Responses to the comments of all reviewers

COMMENTS TO ALL REVIEWERS:

We appreciate the efforts of all of the reviewers. We would like to further emphasise the new and novel aspects at the end of the introduction in the revised manuscript, by including the following points

1. We believe that this is the first ever study to systematically compare subgrid models of quasi-geostrophic (QG) turbulence in the atmosphere and ocean. In particular it is the first study were simple unified scaling laws have been presented that apply to both media.
2. The study uses a much larger set of simulations covering a much broader range of flow parameters, including an order of magnitude change in the Rossby radius of deformation and the energy containing scale, compared with previous studies.
3. By focussing on the enstrophy cascading inertial range in both media, the large number of simulations and wide parameter range has enabled the establishment of robust scaling laws.
4. The scaling laws presented here are particularly simple with eddy viscosity magnitudes that are proportional to T_R^{-1} and power exponents that are approximately proportional to T_R. These results, and the fact that $\boldsymbol{\nu}_{\mathbf{d}} \approx 2\boldsymbol{\nu}_{\mathbf{b}}$, are suggestive of robust fundamental properties of QG turbulence.

We address the specific comments of each reviewer below. Associated changes to the manuscript are marked in blue text.

SPECIFIC POINTS OF REVIEW 1:

- *You say that 'an increase in resolution will not necessarily improve the accuracy'. This seems like a secondary consideration; isn't the primary concern that at low (fixed) resolution the accuracy may be poor?*

The resolution dependence problem is an important issue for all model resolutions, both in terms of accuracy and computational efficiency. If the subgrid model is not applied in a self-similar manner, then the small scales (high wavenumbers) are not modelled correctly, and are typically overly dissipated. This range extending from wavenumber, k_D, to the truncation wavenumber, T_R, is referred to as the dissipation range. If we are interested in scales up to wavenumber k_I, then one must ensure that $k_D > k_I$. To ensure that the dissipation range does not overlap with the scales of interest, overly high resolutions are required which increase T_R and in doing so also k_D. If the subgrid model is applied in a self-similar manner (using the prescribed scaling laws for instance) then there is no artificial dissipation range and all scales can be trusted (see figure 5 of the manuscript). This is becoming increasingly important as the research community is increasingly using smaller scale (higher wavenumber) information to study aspects such as extreme events.

- *It might help to note that alpha, D0, and kappa in equation (1) are not constants but operators.*

The updated version of the manuscript now states the following in paragraph 2 of section 2:

Using standard fluid mechanical nomenclature, D_0^j is the bare dissipation operator representing the unresolved eddy-eddy (or inter-eddy) interactions in the benchmark simulation (McComb, 1990). The constant α^j parameterises the drag by dampening the large scales of motion. Simulations are nudged toward a climate $\tilde{q}^j$ by the constant relaxation parameter κ^j.

- *Its not clear to me why you would ever have j not equal to k in equations 6 and 7. Why not use the version from the Kitsios et al 2012 paper (eqns 8 and 9)?*

This is the more general case, where one can calculate the flux of enstrophy (and hence energy) from level 1 to level 2 (or from level 2 to level 1). This is particularly important for the situation in which baroclinic instability is not resolved, which is addressed in Kitsios et al. (2013). You are correct, however, in that our current study we are primarily concerned with the case in which baroclinic instability is resolved, and hence the only components of $\eta^{jk}(n)$ that are important are for those where $j = k$.

- *In the discussion between equations 11 and 12, and in equation 12 I suspect that there are some places where a subscript 0 is missing on t.*

In fact on page 1687 line 15 and line 16, t_0 should be t. This is corrected in the updated version of the manuscript.

- *page 1689 line 20: The fact that the drain is negative in this regime is very interesting; it might be worth mentioning the recent related work by Jansen & Held (2014) who have proposed a parameterization in the oceanic regime also have negative viscosity.*

The updated version now includes the following sentence in the above referenced place:

Jansen and Held (2014) developed heuristic general purpose oceanic subgrid models for this regime that also have negative viscosity.

- *page 1690 line 9 you say "as more eddies are being explicitly resolved, less enstrophy is transferred to fewer subgrid eddies". This statement seems to conflict with the fact that the enstrophy flux is approximately constant through this range of wavenumbers?*

Yes you are correct. The statement in the updated version of the manuscript now reads as follows:

as more eddies are being explicitly resolved, the enstrophy (and energy) is being transferred to fewer subgrid eddies.

- *Although the energy spectra of the LES show excellent agreement with the DNS, this is only one measure of accuracy. One might also check things like meridional heat flux or large-scale EOFs.*

The Lyapunov equation (equation 12 in the manuscript) ensures that the subgrid terms (effectively Reynolds stresses) balance exactly; therefore, the mean heat and momentum fluxes must also be appropriate in order to get the mean flow correct. In our studies the mean climate state of each of the LES variants matches with that of the DNS with pattern correlations of at least 0.9994. Recall that the subgrid modelling is undertaken in terms of the potential vorticity variable. As defined in equation 2 of the manuscript, the potential vorticity by definition incorporates both the effects of vertical shear ($\zeta_1 - \zeta_2$) and also horizontal shear, via the relationship between the streamfunction and vorticity. The potential vorticity fluxes are essentially equivalent to heat and momentum fluxes.

GENERAL COMMENTS OF REVIEWER 2:

With regard to the reviewer's comments concerning the possible effects of topography, we are currently working on the subgrid modelling studies of atmospheric simulations with topography. We find that the scaling laws governing the eddy-eddy interactions, as presented in the present manuscript, are largely independent of the the topography. In recently published work, Frederiksen et al. (2015) have recovered analogous subgrid coefficients for more complex primitive equation models. These results suggests that the findings from the QG simulations adopted in the present study apply more broadly.

Frederiksen and Kepert (2006) looked at predictability in 10 day simulations with LES, and it was found that model errors and initial conditions error are in fact much larger than any errors associated with the type of subgrid model adopted in both in Frederiksen and Kepert (2006) and also in our present study. It has been found that correct representation of the kinetic energy spectrum, and the addition of stochastic backscatter in the subgrid models improves the ensemble spread and hence predictability of the system (O'Kane and Frederiksen, 2004; Shutts, 2005; Tennant et al., 2011).

MINOR POINTS OF REVIEWER 2:

- *The choice of atmospheric Rossby deformation radius is rather small at 447 km. This may not be very important but I would prefer to see the value at 1000 km.*

The study covers the sensitivity of the subgrid model coefficients to k_R and hence the Rossby radius across an order of magnitude. As you we suggest that the specific value of the atmospheric Rossby radius is not necessarily important.

- *page 1682 line 17 - perhaps a comment could be made on the origin of the Error function dependence ?*

The error function dependence for the drag in the oceanic simulations is used to control the wavenumber location of k_E. Using this approach we are able to study the influence of k_E on the subgrid coefficients, and in the manuscript we develop scaling laws to represent the variability in both k_R and k_E. The updated version now includes the following sentence in the above referenced place:

This functional form allows us to control the location of the energy containing wavenumber

- *In the conclusions, the authors should discuss how their ideas could be modified to take account of the differing dynamical cores used in real NWP and climate models. Their advection schemes are typically quite dissipative and its difficult to see how the turbulence theory could be amended to account for each specific scheme (e.g. semi-Lagrangian/semi-implicit or finite element).*

We would first like to reiterate that the focus of this manuscript is to accentuate the similarities between the properties of subgrid turbulence in the atmosphere and ocean, and not on addressing issues associated with specific numerical implementations of various dynamical cores. Having said that, as you request, we have addressed the issues associated with semi-Lagrangian and semi-implicit time stepping schemes below.

We are actually currently implementing these scaling laws into a complex GCM, which is used for national weather prediction and climate studies. This grid point based GCM is called ACCESS, which is a version of the Hadley Centre model. The subgrid models are being implemented in the ACCESS GCM via grid to spectral transforms, with the specifics discussed in response to the comments of reviewer 3. This GCM has a Lagrangian time stepping scheme, which as the reviewer correctly points out has its own resolution dependent dissipation, and deformation of the kinetic energy spectra. We have run this GCM at various resolutions and do indeed observe resolution dependence associated with the Lagrangian time stepping. Whilst the Lagrangian time stepping allows an order N increase in time step size, it does introduce an significant artificial dissipation range, which to cure requires an order N increase grid resolution, conservatively resulting in an order N^3 increase in computational effort. We can (and have) used the stochastic modelling framework outlined in the current paper to modify the dissipation associated with the Lagrangian time stepping, and avoid such required increases in grid resolution. This, however, is not ideal and a semi-implicit time stepping scheme with known dissipation characteristics would be preferred. The issue of deformation of the kinetic energy spectra may not be a significant one in weather prediction since over the course of 7 days the spectra remains relatively constant. However, for climate simulations undertaken over a period of many years, spectral deformation and resolution dependence is a significant problem.

The following trimmed down version of the above discussion pertaining mainly to implementation of the scaling laws into grid point/finite element codes has been added to the conclusions in the updated version of the manuscript.

The scaling laws developed here can be implemented directly into spectral simulations, and are expected to improve the efficiency and accuracy of numerical weather and climate simulations (Frederiksen et al., 2003, 2015). There are also two possible approaches to implement these scaling laws into grid point codes. The simplest approach is to apply the subgrid model directly in grid-point space via a Laplacian operator of the appropriate power, as outlined in Table 1. More generally it is also possible to employ grid to spectral transforms, where the subgrid model is calculated in spectral space, and then applied in physical space.

SPECIFIC COMMENTS OF REVIEWER 3:

- *The scaling laws for the inverse energy cascade range are not included*

The scaling law behaviour within the inverse energy cascade is different to that within the forward enstrophy cascading inertial range, with different resolution dependence. The subgrid dissipation matrices in the inverse energy cascade are in fact more complex as they are non-diagonal in level space, while the matrices are diagonal in the enstrophy cascading inertial range.

Within the enstrophy cascading inertial range the resolution dependence of the scaling laws also suggests something more fundamental about turbulence in general. The scaling laws illustrated in figure 4, indicate that the magnitude of the eddy viscosity is proportional to T_R^{-1}, which means that as resolution doubles the eddy viscosity halves. The power exponents are approximately proportional to T_R. These exponents indicate that the wavenumber extent that the subgrid interactions can span is fixed and equal to k_E. This concept is discuss further on page 1691 of the previous (and current) version of the manuscript, and is restated below.

It is the extent of the energy containing scales (k_E) that defines how far nonlinear interactions can span in wavenumber space (Kraichnan, 1976), which effectively sets the size of the largest eddy that can interact with the subgrid scales. This wavenumber distance is inversely proportional to the power exponents, and is represented by the span of wavenumbers over which the eddy viscosity profiles are non-zero in Fig. 3(c) and Fig. 3(d).

- *The title is a bit misleading.*

Your point is duly noted. Perhaps the following title would be more appropriate:

Theoretical comparison of subgrid turbulence in atmospheric and oceanic quasi-geostrophic models

- *Justification for drag in atmospheric and oceanic regimes.*

As discussed with reference to a comment by reviewer 2, the drag applied in both the atmospheric and oceanic cases is used to control the extent of the energy containing scales of wavenumber, k_E. In the atmospheric case the a constant drag is applied from the first wavenumber to the Rossby wavenumber, k_R, to ensure that the enstrophy cascade starts at the end of the energy containing range, consistent with observations of the real atmosphere. As mentioned in response to the comments of reviewer 2, in the oceanic case a more complex wavenumber dependent drag is used to control the position of k_E. Using this approach we are able to study the influence of k_E on the subgrid coefficients, and in the manuscript we develop scaling laws to represent the variability in both k_R and k_E. The updated version of the manuscript now includes the following sentence after the definition of the functional form of the oceanic drag:

This functional form allows us to control the location of the energy containing wavenumber.

- *Why does the benchmark calculation have a viscosity that is determined from the scaling laws obtained using the benchmark calculation.*

There are theoretically two ways to ensure that the benchmark simulations accurately represent the associated physics. The first is to resolves all scales of turbulence down to the Kolmogorov scale, such that no subgrid model is required. However, this is obviously not possible since the Kolmogorov scale in the atmosphere and ocean is approximately 1mm and 1cm respectively. The second and chosen approach, is as you say, to employ benchmark simulations that use a subgrid model obtained consistent with the scaling laws determined from it and other benchmark simulations. This has required that the entire exercise to be undertaken twice. The first time an estimate of the subgrid model in the initial benchmark simulations was used. The subgrid coefficients, and hence scaling laws were determined from these initial benchmark simulations. The subgrid model used in the benchmark simulations were then replaced with a subgrid model consistent with the scale laws, and the process repeated again. Minimal changes to the scaling laws were observed, however, this exercise ensured that the subgrid model used in the benchmark simulation is self-consistent with the subgrid models determined from it.

- *What is the physical justification for the power law? How can you apply the spectral power law to grid-point models?*

As mentioned in response to the comments of reviewer 2, we would first like to reiterate that the focus of this manuscript is to accentuate the similarities between the properties of subgrid turbulence in the atmosphere and ocean, and not on addressing issues associated with specific numerical implementations. Having said that, as you request, we have addressed the issues associated with the implementation of such scaling laws into grid point models.

Firstly the cusp-like nature of subgrid turbulence was first discussed in the seminal works of Kraichnan (1976), and in many articles on fundamental turbulence since. There are two possible approaches to implementing these scaling laws in grid point models, which are now outlined in the following new paragraph in the conclusions.

The scaling laws developed here can be implemented directly into spectral simulations, and are expected to improve the efficiency and accuracy of numerical weather and climate simulations (Frederiksen et al., 2003, 2015). There are also two possible approaches to implement these scaling laws into grid point codes. The simplest approach is to apply the subgrid model directly in grid-point space via a Laplacian operator of the appropriate power, as outlined in Table 1. More generally it is also possible to employ grid to spectral transforms, where the subgrid model is calculated in spectral space, and then applied in physical space.

- *p1692, line 16-18: If the enstrophy flux is required to determine the eddy viscosity, how would this parameterization be used in a coarse resolution ocean model where the enstrophy flux is not known a priori?*

The enstrophy flux could be estimated from either previous oceanic GCMs or from measurements of ocean spectrum as those in Stammer (1997). In either case, this is arguably a more sophisticated and self-consistent approach than subgrid model parameter tuning that is typically undertaken in both atmospheric and oceanic GCM, to attain numerical stability and produce desirable results (Griffies et al., 2005). The efforts taken to get appropriate measures of the enstrophy flux is a judgement call that the modeller themselves need to make.

- *Figure 4: what is the dashed line in b and d?*

The dashed lines in figures 4b and 4d and the same as the solid lines in figures 4a and 4c respectively, to serve as a direct comparisons. The following sentence has been added to the caption of figure 4.

The dashed lines in figures **b** and **d** represent the drain scaling laws to serve as a direct comparison to the backscatter scaling laws represented by the solid lines.

References

Frederiksen, J. S., Dix, M. R., and Davies, A. G. (2003). The effects of closure-based eddy diffusion on the climate and spectra of a GCM. *Tellus*, 55:31–44.

Frederiksen, J. S., Dix, M. R., Osbrough, S. L., and Kitsios, V. (2015). Stochastic subgrid parameterisations for primitive equation atmospheric circulation models. *ANZIAM J.*, 56:C83–C100.

Frederiksen, J. S. and Kepert, S. M. (2006). Dynamical subgrid-scale parameterizations from direct numerical simulations. *J. Atmos. Sci.*, 63:3006–3019.

Griffies, S., Gnanadesikan, A., Dixon, K., Dunne, J., Gerdes, R., Harrison, M., Rosati, A., Russell, J., Samuels, B., Spelman, M., Winton, M., and Zhang, R. (2005). Formulation of an ocean model for global climate simulations. *Ocean Sci.*, 1:45–79.

Jansen, M. and Held, I. (2014). Parameterizing subgrid-scale eddy effects using energetically consistent backscatter. *Ocean Model.*, 80:36–48.

Kitsios, V., Frederiksen, J. S., and Zidikheri, M. J. (2013). Scaling laws for parameterisations of subgrid eddy-eddy interactions in simulations of oceanic circulations. *Ocean Model.*, 68:88–105.

Kraichnan, R. (1976). Eddy viscosity in two and three dimensions. *J. Atmos. Sci.*, 33:1521–1536.

McComb, W. D. (1990). *The Physics of Fluid Turbulence*. Clarendon Press, Oxford.

O'Kane, T. J. and Frederiksen, J. S. (2004). The QDIA and regularized QDIA closures for inhomogeneous turbulence over topography. *J. Fluid. Mech.*, 504:133–165.

Shutts, G. (2005). A kinetic energy backscatter algorithm for use in ensemble prediction systems. *Q. J. R. Meteor. Soc.*, 131:3079–3102.

Stammer, D. (1997). Global characteristics of ocean variability estimated from regional TOPEX/POSEIDON altimeter measurements. *J. Phys. Oceanogr.*, 27:1743–1769.

Tennant, W., Shutts, G., and Arribas, A. (2011). Using a stochastic kinetic energy backscatter scheme to improve MOGREPS probablistic forecast skill. *Mon. Wea. Rev.*, 139:1190–1206.