delta t or for successive Delta t's" – Delta t is used in the paper as describing the sampling interval, not the time index; so this part should read "for one or more time steps"

(1) We agree with the change suggested by the referee.

(2) p.5, l.4: please add a reference for the presence of intermittency

(2) We have inserted the relevant reference here.

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(3) At many points throughout the manuscript, the authors use a very sloppy language by just skipping some important words. For example, on p.7, l.6, “the number of occurrences from state I to state j” should read “the number of occurrences of transitions from...” I recommend careful proofreading of the whole manuscript regarding this aspect prior to delivering the final version to the publisher.

(3) We hope to have responded to the referee’s comments. We have paid special attention to the proofreading of the manuscript.

(4) p.7, ll.11-13 and 14-16: literal repetition due to a copy-paste error

Corrected.

(5) p.7, ll.19-20: Requirement 2 is not clear from the text, please phrase more clearly

With the rearrangement of the text, the state definition precedes the requirement 2. So, it should be clear.

(6) p.7, l.21: The authors use the state numbering that is only introduced on p.8 – please rearrange the material in this section accordingly

The rearrangement is done.

(7) p.9, ll.4-5: Do you mean “within the spatial distance  $d_{ij}$  between I and j around I”? Please specify.

We have made the change.

(8) p.10: Given the actual contents of Section 3, I recommend changing the section title to “Analysis methods and results”.

(8) Done.

(9) Section 3.1: The advantage of EEMD in comparison with classical EMD still does not become fully obvious. Please add one sentence or so to make this point explicit.

(9) It turns out that in the time-series we dealt with in this paper, there are at least two-successive state-to-state transitions that do not have any earthquake occurrences of magnitudes considered in any of the studied zones. We define them as intermittencies in the data. Wu and Huang (2009) have discussed this aspect of intermittency in classical EMD and concluded that the ensemble approach to EMD (EEMD) would adequately treat the intermittency problem. Hence, we followed this approach.

(10) p.12, l.22: Is there any argument justifying this assumption?

(10) Based on the arguments by Wu and Huang (2009), their EEMD approach represents a substantial improvement over the original EMD in that it utilizes the

full advantage of the statistical characteristics of white noise to perturb the signal in its true solution neighbourhood, and to cancel itself after serving its purpose.

(11) p.13, ll.2-4: It is still not fully clear what is done here. Previously, the authors take the EQ catalog data, compute time series of transition frequencies between all states and sum them up. I assume that at this point, the summation is skipped, and EEMD is performed to the transition frequency time series for all pairs of states first, and then the corresponding IMFs are summed up – is this correct? If so, please write this more clearly; otherwise explain what you are actually doing. I agree that due to the additivity imposed in both the EMD and total weight calculation, both steps would commute. However, I am wondering if the natural time-scales of the IMFs for each pair of states are always the same. The author state elsewhere that EMD provides a dyadic decomposition – in my personal experience, this is not necessarily the case. Is there something special about the implementation of EMD used in this work?

(11) Summing the weights of the recurrence arcs has nothing to do with summing of the IMFs to verify the EEMD (or classical EMD) procedure. It is not correct to say that the additivity imposed in both the EMD and total weight calculation would commute.

In this particular case, the decomposition is dyadic in that successive IMFs are sampled according to  $\Delta T$  (first IMF),  $2\Delta T$  (second IMF),  $4\Delta T$  (third IMF),  $8\Delta T$  (fourth IMF),  $16\Delta T$  (fifth IMF) and so on.

(12) p.13, ll.16-18: It would be good if the authors could explicitly refer to certain times (calendar dates) here. If I interpret this sentence correctly, the authors mean a certain time period of enhanced activity at the end of the record (“bursts” in Fig. 2a) – if so, it is not surprising that such a bursty (intermittent) behavior shows up at various time scales of the decomposition (you would expect something similar in other time series decomposition techniques as well). In this spirit, the broad signal across various scales is not too surprising from the original data. I feel that this aspect should be mentioned briefly.

(12) We have included the calendar dates in place of time steps in the requested figures except Figure 3. To avoid cluttering of the x-axes in this figure, we have included in the figure caption the time-steps and the corresponding calendar states.

The intermittent behaviour as we defined in our response to an earlier question should not be confused with the enhanced activity at the end of the record.

(13) p.14, l.1: From the context, it is not clear why the authors speak of a “periodic trend” here.

(13) An apparent “periodic” trend we discuss is the observation we make when we look at the first few intrinsic mode functions. They are characterised by (1) a certain number a pattern of rise and fall of the arc weights and (2) by a systematic decrease in the frequency of the number of the patterns. Since the rise and fall of

the arc weights covers the entire catalogue of data, the periodicity that we note could be intrinsic to earthquake processes.

(14) p.14, l.2: Please give calendar dates instead of numbers of sampling intervals.

Done.

(15) Figure captions: The captions of Fig. 3 and Fig. 6 could be drastically reduced by combining repeated contents. For example, in Fig. 3, you do not need to explicitly count the IMFs. In Fig. 6, the text describing the contents of panels (a,b) and (c,d) could be easily combined.

(15) We have made the relevant changes suggested by the referee.

(16) In general, I strongly recommend to redraw several of the figures replacing the rather arbitrary time axis by calendar dates as values (Figs. 2a, 3, 4,6).

(16) We have redrawn Figures 2a, 4, and 6 with the arbitrary time axis replaced by calendar dates. Since adding calendar dates information to all the panels of Figure 3 would make them look cluttered. Instead, we have provided the calendar dates to the time markers that appear in all the panels in the caption of Figure 3.

(L) Although this is a list with quite some items, I think that addressing these points only requires minor amendments to the text, so that I can actually recommend publication of this work without further review. In addition to the specific points raised above, an additional careful proofreading is recommended.

(L) We have paid special attention to the proof-reading of the text.

2 **Earthquake sequencing: Analysis of time-series constructed from**  
3 **the Markov chain model**

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12 **M. S. Cavers<sup>1</sup> and K. Vasudevan<sup>1,2</sup>**

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**Abstract.** Directed graph representation of a Markov chain model to study global earthquake sequencing leads to a time-series of state-to-state transition probabilities that includes the spatio-temporally linked recurrent events in the record-breaking sense. A state refers to a configuration comprised of zones with either the occurrence or non-occurrence of an earthquake in each zone in a pre-determined time interval. Since the time-series is derived from non-linear and non-stationary earthquake sequencing, we use known analysis methods to glean new information. We apply decomposition procedures such as ensemble empirical mode decomposition (EEMD) to study the state-to-state fluctuations in each of the intrinsic mode functions. We subject the intrinsic mode functions, derived from the time-series using the EEMD, to a detailed analysis to draw information-content of the time-series. Also, we investigate the influence of random-noise on the data-driven state-to-state transition probabilities. We consider a second aspect of earthquake sequencing that is closely tied to its time-correlative behavior. Here, we extend the Fano factor and Allan factor analysis to the time-series of state-to state transition frequencies of a Markov chain. Our results support not only the usefulness the intrinsic mode functions in understanding the time-series but also the presence of power-law behaviour exemplified by the Fano factor and the Allan factor.

20



## 1 Introduction

2 Earthquake sequencing has been the subject of detailed research (Nava et al., 2005;  
Ünal and Çelebioğlu, 2011, 2014; Telesca et al., 2001, 2008, 2009, 2011; Cavers  
4 and Vasudevan, 2013, 2015; Vasudevan and Cavers, 2012, 2013) both in the  
regional and global sense in recent years. Nava et al. (2005) have introduced the  
6 Markov chain model to study the earthquake sequencing in a seismogenically active  
region where the region is partitioned into zones. The functionality of the method  
8 is determined by the characteristics of the state-to-state transitions where each state  
is described by the earthquake occupancy of the zones. In particular, for a given  
10 number of zones,  $N$ , a state corresponding to a time interval is expressed as a  
concatenation of binary digits  $b_{N-1} \dots b_1 b_0$ , where  $b_L = 1$  (or  $b_L = 0$ ) indicates there  
12 was (or was not) an earthquake occurrence in zone  $L$  during the specified time  
interval. Thus, states can fall into zones of no occupancy to full occupancy at the  
14 extreme and into zones where some are occupied and some are not. The approach  
of Nava et al. (2005) was immediately extended to other regions (Herrera et al.,  
16 2006; Ünal and Çelebioğlu, 2011, 2014). Cavers and Vasudevan (2013) adapted  
the method of Nava et al. (2005) to a global catalogue which was partitioned into  
18 zones on the basis of the tectonic boundaries (DeMets et al. (1990, 2010), Bird  
(2003), Kagan et al., 2010). The existing Markov chain model was refined by  
20 incorporating the record-breaking recurring events for each event in the catalogue  
under certain constraints. A directed graph representation of the modified Markov  
22 chain model was then subjected to detailed analysis for forecasting purposes (Cavers  
and Vasudevan, 2015).

24 One consequence of the approach taken by Cavers and Vasudevan (2015) and  
Vasudevan and Cavers (2013) is that it results in a time-series of state-to-state

transition frequencies of the modified Markov chain model,  $x_{sst}(t)$ . This time-series  
2 is for an optimized time-interval,  $\Delta t$ . The fluctuations in state-to-state transitions  
are  $\Delta t$  sampled. The time-series is a comprehensive representation of earthquake  
4 sequencing in which interaction of seismic events within and among zones are  
considered. Therefore, it can be subjected to a detailed analysis.

6 Earthquake sequencing may be considered a non-linear and non-stationary  
process (Kanamori, 2003; Telesca et al., 2001, 2008, 2009, 2011; Flores-Marquez  
8 and Valverde-Esparza, 2012). In earthquake sequencing, earthquakes are viewed as  
part of a point process, with earthquake events occurring at some random locations  
10 in time. This means that the earthquake sequencing is dictated by the set of event  
times, and can also be expressed by the set of time-intervals between events. The  
12 time-series of earthquakes for any time-interval can be analyzed in many ways  
(Telesca et al., 2001, 2008, 2009, 2011).

14 We postulate here that the non-linear and non-stationary behavior in the time-  
series should also be present in the time-series of the state-to-state transition  
16 frequencies derived from earthquake sequencing. Hence, we consider the  
approaches of Telesca et al. (2001, 2008, 2009, 2011) to be appropriate for a study  
18 here.

Non-linear and non-stationary time-series have been examined in recent years  
20 with a method known as empirical mode decomposition (EMD) and the intrinsic  
mode functions derived from this are useful in this regard (Huang et al., 1998). The  
22 present time-series of state-to-state transition frequencies is suited for such a study.

In general, the time-series has non-zero amplitudes for the state-to-state  
24 transition frequencies (Cavers and Vasudevan, 2015). In this particular case, there

are instances where there are no earthquakes exceeding the magnitude of 5.6 in all  
2 zones for one or more time steps. This introduces “intermittency” in the time series.

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However, because of the presence of intermittency in it, an ensemble approach  
4 to empirical mode decomposition, EEMD (Wu and Huang, 2004, 2009; Flandrin et  
al., 2004, 2005) is applied here. The intermittency problem is handled with the  
6 addition of random noise to the time-series before carrying out the EEMD  
(REFERENCE Wu and Huang, 2009). We examine the criteria used for the  
8 selection of the added noise and the ensemble number for the EEMD.

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Another aspect of the study here is to ask a question if the time-series resulting  
10 from a directed graph representation of the Markov chain model of earthquake  
sequences exhibits power-law statistics similar to a description of fractal stochastic  
12 point processes (Telesca et al., 2001, 2009, 2011) to model the time-occurrence-  
sequence of seismic events. Quantifying the earthquake sequencing in terms of its  
14 fractal properties was done by means of the Fano factor and the Allan factor (Allan,  
1966; Barnes and Allan, 1966; Lowen and Teich, 1993, 1995; Thurner et al., 1997;  
16 Telesca et al., 2001, 2009, 2011; Flores-Marquez and Valverde-Esparza, 2012;  
Serinaldi and Kilsby, 2013). Since the fractal properties of the time-series studied  
18 here has never been investigated, we calculate the Fano factor and the Allan factor  
for the purpose of quantitative analysis.

20 The remainder of the paper is divided into three sections. In the next section, we  
show how the time-series of the state-to-state transition frequencies for a modified  
22 Markov chain model as described in Cavers and Vasudevan (2015) is generated. In  
the following section, we describe the EEMD procedure used and the analysis of  
24 the results that accrue from this procedure. We extend the approaches of Telesca et  
al. (2001, 2008, 2009, 2011) to calculate the Fano factor and the Allan factor with a

view to study the fractal properties of the time-series. In the last section, we discuss the results of the analysis methods and draw certain inferences about the state-to-state transition frequencies.

## 2 Directed graph representation of earthquake sequencing

A Markov chain is a discrete-time stochastic process  $X = \{X_0, X_1, X_2, \dots\}$  with state space  $S$  where  $\Pr\{X_{n+1} = j \mid X_0, \dots, X_n\} = \Pr\{X_{n+1} = j \mid X_n\}$ , for all  $j$  in  $S$  and  $n$  in  $\{0, 1, 2, \dots\}$ . For each  $n$ , the state of  $X_{n+1}$  is independent of  $X_0, X_1, \dots, X_{n-1}$  given  $X_n$ , and furthermore, we assume  $\Pr\{X_{n+1} = j \mid X_n = i\}$  is independent of  $n$  (Çınlar, 1975). To build a Markov chain model we first partition the region, either local or global, into zones. Typically these zones are made up of rectangles that divide the region (Nava et al., 2005; Ünal and Çelebioğlu, 2011). Recently, other partitions have been used. In particular, Cavers and Vasudevan (2015) used a simplified 5-zone plate boundary template as given by Kagan et al. (2010) to study global seismicity, while Ünal et al. (2014) used a seismic zones map that uses geographic information system analysis to divide Turkey into regions. For this particular study, we used the five-zone model described in Cavers and Vasudevan (2015) and give an overview of its construction here.

Kagan et al. (2010) partitioned the shallow ( $\leq 70$  km-depth) events with moment magnitude,  $M_w > 5.6$  from the Global CMT catalogue (1982/01/01-20082007/03/31) into 5 zone sub-catalogues using their grid-assignment schemes (Table 1). The selected catalogue consists of 6752 earthquakes with 4407 from **Zone 4** (Trenches), 723 from **Zone 3** (Fast-spreading ridges), 487 from **Zone 2** (Slow-spreading ridges), 898 from **Zone 1** (Active continent), and 237 from **Zone 0** (Plate interior) respectively. For these five zones, we express a state,

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Commented [m4]: The catalogue is 1982/01/01 to 2007/03/31. This was a typo on our part.

corresponding to a time interval  $\Delta t$ , as a concatenation of binary digits  $b_4 b_3 b_2 b_1 b_0$ ,

2 where  $b_L = 1$  indicates an earthquake occurrence in zone  $L$  during the specified time  
interval  $\Delta t$ , and  $b_L = 0$  indicates the lack of an earthquake occurrence in zone  $L$   
4 during the specified time interval  $\Delta t$ . We use  $\Theta = [\theta_{ij}]$  to denote the transition  
frequency matrix, where  $\theta_{ij}$  is the number of occurrences of transitions from  
6 state  $i$  to state  $j$ . Letting  $s(n)$  represent the state for interval number  $n$ , the probability  
transition matrix,  $P = [p_{ij}]$ , consists of transition probabilities,  $p_{ij}$ , given as

$$8 \quad p_{ij} = \Pr \{s(n+1) = j \mid s(n) = i\} = \Pr \{j|i\}, \quad (1)$$

$$p_{ij} = \theta_{ij} / \zeta_{ij}, \text{ where } \zeta_{ij} = \sum_j \theta_{ij}. \quad (2)$$

10 ~~For a Markov chain structure given earlier for the five zones, the computation of  
transition frequencies and hence, transition probabilities, depend on the chosen  
12 time interval  $\Delta t$ .~~

~~For a Markov chain structure given earlier for the five zones, the computation of  
14 transition frequencies and hence, transition probabilities, depend on the chosen  
time interval,  $\Delta t$ . We use the simple rules outlined by Nava et al. (2005) to choose  
16  $\Delta t$ :~~

~~1.  $\Delta t$  should be small enough such that the hazard estimations are useful; □~~

18 ~~2.  $\Delta t$  should not be too small that the most frequently occurring transition is from  
state 0 to state 0; □~~

20 ~~3.  $\Delta t$  should not be too large that state 31 to state 31 transitions are dominant.~~

~~So, for the threshold magnitudes chosen,  $\Delta t$  should be large enough to allow  
22 interaction among regions and make estimates of Markov chain transition  
probabilities robust. Following the selection rules given elsewhere (Nava et al.,~~

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2005; Ünal and Çelebioğlu, 2011; Cavers and Vasudevan, 2015), we used a  $\Delta t$  value of 9 days for the construction of the Markov chain of transition probabilities.

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A finite-state Markov chain can be depicted using a digraph representation,  $G$ , where the set of possible states (binary strings of length 5) are the nodes, and an arc  $(i, j)$  connects two states  $i$  and  $j$  if and only if  $p_{ij} > 0$  (Jarvis and Shier, 1996). Figure 1 shows an example of a digraph representing a Markov chain with a three zone partition, hence, there there are  $2^3 = 8$  states  $\{000, 001, 010, 011, 100, 101, 110, 111\}$  that we write in decimal format  $\{0, 1, 2, 3, 4, 5, 6, 7\}$ , respectively. In this figure, we do not show all of the possible transitions between states and typically an arc  $(i, j)$  is omitted when  $p_{ij} = 0$ . We follow the same decimal state labelling format as in Figure 1 for our  $2^5 = 32$  states, that is, state '0' (representing 00000 in binary) corresponds to no earthquake occurrence in all five zones in the chosen time interval,  $\Delta t$ , and state '31' (representing 11111 in binary) points to earthquake occurrences in all five zones. Table 2 shows details for defining all other states, '1' to '29'.

For a Markov chain structure given earlier for the five zones, the computation of transition frequencies and hence, transition probabilities, depend on the chosen time-interval,  $\Delta t$ . We use the simple rules outlined by Nava et al. (2005) to choose  $\Delta t$ :

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1.  $\Delta t$  should be small enough such that the hazard estimations are useful;

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2.  $\Delta t$  should not be too small that the most frequently occurring transition is from state 0 to state 0;

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3.  $\Delta t$  should not be too large that state 31 to state 31 transitions are dominant.

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So, for the threshold magnitudes chosen,  $\Delta t$  should be large enough to allow interaction among regions and make estimates of Markov chain transition probabilities robust. Following the selection rules given elsewhere (Nava et al., 2005; Ünal and Çelebioğlu, 2011; Cavers and Vasudevan, 2015), we used a  $\Delta t$  value of 9 days for the construction of the Markov chain of transition probabilities.

- The combinatorial structure of a digraph representation of the Markov chain model contains important information for earthquake sequencing (Cavers and Vasudevan, 2015). It is often useful to use a weight,  $w_{ij}$ , for each arc  $(i, j)$  of the digraph to get a weighted digraph. The weights have the form  $w_{ij} = \theta_{ij}$ ,  $w_{ij} = p_{ij}$ , or can be empirically derived from the Markov chain. To introduce spatial-temporal complexity into the model so that transitions with earthquake occurrences at large distances have less of an impact on our model than transitions with earthquake occurrences at short distances, we follow the approach by Cavers and Vasudevan (2015) to modify the weights  $w_{ij}$  in the weighted digraph by considering recurrences. Each earthquake (event) in a zone may have several recurring events in the record-breaking sense (Davidsen, 2008). For example, an event  $j$  is treated as a record with respect to an earthquake  $i$  if no event takes place within the  $i$ - $j$  spatial distance,  $d_{ij}$ , between  $i$  and  $j$  around  $i$  during the time interval  $[t_i, t_j]$  with  $t_i < t_j$ . The next record-breaking event,  $k$ , in the catalogue with reference to the original event,  $i$ , during the time interval  $[t_i, t_k]$  with  $t_i < t_k$  will have a spatial distance,  $d_{ik}$ , less than  $d_{ij}$ . The recurring events for one event in a given zone may fall into other zones or may be in the same zone. This flexibility adds to the possibility of interactions among zones. We first form the network of recurrences as described by Davidsen et al. (2008). The weight applied to each arc in the network of recurrences is derived empirically by using a total count of record breaking events between the

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corresponding earthquake zones and the distance involved (Cavers and Vasudevan, 2015; Vasudevan and Cavers, 2013). Each recurrence from an earthquake  $a$  to an earthquake  $b$  in the sequence is given a weight between 0 and 1, with a weight equal to 1 if the distance between  $a$  and  $b$  is less than 50 km. If the distance is  $r$  with  $r > 50$  km and earthquakes  $a$  and  $b$  occur in Zones  $j$  and  $k$  respectively, a weight of

$$[L_{jk}(20000) - L_{jk}(r)] / [L_{jk}(20000) - L_{jk}(50)] \quad (3)$$

is given, where  $L_{jk}(r)$  defined by Cavers and Vasudevan (2015) is the number of record-breaking events from zone  $j$  to zone  $k$  at distance at most  $r$  in the network of recurrences. The function in Equation (3) is a decreasing function in  $r$  giving a weight close to 0 when the distance  $r$  is large. Note that for  $r = 50$  km, an output of 1 is given while for  $r = 20,000$  km, an output of 0 is given. As described by Cavers and Vasudevan (2015), a Markov chain with the inclusion of spatio-temporal complexity of recurring events is derived by summing the weights of the recurrence arcs corresponding to occurrences from state  $i$  to state  $j$  in consecutive time-intervals. Here, we calculated the time-series of the resulting state-to-state sequence (Figure 2a) and the corresponding transition frequency matrix (Figure 2b). There is one comment in order here. Figures 2a and 2b provide different representations of the same Markov chain. The first can be considered “dynamic”, because it shows the time evolution of the transition from one state to another in consecutive time intervals of 9 days each. The second can be considered “static” because it shows the transition probabilities from one state to another but considering the whole earthquake sequence occurred during the whole observation period. However, they are not equivalent. We can go from the time-series data to transition-frequency matrix. We cannot go from the timetransition-frequency matrix to time-series without the additional information such as the catalogue and the record-breaking



statistics of recurrences. Since it is obtained from the non-linear, non-stationary  
2 global earthquake sequence, we consider it non-linear and non-stationary as well,  
and hence, can be subjected to analysis methods. Although it is not shown here, the  
4 approach equally applies to earthquake catalogues from localized seismogenic  
zones.

6

### 3 Analysis methods **and results**

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8 Each sample in the time-series shown in Figure 2a represents a “zone-configuration”  
state (Table 2). By definition, a zone-configuration has no zone or some zones or  
10 all zones highlighted by an earthquake or more in the optimally chosen time-  
interval. Going from one sample to the next does not only represent going from one  
12 state to the next but also shows the amplitude fluctuation between them. The  
adjacent states could represent the same zone-configuration or different zone-  
14 configurations. The time-series deduced from using the present approach with the  
five-zones marks the state-to-state fluctuations arising out of the fluctuations of  
16 oscillations or earthquake occurrences in the five-zones. We present in the  
following two analysis methods to glean an insight into the characteristics of the  
18 time-series.

#### 20 **3.1. Ensemble empirical mode decomposition as applied to state-to-state transition frequency sequence**

22

For non-linear and non-stationary time-series, the method of empirical mode  
24 decomposition (EMD) has been recently proposed as an adaptive time-frequency  
analysis method (Huang et al., 1998, 1999) to decompose the original data into a  
26 basis set of intrinsic mode functions. Since the process that leads to the state-to-  
state transition frequency sequence or time-series is inherently non-linear and non-

17

stationary, it is appropriate to apply the EMD to this data to understand the behavior  
2 of the intrinsic mode functions. The time-series (Figure 2a) reveals the fluctuations  
in the state-to-state transition frequencies arising out of varying occupancy of the  
4 zones from one time interval to the next. A situation would easily arise when two  
or three successive state-to-state transitions do not have earthquake occurrences in  
6 any of the zones studied. This would translate into intermittency in the time-series.  
Recent studies (Flandrin et al., 2004, 2005; Gledhill, 2003; Wu and Huang, 2004,  
8 2009) support the idea of carrying out noise-added analyses with the EMD. The  
noise added analyses involves multiple realization of added noises to the time-series  
10 in question, leading to the ensemble EMD (EEMD), as proposed by Wu and Huang  
(2004, 2009).

12 In the EEMD, the signal or the time-series in question with the added Gaussian  
white noise, denoted as one trial, would populate the whole time-frequency space  
14 uniformly with the constituting component of different scales. Since the noise  
added in each trial is different, the ensemble mean of the noise cancels out and,  
16 hence, the signal resides in the intrinsic mode functions generated from the EEMD  
(Wu and Huang, 2009).

18 The time-series of state-to-state transition frequencies of the modified Markov  
chain model,  $x_{ssf}(t)$ , is taken as the signal. In each realization of the experiment,  
20 white noise,  $w(t)$ , is added to the signal. One might interpret the added Gaussian  
white noise as the possible random noise that would be encountered in the  
22 measurement process or in certain restrictions applied to the calculation of edge  
weights in the modified Markov chain. So, for the  $i^{\text{th}}$  realization,

24 
$$x_{ssf,i}(t) = x_{ssf}(t) + w_i(t). \quad (4)$$

For each realization, we decompose the data with the added Gaussian white noise into intrinsic mode functions (IMFs). We consider the ensemble means of the IMFs of the decompositions as the final result.

Wu and Huang (2009) recommended that the ensemble size should be kept large and the amplitude of the added noise should not be small. We set the ensemble number for the number of realizations in EEMD large such that the noise series cancel each other in the final mean of the corresponding IMFs. For the two parameters, we used an ensemble size of 1000 and added noise with an amplitude of 0.2 times the standard deviation of the original data. We assume that the IMFs

resulting from the EEMD ~~truly represent the true IMFs,~~ represent a substantial improvement over the IMFs of the original EMD in that it utilizes the full advantage of the statistical characteristics of white noise to perturb the signal in its true solution neighbourhood, and to cancel itself after serving its purpose (Wu and Huang, 2009).

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EEMD results are summarized in Figures 3a to 3t with each intrinsic mode functions followed by ~~its~~ their state-to-state relative weight matrices\* derived from the basis set ~~of~~ the intrinsic mode functions of the time-series in a fashion identical to the original time-series. By summing the weights of the recurrence arcs corresponding to occurrences from state  $i$  to state  $j$  in consecutive time-intervals, we calculate the weighted matrix for state-to-state transitions for each intrinsic mode function. Since the intrinsic mode functions are the mathematical basis set of the original time-series, their static displays or the weighted matrices show negative values. Identical to the sum of the intrinsic mode functions yielding the original time-series, the sum of the weighted matrices yields its transition frequency matrix. Similar to what Huang et al. (1998, Figure 6 in their paper) have observed with the wind data, all of the intrinsic mode functions excluding the trend for the wind data contain both

positive and negative values. We observe the same thing with the time-series in  
2 that the transition probability values for the ~~intrinsic mode function~~IMFs show both  
positive and negative values except that the first two ~~intrinsic mode function~~IMFs  
4 have negative values larger than the lowest positive value of the trend. So, it is not  
surprising that the transition frequency matrices of the ~~intrinsic mode function~~IMFs  
6 contain the positive and negative numbers. However, viewing each ~~intrinsic mode~~  
~~function~~IMF with the trend starting with the third IMF will obviate this difficulty in  
8 that the high or low frequency fluctuations ride on the trend with no negative values  
and the corresponding transition-frequency matrices are positive. A similar  
10 observation has been made by Huang et al. (1998, Figure 7) with their wind data.  
The observation made with the first two ~~intrinsic mode function~~IMFs suggests a  
12 limit on the proposed method, and it would require further investigation.

\_\_\_The decomposition of the original time series into intrinsic mode functions and  
14 the trend is dyadic in nature, as shown in Figure 3. This means that as we go from  
the first intrinsic mode function to the second and so on, the interval increases by a  
16 factor of 2 from  $\Delta t = 9$  days to  $\Delta t = 18$  days and so on. With an increase in the time  
interval from one IMF to the next, we observe the relative weights of the state-to-  
18 state transitions to vary. We also find that the state-to-state transitions within each  
IMF occur in packets, and the number of packets progressively decreases. The last  
20 packet of state-to-state transitions is persistent over the first 8 IMFs corresponding  
to a time-interval of 9 days to 1152 days suggests the importance of the zone 4  
22 earthquakes in understanding the earthquake sequencing. Although zone 4  
earthquakes persist in the state-to-state transitions in the first few intrinsic mode  
24 functions, the participation of other zones in state-to-state transitions becomes  
significant in the higher intrinsic mode functions, IMFs 6 to 9.

In general, the intrinsic mode functions are characterized by (1) a certain number of  
2 a pattern of rise and fall of the arc weights and (2) by a systematic decrease in the  
frequency of the number of such patterns as one goes intrinsic mode function 1 to  
4 the intrinsic mode function 9. Since the rise and fall of the arc weights covers the  
entire catalogue of data, the periodicity that we notice could be intrinsic to  
6 earthquake processes.

The Hilbert-Huang amplitude spectrum of the time series, shown in Figure 4,  
8 reveals at least two important features: (1) The temporal fluctuations in amplitudes  
occur in packets, each packet containing a set of zone to zone interactions. The  
10 oscillatory behaviour of packets contains certain periodicity within the earthquake  
sequence. A periodic trend at low frequencies suggests the role of zone 4 (Trenches)  
12 and zone 0 (Intraplate). A higher power at ~~900-2004/03/06~~ and ~~950-2005/05/30~~  
~~time interval~~ **sindicates** the importance of zone 4 with earthquakes of larger  
14 magnitude prompting a cascade of aftershocks in zone 4 and main shocks in zones  
that are in close proximity to zone 4. (2) The frequency-dependence of amplitude  
16 packets encapsulates the relative importance of the interaction among multiple  
zones over different time intervals. We interpret them to mean that certain state-to-  
18 state transitions involving zone 4 are important over a range of frequencies.

### 3.2 Evaluation of fractality in a state-to-state transition frequency sequence

20 Earthquake occurrences have been modelled to be stochastic point processes  
(Thurner et al., 1997; Telesca et al., 2001, 2005, 2009 and 2011; Flores-Marquez  
22 and Valverde-Esparza, 2012). One representation of the point process is to examine  
the inter-event time-intervals. The resulting inter-event interval probability density  
24 function says something about the behavior of the times between events. We do not  
know anything about the information contained in the relationships among these

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items. Since successive events do not occur in constant time-intervals, another  
 2 representation of a point process is given by dividing the time-axis into equally  
 spaced contiguous counting windows of duration  $\tau$ , and producing a sequence of  
 4 counts that fall within each time-window. For example, for the  $k^{\text{th}}$  time-window,  
 the expression for the number of counts,  $N_k(\tau)$ , is given by

$$6 \quad N_k(\tau) = \int_{t_{k-1}}^{t_k} \sum_{j=1}^n \delta(t - t_j) dt \quad (5)$$

where  $N_k(\tau)$  is the number of earthquakes in the  $k^{\text{th}}$  window (Figure 5; panels a to  
 8 d). The correlation in the process  $\{N_k(\tau)\}$  is the correlation in the underlying point  
 process (Lowen and Teich, 1993a, 1993b; Thurner et al., 1997; Telesca et al., 2001,  
 10 2005, 2009, 2011) have accessed such a representation of the point-processes to  
 underscore the existence or non-existence of fractality in them. They have two  
 12 calculable measures, Fano factor (FF) and Allan factor (AF), to quantify the  
 fractality of the process (Lowen and Teich, 1993a, 1993b; Thurner et al., 1997;  
 14 Telesca et al., 2001, 2005, 2009, 2011; Flores-Marquez and Valverde-Esparza,  
 2012).

16 The Fano factor is a measure of correlation over different timescales (Thurner et  
 al. 1997). It is defined as the ratio of the variance of the number of events in a  
 18 specified counting time  $\tau$  to the mean number of events in the counting time, as is  
 given by

$$20 \quad FF(\tau) = \frac{\langle N_k^2(\tau) - N_k(\tau) \rangle}{\langle N_k(\tau) \rangle} \quad (6)$$

22 where  $\langle \rangle$  denotes the expectation value. Lowen and Teich (1995) point out that  
 the  $FF$  of a fractal point process follows a power law with the power-law exponent,  
 24  $\alpha$ , obeying  $0 < \alpha < 1$ . In other words, the  $FF$  is always greater than 1. For Poisson

processes, the  $FF$  is always near unity for all counting times, and the fractal  
2 exponent is approximately equal to zero.

The Allan factor is a relation with the variability of successive counts (Allan,  
4 1996; Barnes and Allan, 1966). It is the ratio of the variance of successive counts  
for a specified counting time  $\tau$  divided by twice the mean number of events in the  
6 counting time. The expression of  $AF$  is given as

$$AF(\tau) = \frac{\langle N_{k+1}(\tau) - N_k(\tau) \rangle^2}{2\langle N_k(\tau) \rangle} \quad (7)$$

8 Similar to the  $FF$ , the  $AF$  assumes values near unity for Poisson processes. Telesca  
et al. (2009, 2011; henceforth, referred to as Telesca's approach) and Flores-  
10 Marquez and Valverde-Esparza (2012) have shown the power-law exponent for the  
 $AF$  to be  $0 < \alpha < 1$ .

12 In this paper, we examine both the results of Telesca's approach to the initial  
catalogue of the data used and of the new representation of the point process with a  
14 Markov chain model. For the working model, we compute the state-to-state  
transition frequencies as described by Nava et al. (2005) and as applied to global  
16 seismicity (Vasudevan and Cavers, 2012; Cavers and Vasudevan, 2013).  
Expressions similar to equations (6) and (7) can be derived if we know the optimal  
18 time-interval for the Markov chain model. Since we know the optimal time-interval,  
we introduce a sequence of state-to-state transition frequencies,  $\{N_{ssff,k}(\tau)\}$ , with  
20  $N_{ssff,k}(\tau)$  referring to the weight of state-to-state transitions over the  $k^{\text{th}}$  window for  
the optimal time-interval, as is shown in Figure 5f. For an easy understanding of  
22 Figure 5f, we have included Figure 5e.

There are a few observations to be made. First,  $N_{ssff,k}(\tau)$  is not necessarily an  
24 integer number for any  $k^{\text{th}}$  window. Following the definition of a state, in the context  
of a directed graph of a Markov chain model, a state-to-state transition refers to an

edge of a graph. It is the weight associated with the edge of the directed graph that plays an important role. Since we have used a modified Markov chain model which includes the influence of the event recurrences in the record-breaking sense, the above expression includes their weights as well in the computation of  $N_{sstf,k}(\tau)$ . The sequence of state-to-state transition frequencies,  $\{N_{sstf,k}(\tau)\}$ , yields a time-series. This time-series is the new expression of the point-process where the weighted edges of directed graph of the modified Markov chain represent the significance of the earthquakes between states. This new alternative representation signifies the behavior of the state-to-state transition frequencies over a large time window. Here, seeking to find the time-correlative behavior of the time-series would be of great importance since this would give us an opportunity to see the interaction of zones considered in a collective sense.

Here, we seek to understand the correlative behavior by looking at the two statistical measures,  $FF_{sstf}$  and  $AF_{sstf}$ , as defined below:

$$FF_{sstf}(\tau) = \frac{\langle N_{sstf,k}^2(\tau) - N_{sstf,k}(\tau) \rangle}{\langle N_k(\tau) \rangle} \quad (8)$$

$$AF_{sstf}(\tau) = \frac{\langle N_{sstf,k+1}(\tau) - N_{sstf,k}(\tau) \rangle^2}{2\langle N_{sstf,k}(\tau) \rangle} \quad (9)$$

The behavior of the two measures,  $FF_{sstf}$  and  $AF_{sstf}$ , with respect to the optimal time-interval should shed some light on the correlative behavior of the time-series but also on the selective clustering of the certain state-to-state transitions. We consider this knowledge to be useful for forecasting purposes.

In our adaptation of the sum of edge weights for the state-to-state transition frequencies as a new representation of a point-process embedded in the modified Markov chain here, the arguments of Thurner et al. (1997) and Telesca et al. (2001, 2005, 2009, 2011) would apply. This means that the FF of the modified Markov



chain sequence would follow a power-law with the power-law exponent,  $\alpha$ ,  
2 satisfying  $0 < \alpha < 1$ .

Extending this to  $FF_{sstf}$  and  $AF_{sstf}$ , as is shown in Figure 6 (panels 6c and 6d), we  
4 find that the power law exponent calculated, corresponding to the least-squares fit  
of the data is greater than zero (0.27 and 0.30 respectively). They suggest not only  
6 the fractality of the modified Markov chain sequence for optimal time-interval but  
also the deviation from the Poissonian behavior of earthquake sequencing  
8 considered in this present study.

#### 10 4 Discussion and conclusions

Thurner et al. (1997) pointed out that the sequence of counts, generated by recording  
12 the number of events in successive counting time-windows of certain length,  
contained information about the point process depicted by the set of event times.  
14 This idea was further tested in understanding the dynamics of earthquake  
sequencing (Telesca et al., 2009, 2011; Flores-Marquez and Valverde-Esparza,  
16 2013), and in particular, the fractal behavior of the sequence of counts. We know  
that this idea was initially restricted to the sequence of counts for varying windows  
18 of interval-times. However, for comparison purposes, we calculated the Fano factor  
and the Allan factor for the initial catalogue of data using equations (6) and (7). We  
20 include their graphs in Figure 6 (panels 6a and 6b). Similar to observations made  
by Telesca et al. (2009) with the earthquake data from the Taiwan region, we find  
22 the presence of two distinctly different regions of scaling behaviour. For small time-  
intervals, we also observe the Poisson behaviour. Since very poor statistics at time-  
24 scales larger than  $10^8$  seconds would influence the Fano factor and the Allan factor,  
we have restricted our analysis to ~~2-53.17~~ years or roughly  $10^8$  seconds.

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In our description of the directed graph of the Markov chain model of any earthquake sequencing, regional or global, we stress the significance of the state-to-state transition probabilities for multiple zones that span the sequence of earthquakes over an optimal time window (Cavers and Vasudevan, 2013; Vasudevan and Cavers, 2013). In other words, the edges of the directed graph carry weights. We conjecture that these weights represent a new definition of the point process. Furthermore, a consideration of the earthquake recurrences within each zone and among zones, following the concept of recurrences in the record-breaking sense (Davidsen et al., 2008), leads to an empirically-determined distance-dependent weights for the edges. Unlike extending the idea of the sequence of counts where every event occurrence augments the counting value by unity (Turner et al., 1997; Telesca et al., 2009, 2011; Flores-Marquez and Valverde-Esparza, 2013), we consider the summing of the weights for each edge such that the sum represents a “pulse” for each state-to-state transition. We analyse the resulting time-series from the point of view of its Fano factor and Allan factor. There is evidence for fractality of the multi-state modified Markov chain to represent the earthquake sequencing, as is revealed by the power-law scaling behavior present in the Fano and Allan factors with their respective exponents of 0.27 and 0.30 (Figure 6, panels 6c and 6d). However, it is important to note that the exponents of the power-laws in both cases have a smaller value than those observed for the initial catalogue.

Cavers and Vasudevan (2013) interpreted the Markov chain of 32-states for five-distinctly different zones to contain the basic combinatoric structure superimposed by the thumb-print of the undulatory structure of the recurrence weights. Since the earthquake sequencing is in general non-linear and non-stationary, we contend that the time-series representing the above Markov chain is also non-linear and non-

stationary, and is conducive to an ensemble empirical mode decomposition (EEMD) procedure to understand its intrinsic mode functions (IMFs). The ensemble empirical model decomposition of the time-series leads to nine intrinsic mode functions and a trend. Each one of the IMFs reveals the amplitude fluctuation of the state-to-state transitions. While there is a commonality in the relative dominance of the subduction-style earthquakes, represented by the top right corner grid of the relative weight matrices (Figure 3), the presence or absence of certain state-to-state transitions in certain IMFs reveals the importance of integral multiples of the optimal time-interval.

A simple observation of the first 6 or 7 IMFs stresses the importance of multiple-zone approach to global seismicity problem in that the earthquake sequencing for the time period we considered has similar oscillatory behavior of the state-to-state transition probabilities from the point of view of the amplitude scaling and the oscillating period. The growth and decay of oscillations in easily identifiable packets in each IMF following certain periodicity is an intrinsic signature of the role of multiple zones in earthquake sequencing.

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## List of tables

2

4 **Table 1.** Tectonic zone identifier, tectonic zone and the number of earthquakes considered for  $M_w$   
4 > 5.6 and depth < 70 km from 1982/01/01 to 20078/03/31. |

6 **Table 2.** Zone and state definition used in the construction of a directed graph of a Markov chain.  
8 '0' and '1' refer to the no occurrence or occurrence of an earthquake for a given zone. For five  
8 zones, there are 32 states.

**Table 1.**

<b>Zone identifier</b>	<b>Tectonic zone</b>	<b>N</b>	<b>N/N<sub>total</sub></b>
0	Plate-interior	237	0.0351
1	Active continent	898	0.1330
2	Slow-spreading ridges	487	0.0721
3	Fast-spreading ridges	723	0.1071
4	Trenches	4407	0.6527
	Global (or N <sub>total</sub> )	6752	1.0000

2



**Table 2.**

<b>State</b>	<b>Zone 4</b>	<b>Zone 3</b>	<b>Zone 2</b>	<b>Zone 1</b>	<b>Zone 0</b>
0	0	0	0	0	0
1	0	0	0	0	1
2	0	0	0	1	0
3	0	0	0	1	1
4	0	0	1	0	0
5	0	0	1	0	1
6	0	0	1	1	0
7	0	0	1	1	1
8	0	1	0	0	0
9	0	1	0	0	1
10	0	1	0	1	0
11	0	1	0	1	1
12	0	1	1	0	0
13	0	1	1	0	1
14	0	1	1	1	0
15	0	1	1	1	1
16	1	0	0	0	0
17	1	0	0	0	1
18	1	0	0	1	0
19	1	0	0	1	1
20	1	0	1	0	0
21	1	0	1	0	1
22	1	0	1	1	0
23	1	0	1	1	1
24	1	1	0	0	0
25	1	1	0	0	1
26	1	1	0	1	0
27	1	1	0	1	1
28	1	1	1	0	0
29	1	1	1	0	1
30	1	1	1	1	0
31	1	1	1	1	1

2

## List of figures

2

**Fig 1.** A graph representation of earthquake sequencing with arcs (with weights  $w_{ij}$ ) representing transitions between states.

4

**Fig 2.** (a) A time-series of the state-to-state transition frequencies of the modified Markov chain model of the earthquake sequencing. The sampling time ( $\Delta t$ ) of 9 days is used. (b) The state-to-state transition frequencies of the modified Markov chain model of the earthquake sequencing.

8

10 **Fig 3.** Ensemble empirical mode decomposition of the time series. (a-i) ~~First~~ intrinsic mode functions from the first to the ninth; (j) ~~Intrinsic mode function of the trend;~~ (k-s) ~~(b)~~ State-to-state relative weight matrices for the ~~first~~ intrinsic mode functions from the first to the ninth; (e) ~~State-to-state relative weight matrix of the trend.~~ Time steps and the corresponding calendar dates: ~~0  $\Delta t$  -> 1982/01/01; 200  $\Delta t$  -> 1986/12/06; 400  $\Delta t$  -> 1991/11/10; 600  $\Delta t$  -> 1996/10/14; 800  $\Delta t$  -> 2001/09/18; 1000  $\Delta t$  -> 2006/08/23; 1024  $\Delta t$  -> 2007/03/27.~~ We provide this information here to avoid any cluttering of the plots. ~~Second intrinsic mode function; (d) State-to-state relative weight matrix for the second intrinsic mode function; (e) Third intrinsic mode function; (f) State-to-state relative weight matrix for the third intrinsic mode function; (g) Fourth intrinsic mode function; (h) State-to-state relative weight matrix for the fourth intrinsic mode function; (i) Fifth intrinsic mode function; (j) State-to-state relative weight matrix for the fifth intrinsic mode function; (k) Sixth intrinsic mode function; (l) State-to-state relative weight matrix for the sixth intrinsic mode function; (m) Seventh intrinsic mode function; (n) State-to-state relative weight matrix for the seventh intrinsic mode function; (o) Eighth intrinsic mode function; (p) State-to-state relative weight matrix for the eighth intrinsic mode function; (q) Ninth intrinsic mode function; (r) State-to-state relative weight matrix for the ninth intrinsic mode function; (s) trend; (t) State-to-state relative weight matrix for the ninth intrinsic mode function.~~

28 **Fig 4.** Hilbert-Huang amplitude spectrum of the intrinsic functions.

30 **Fig 5.** Representation of a point process (panels a to d) versus representation of a state-to-state transition (panels e and f). (Adapted from Thurner et al. (1997))

32

34 **Fig 6.** (a) ~~and (b) show the Fano and Allan factor graphs, respectively,~~ derived from the earthquake catalogue data using the approach of Telesca et al. (2001, 2008, 2009, 2011); ~~(b) Allan-factor graph derived from the earthquake catalogue data using the approach of Telesca et al. (2001, 2008, 2009, 2011);~~ (c) ~~and (d) show the Fano and Allan factor graphs, respectively,~~ for the time series of the state-to-state transition frequencies of the modified Markov chain model of the earthquake sequencing; ~~(d) Allan factor graph for the time series of the state-to-state transition frequencies of the modified Markov chain model of the earthquake sequencing.~~

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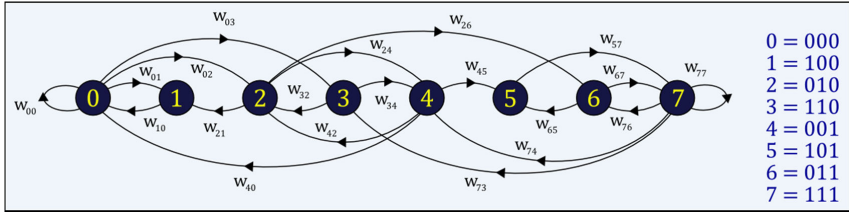
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Figure 1.

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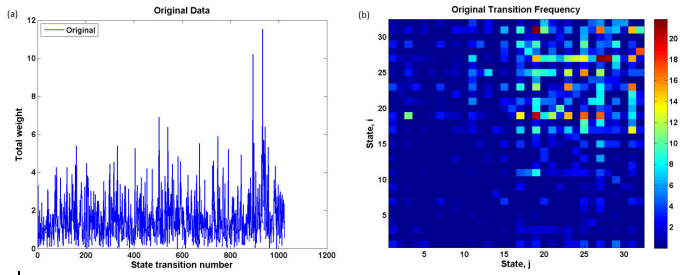
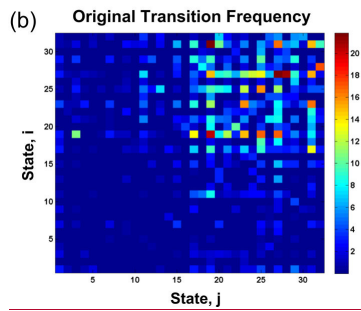
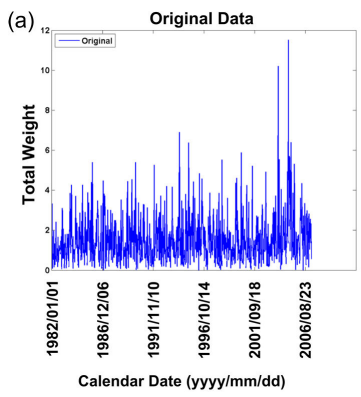


Figure 2.

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Figure 3.

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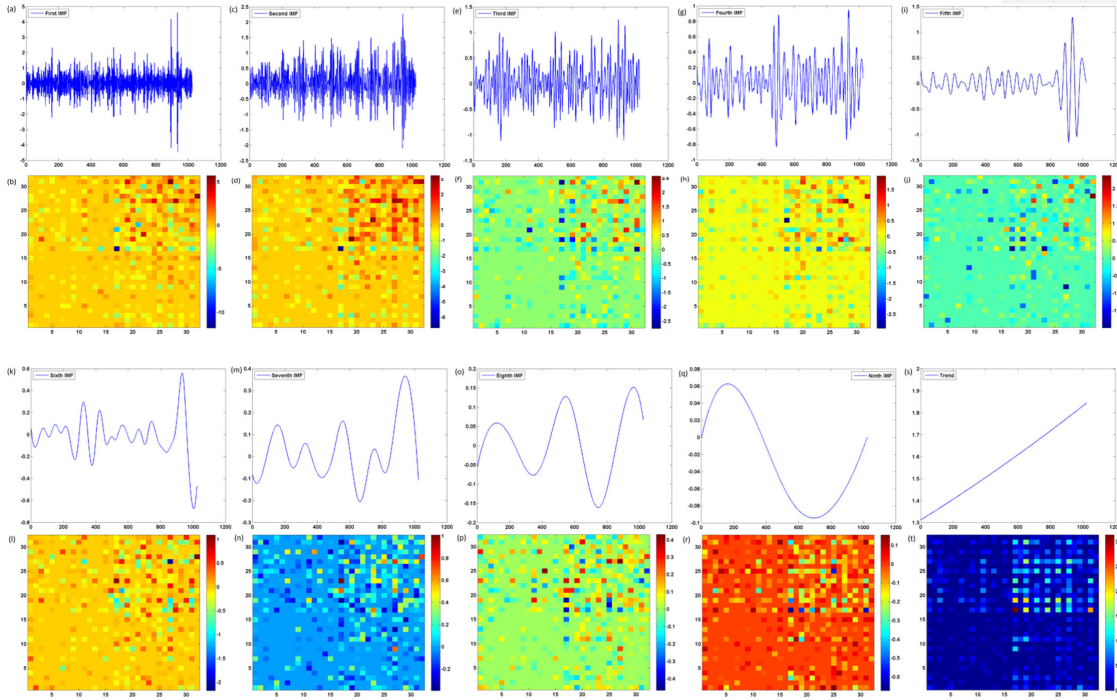
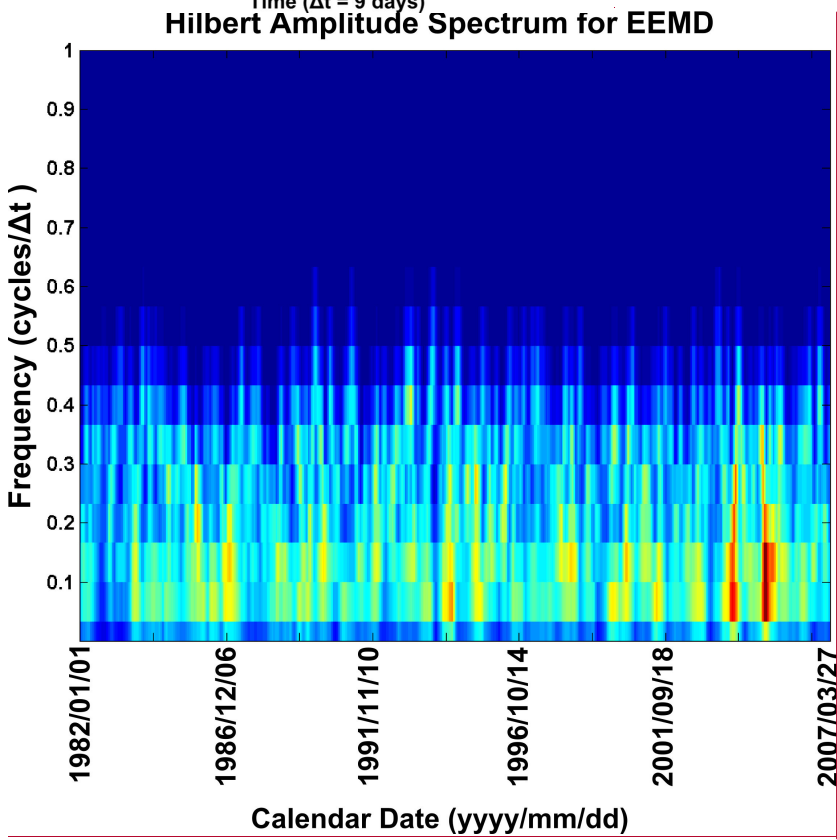
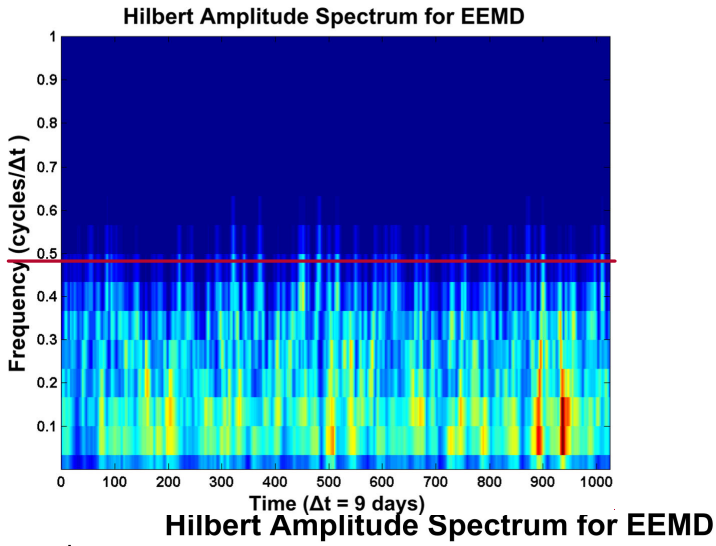


Figure 4.

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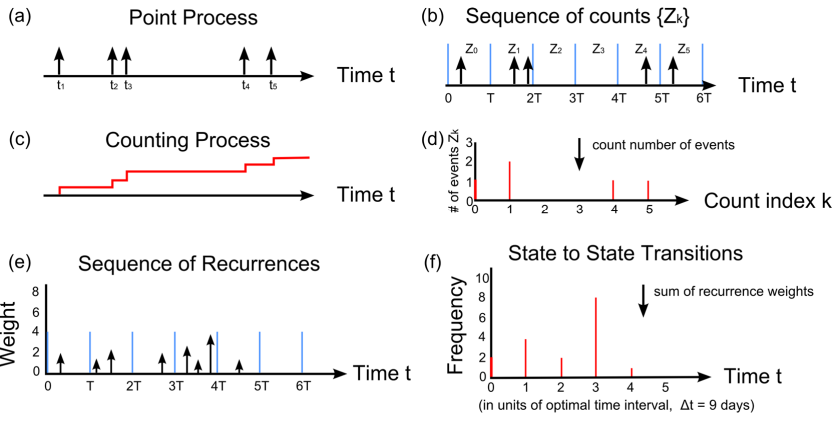
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Figure 5.

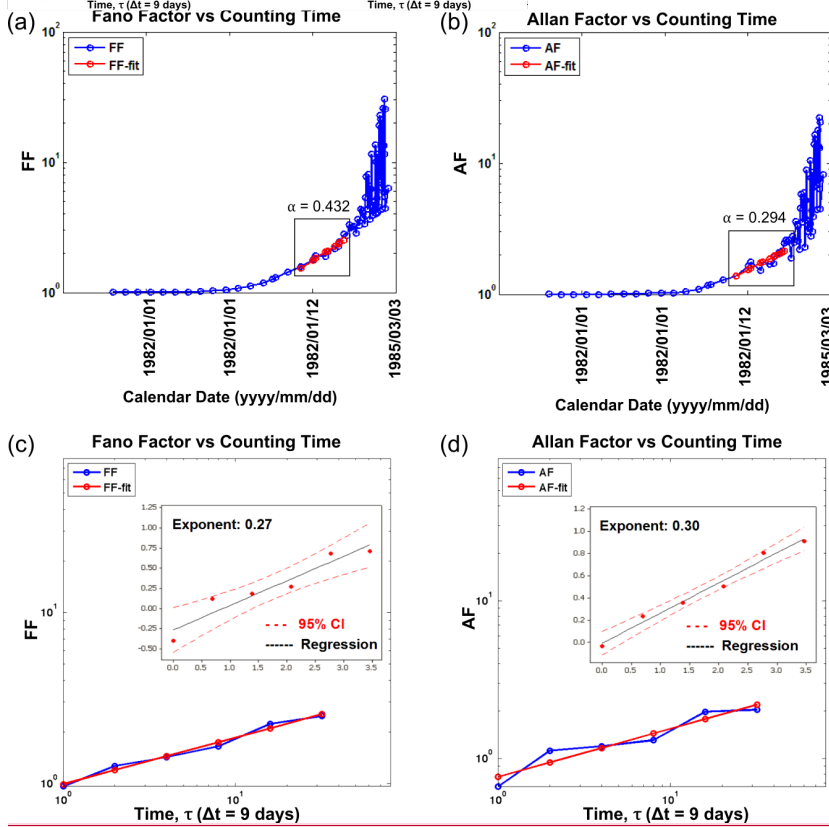
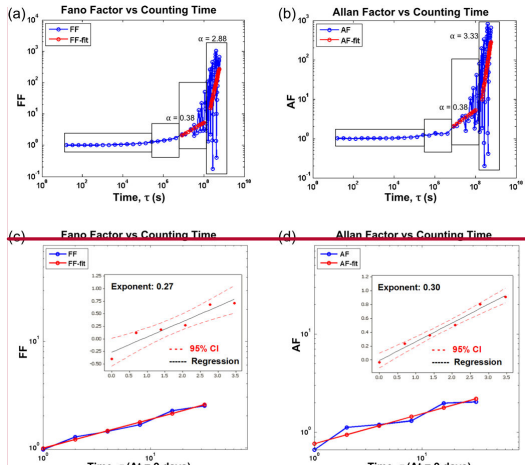
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Figure 6.

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