ANSWERS TO
Interactive comment on “Study of the overturning length scales at the Spanish planetary boundary layer” by P. López and J. L. Cano

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We thank the referee for his constructive remarks and sugestions. They are very useful in preparing a revised version of our paper. In the following, we present a response to major comments.

The manuscript deals with an interesting subject and presents a set of very high resolution methods, wind velocity and temperatura and local density.
About the wind and temperature results, maybe it could be possible to calculate the structure functions and the intermittency.

We agree that it would be interesting to analyze the relation between the Thorpe scale or the maximum Thorpe displacement and the structure functions exponents and intermittency. Really, this work has already been done. See *Turbulent intermittent structure in non-homogeneous non-local flows* by Mahjoub, O. B.; Castilla, R.; Vindel, J. M.; Redondo, J. M.. They used data from SABLES98 experimental campaign (as we do) in order to study the influence of stability on intermittency. They used SABLES98 data in order to evaluate structure functions and the scale to scale characteristics. They got differences in structure and higher order moments between stable, convective and neutral turbulence which were used to identify characteristics in turbulent intermittent mixing and velocity PDF's. These authors studied the intermittency of atmospheric turbulence in strongly stable situations which modifies the structure functions exponents. The topological aspects of the turbulence affected by stratification reduce the vertical length-scales to a maximum described by the Thorpe and the Ozmidov lenth-scales. Moreover, the paper entitled *Structure function analysis and intermittency in the ABL* by Vindel J.M., Yagüe C. and J.M. Redondo deduced that the relationship existing between the structure functions and stratification shows that as stability increases the structure functions decrease, and the same happens with the maximum Thorpe displacement and the Thorpe scale. Another of their conclusions is that the overall results show that for convective, unstable turbulence intermittency increases while neutral conditions exhibit low intermittency.
But we reaffirm that the main purpose of the article is to present the behaviour of the maximum Thorpe displacement and the Thorpe scale under different conditions in order to choose the best scale the best length scale and not seek relations with the structure functions exponents and the intermittency.

An alternative easier systems would be to present the evolution of Kurtosis in time or its statistical correlation with the Thorpe scale.

Yes, of course, it is a very interesting and easy way to do it. There are three ways of describing the intermittency: the evolution of flatness with the scale, the evolution of PDFs with the scale and the values of the absolute scaling exponents.
In the mentioned paper by Vindel J.M., Yagüe C. and J.M. Redondo, they perform an analysis of the PDFs of the horizontal velocity differences and they study the evolution of flatness. The variation of flatness with scale shows that the most stable and unstable situations have the highest values of flatness (for stably stratified flows, this happens at large scales).

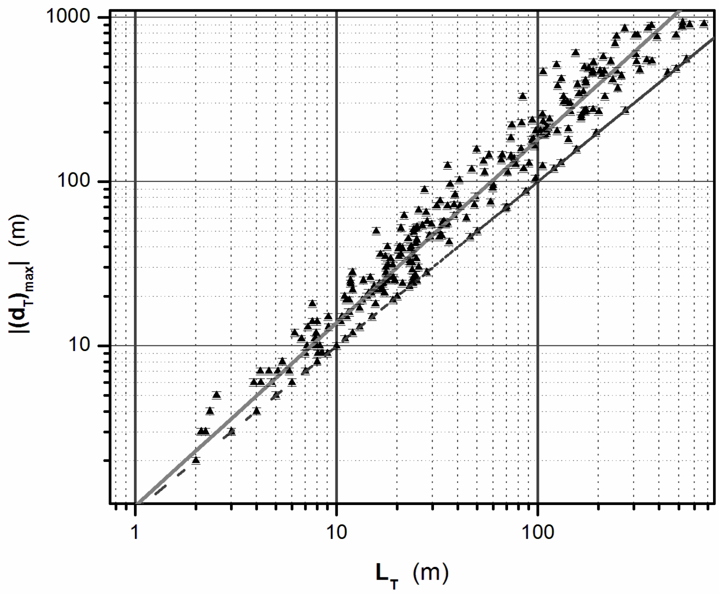
Questions and comments:

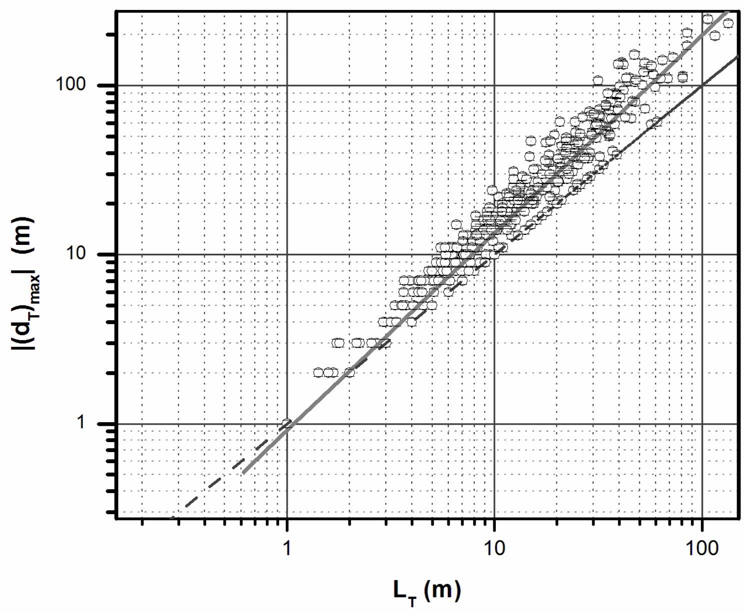
- In the figure 1: Why there is not data between aproximately 12 - 15 hours?

It is true (and also in Figure 2). There is not data between 12:00 and 15:00 hours because the team had to rest. The data were registered mainly by a tethered balloon which needs to be watched and monitored to prevent its breakage (for example, the balloon must be collected if there is a storm).

- Can you indicate the RMS error values in the figures?

Yes, I agree. You can see the error bars in the figures below (corresponding to Figure 3 and 4). Respect to figures 1 and 2, the error of Thorpe displacements is ±1 m (related to the experimental precision).





- It should be interesting to define in the paper Ozmidov scale and present some formal relationships between this scale and Elison scale and Monin-Obukhov lenghtscale.

We agree and we have initiated this study before. See *Thorpe method applied to planetary boundary layer data by* P. López González-Nieto, J. L. Cano, D. Cano and M. Tijera. By setting the buoyancy forces equal to the inertial forces, Ozmidov derived the Ozmidov length scale which would describe the largest possible overturning turbulent scale allowed by buoyancy. This scale is is helpful to estimate mixing, at least that

associated with patches of high turbulent activity. Various measurements have shown that the Thorpe scale is nearly equal to the Ozmidov scale. For example, far from the surface in wind-forced mixing layers in the seasonal thermocline the overall relationship $L_T / L_O\_ = 1.25$ has been reported. Other results present a wider range: $L_T / L_O = [0.9, 1.4]$ for measurements of turbulence during conditions of wear overflow. This is very important because it can be used to calculate the dissipation rate ε from L_T and the stability N^2 . Therefore, the Thorpe scales can be used to estimate rates of dissipation of turbulent kinetic energy and this is an essential result. Moreover, the length scale ratio L_T / L_O can be interpreted as a "clock", which increases monotonically as the turbulent event evolves.
The paper titled *Evolution of the Thorpe and Ozmidov scales at the lower atmospheric and planetary boundary layer* by P. López, J. L. Cano, and J. M. Redondo could be consulted in Academia.edu. This paper also analyzes the time evolution of the Thorpe and the Ozmidov scales during a day cycle. Both scales are always positive during a day cycle but they have not a similar behaviour, almost an opposite behaviour. This paper briefly mentions the Ellison scale that is another dynamical quantity used to estimate the overturning eddy size. The Ellison scale L_E is based on density ρ instead of temperature T. This length scale descriptor is the typical vertical displacement traveled by fluid particles before either returning towards their equilibrium level or mixing. It is often assumed that there is also a linear relationship between L_T and L_E, but this is not often the case $L_T \approx 1.2 L_E$.

As mentioned before, the main aim of the present paper is to choose the best length scale between the Thorpe scale and the maximum Thorpe displacement. This is the reason why we do not include relations with the Ozmidov scale, the Elison scale and Monin-Obukhov lenghtscale.

- As Thorpe scale is define here both in stable and unstaible atmosphere boundary layer conditions, the situation of convective generation of turbulence in the atmosphere, Could you define the local Rayleigh number for the situation of negative Thorpe scale?

The Thorpe scale L_T is the root mean square *(rms)* of the Thorpe displacements $(L_T)_{rms} = L_T = \langle d_T^2(z) \rangle^{1/2}$. Therefore, it is a statistical measure of the vertical size of overturning eddies and is proportional to the mean eddy size. Therefore, we deduce there is any situation of negative Thorpe scale which is always positive by definition.

- It is interesting to see in Figures 1- 2; How the large values take care in the morning and at sunset? Can you compare the evolution of the Thorpe scale in the sunny or a cloudy day because the overturning efects should be related to the solar radiation.

Yes, it is true. That is a very interesting idea to realize the same procedure with sunny and cloudy days to compare. Then, we will need to analyze the meteorological maps of campaign area and we will have a method to quantify the degree of cloudiness.

ANSWERS TO

Interactive comment on "Study of the overturning length scales at the Spanish planetary boundary layer" by P. López and J. L. Cano

Anonymous Referee #1
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We would like to thank the referee for him/her useful comments towards the improvement of our manuscript.

The paper is addressing a very interesting topic that can have a deep consequences in modelling the ABL. The problem is well exposed and carefully documented by chosen references. The obtained results seem to bring a little more complexity to the problem by obtaining power-law, rather than linear relations between the considered length scales. Also the day-time versus nocturnal period separated statistics seems to be a reasonable approach.

Thank you for your opinion.

When it comes to results, the only thing that puzzles me are the breaks in time series of measured data. These breaks were explained, but I was wondering how these gaps in data could have affected the results and conclusions. It means, would we get somehow different results with complete data, or inversely, would the other authors get different results if they will also have such gaps in data?

There is not data between aproximately 12:00-15:00 hours (as figures 1 and 2 show) because the data were registered mainly by a tethered balloon which needs to be watched and monitored to prevent its breakage (for example, the balloon must be collected if there is a storm or wind suddenly appears).
We consider that these gaps in data would not affect our conclusions, that is, they would be the same if we were able to measure 24 hours a day. There are physical explanations which would support our hypothesis. We consider that at the 12:00-15:00 h time interval, the overturns could be generated by one or several convective burst with different scales (due to the effects of solar heating and the meteorological conditions). These convective situation could make several mixing events which could superimpose and could make greater overturns. Based on two-dimensional visualizations of temperature data, Keller and Van Atta conjectured that overturns could be generated by a localized vertical advection of well-mixed lumps of fluid past their equilibrium position and subsequnt displacement of stable density fronts (Keller, K. H. and Van Atta, C.: An experimental investigation into the vertical temperature structure of homogeneous stratified shear turbulence, J. Fluid Mech. 425, 1-29, 2000). Moreover, it has been studied that the close proximity of adjacent overturns allows them to merge and to generate larger-scale overturns (Diamessis, P. J and Nomura, K. K.: The structure and dynamics of overturns in stably stratified homogeneous turbulence, J. Fluid Mech, 499, 197-229, 2004).
As a consequence, it is possible to expect that the corresponding maximum Thorpe displacement $(d_T)_{max}$ and the Thorpe scale, L_T, would be greater. As a consequence, it

would be possible to get features having an 'eddylike' shape similar, some a random mix of different-scale fluctuations without sharp boundaries as in the following figures.

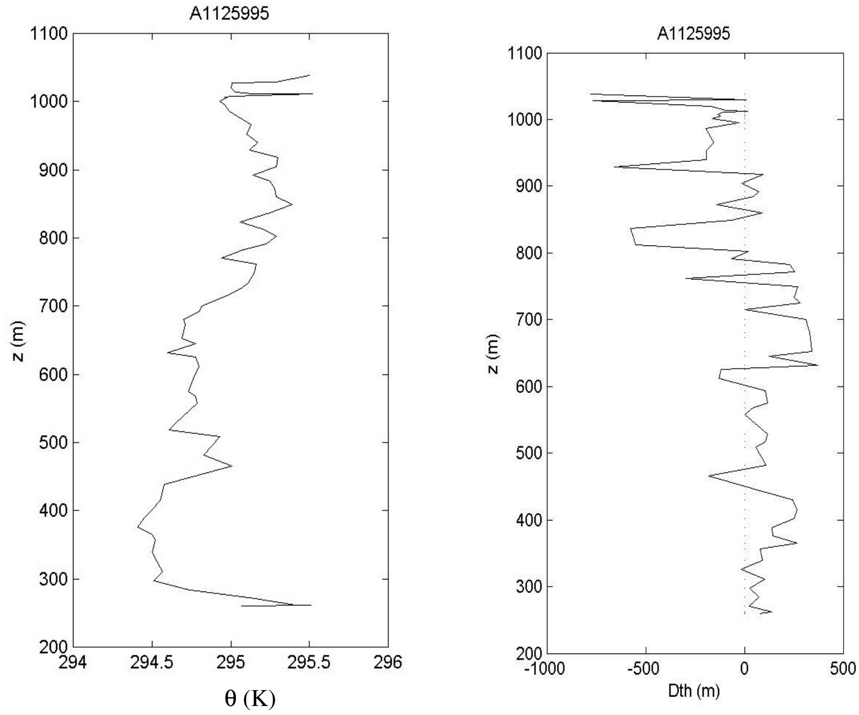


Figure 1. The real potential temperature profile (left curve) and the corresponding Thorpe displacements profile (right curve) corresponding to 11:00 GMT.

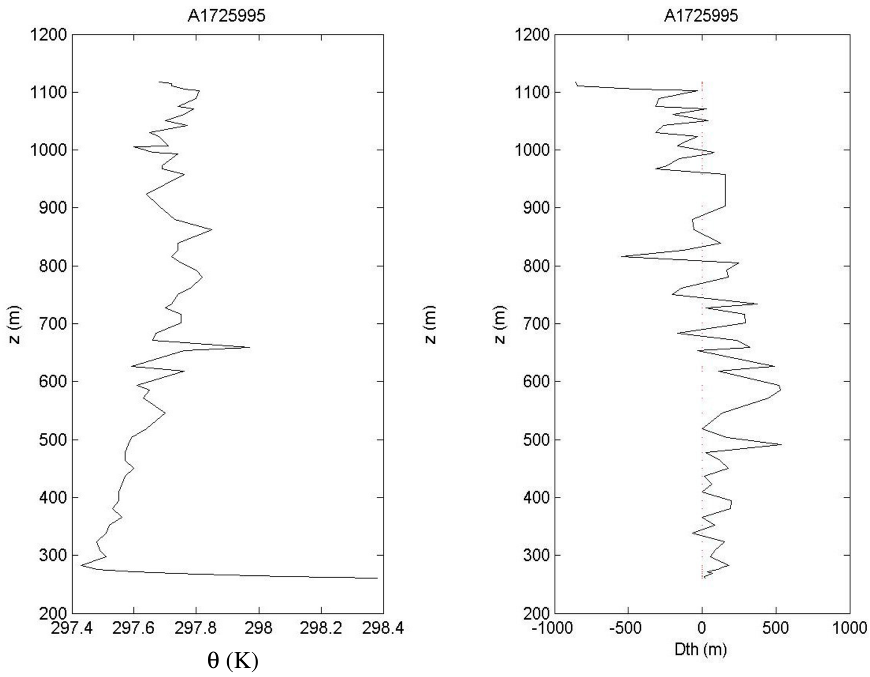


Figure 2. The real potential temperature profile (left curve) and the corresponding Thorpe displacements profile (right curve) corresponding to 17:00 GMT.

The data correspond to the same field campaign made at 25^{TH} September of 1995. We represent the potential temperature profile and the Thorpe displacements profile. Figure 1 shows the behavour at 11:00 GMT, when the convective effects would start. Figure 2 corresponds to 17:00 GMT, when the convective effects would be more developed. We clearly observe the random mix of different-scale fluctuations without sharp boundaries with a vertical extent of the order of 500 m at 11:00 GMT, and greater at 17:00 GMT (of the order of 1000 m). These mentioned fluctuations act as external intermittency that refers instead to the intermittency of the occurrence and variability among different turbulent events (which could generate overturns as sporadic convective processes o baroclinic instabilities).

Finally, we present a new figure (figure 3) which represents the potential temperature profile and the Thorpe displacements profile at 07:00 GMT (without convective effects). We observe a clear *z*-shape overturn that has sharp boundaries with displacement fluctuations of a size comparable to the size of the disturbance itself in the interior, that is, with intense mixing inside (Dillon, T. M., 1982: Vertical Overturns: A Comparison of Thorpe and Ozmidov Length Scales, J. Geophysical Research, 87, C12, 9601-9613). This typical large overturning eddies have sharp upper and lower boundaries with intense mixing inside. This kind of overturn could be probably generated by random breaking of internal waves or Kelvin-Helmholtz instabilities. We also observe that this overturn is not so greater (about 40 m) as the ones of fiures 1 and 2.

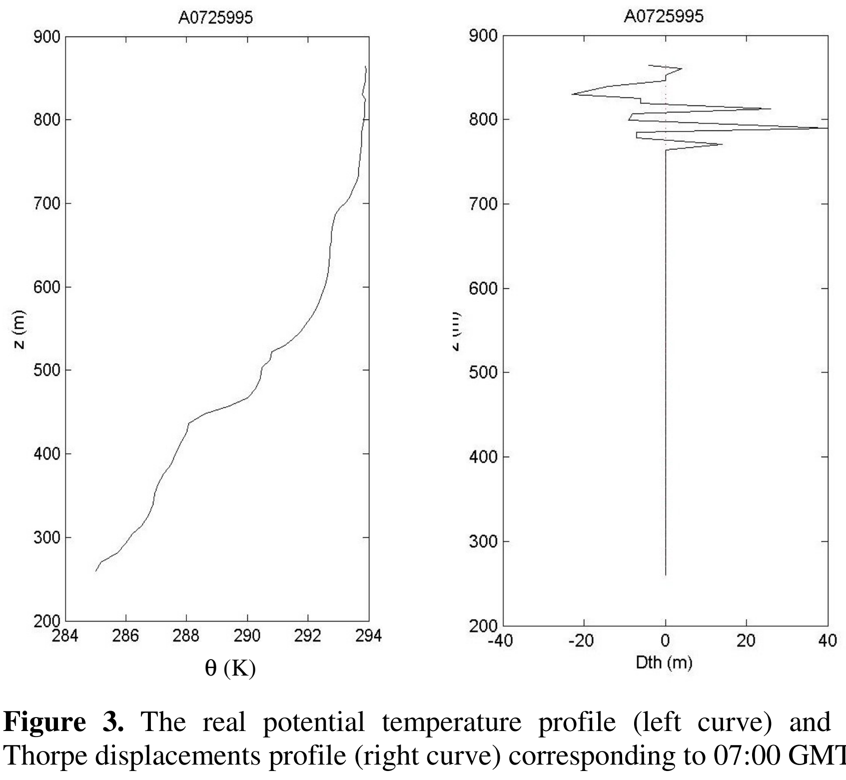


Figure 3. The real potential temperature profile (left curve) and the Thorpe displacements profile (right curve) corresponding to 07:00 GMT.

Finally, there are other reasons. From Figures 1 and 2 (paper figures), it follows that 12:00-15:00 hours missing data should correspond to the greater values of the maximum Thorpe displacements and the Thorpe scales. That is, these missing data should not have small values of $(d_T)_{max}$ and L_T under convective conditions and the typical meteorological situation of this area. Therefore, these missing (no measured) values would be shown in the right part of the graphics (only in figures 3 and 4). Simultaneously, if we were able to measure 24 hours a day, the sample size would be greater and, therefore, the reliability of our results will improve. The reason is statiscal because one of the ways to get an improvement of the hypothesis test power is to increase the sample size. As a consequence, our conclusions would be reinforced.

From technical point of view, I don't like the figures at the end of the paper (which makes it harder to read), but this is probably just the manuscript style, not a choice of authors.

Yes, it is true.

There are few misspelled words in the text, which is easily fixable in the final version of the paper. A little annoying for me was also the use of expressions "P value is ...", "R-squared coefficient is...", "F test for ...", which is probably some common notation use by someone in certain branches of statistics, but for a technical (physical) paper, these terms (and notation) should be explained or rather properly referenced.

We agree (we have used the typical statistical notation) and we are going to describe these terms properly in the revised version of the paper.

The p-value helps us to determine the significance of the results when we perform a hypothesis test which is used to test the validity of a claim that is made about a population. This claim that's on trial, in essence, is called the null hypothesis. The alternative hypothesis is the one we would believe if the null hypothesis is concluded to be untrue. The p-value is defined as the probability of obtaining a result equal to or "more extreme" than what was actually observed, assuming that the null hypothesis is true. We use a p-value (always between 0 and 1) to weigh the strength of the evidence. A small p-value (typically ≤ 0.05) indicates strong evidence against the null hypothesis, so you reject the null hypothesis. A small p-value (typically ≤ 0.05) indicates strong evidence against the null hypothesis.

The R coefficient or linear correlation coefficient is a normalized measurement of how two variables are linearly related. It represents the correlation coefficient of two variables. If the correlation coefficient is close to 1, it would indicate that the variables are positively linearly related. The R-squared coefficient is called the determination coefficient which represents the proportion of the variance (fluctuation) of one variable that is predictable from the other variable. It is a measure that allows us to determine how certain one can be in making predictions from a certain model/graph. The coefficient of determination is a measure of how well the regression line represents the data.

As it was mentioned at the paper, it is necessary to do a multiple regression analysis. The comparison of regression lines procedure is designed to compare the regression lines relating y and x at two or more levels of a categorical factor. Tests are performed to determine whether there are significant differences between the intercepts and the slopes at the different levels of that factor.

Comparing two regression lines is the simplest model of covariance analysis. It uses the independent variable *x* as covariate and dependent variable *y* as outcome in a 2 group analysis of variance (decomposition of the variability of the dependent variable *y* into a model sum of squares and a residual or error sum of squares). Of particular interest is the F-test on the model line which tests the statistical significance of the fitted model. A small p-value (less than 0.05) indicates that a significant relationship of the form specified exists between *y* and *x*. The F-test is any statistical test in which the statistic has an F-distribution under the null hypothesis. It is most often used when comparing statistical models that have been fitted to a data set, in order to identify the model that best fits the population from which the data were sampled.

ANSWERS TO

Interactive comment on “Study of the overturning length scales at the Spanish planetary boundary layer” by P. López and J. L. Cano

Anonymous Referee #2
Received and published: 8 January 2016

The authors study an interesting problem in modelling the atmospheric boundary layer by analysing the relationship between the maximum Thorpe displacement, and the Thorpe scale, which is the statistical mean of displacements. The results are based on the set of detailed observations. The authors argue that the relationship under consideration is not linear, as found previously for the atmospheric boundary layer, but powerlike, and find the corresponding power laws for the complete set of data, and separately for day and night observations.

Thank you for your opinion and the opportunity to revise our paper. I have commented below on each of the points raised by the referee.

First, the authors write, in section 4.1, that they have found two qualitatively different behaviours of Thorpe displacements. It is rather difficult to visualize these cases from the explanation. Perhaps it would be better to illustrate these behaviours with a figure.

The following text mentions that for our ABL studies, Thorpe displacements could be qualitative classified in two groups: the first group represents discrete overturns where the Thorpe displacements are always zero except in a region with an isolated *Z* patterns (usually under neutral and stable stratification conditions); the second group represents a random mix of different scale fluctuations without sharp boundaries, some having an eddylike shape similar to the larger overturns, where the Thorpe displacements rarely are zero for the whole profile (the opposite behaviour that usually happens at convective conditions). The following figures show the two Thorpe displacement groups with different behaviour. The left figure is an example of the first group, that is, an isolated overturn and the right figure is an example of the second group. From these figures it is clear that there is a different behaviour. Both graphs correspond to a campaign made 25th of September of 1995. The left figure is at 07:00 GMT (stable conditions) and the right graph is at 17:00 GMT (convective conditions).
We will probably add this figure to the revised version of the paper although this kind of figures are shown at the references cited at the paper: *López, P., Cano, J. L., Cano, D., and Tijera, M.: Thorpe method applied to planetary boundary layer data, Il Nuovo Cimento, 31C, 881–892, 2008* and *López, P., Redondo, J. M., and Cano, J. L.: Thorpe scale at the planetary boundary layer:comparison of Almaraz95 and Sables98 experiments, Complex Environmental Turbulence and Bio-Fluids Flows, Institute of Thermomechanics AS CR, Prague, 2015 (in press).*

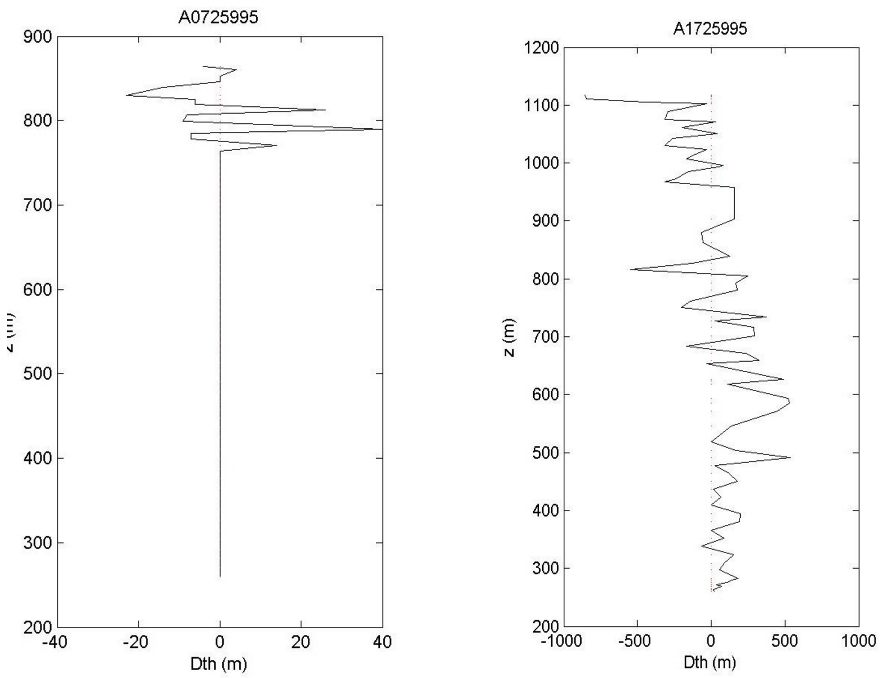


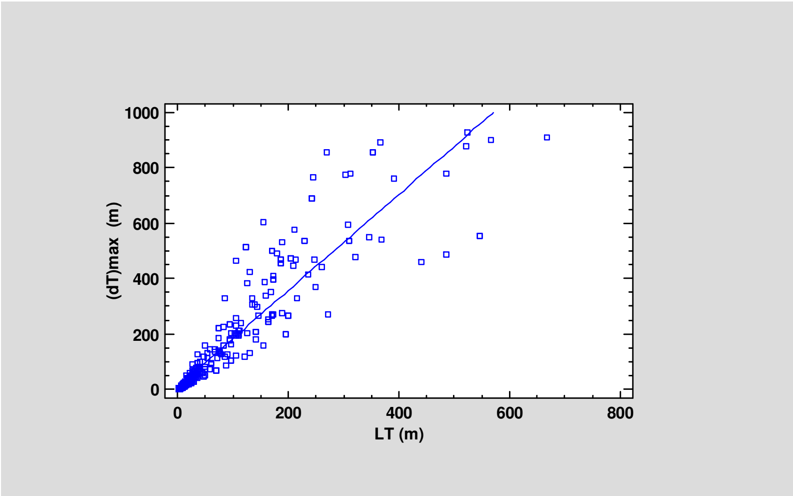
Figure 1. Left curve**,** Thorpe displacements profile with an isolated patch corresponding to 07:00 GMT. Right curve, Thorpe displacements profile with a random mix of fluctuations corresponding to 17:00 GMT.

Second, it does appear from figures 3-5 that a power law fits the data better than a linear one. However, it would be better to explain it clearly in statistical terms. [….] I can trust that they have chosen the best exponent for the powerlike fit, but how much better it is, in statistical terms, than a linear fit with a certain coefficient, similar to what has been found by other authors?

To explain in statistical terms that the the power law fits the data better than a linear one, we have realized a new statistical study. We have made a simple regression procedure to construct a statistical model describing the dependence of $|(d_T)_{max}|$ on L_T considering the different situations, i. e., the daytime data (figure 4) and the nighttime data sets (figure 5). The new study for the whole data (figure 3) is described in the following comment.

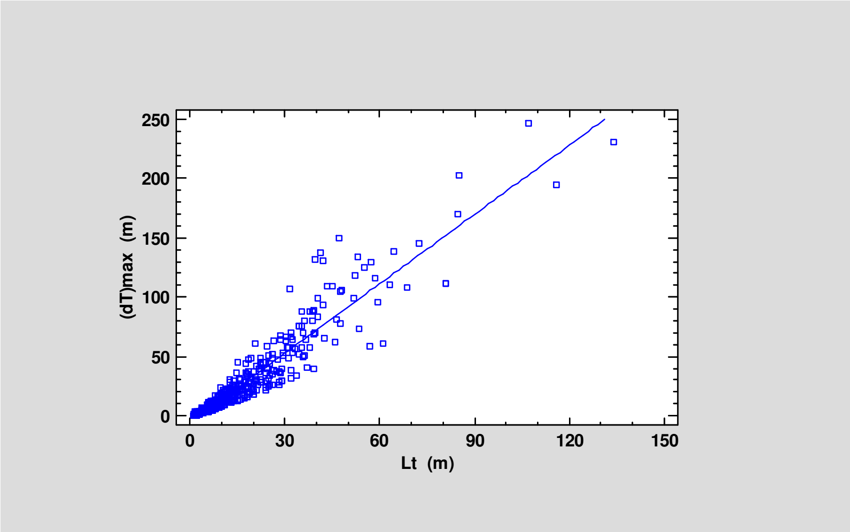
First, we analyze the behaviour of the daytime data set (figure 4). The linear model was fit using least squares and tests were run to determine the statistical significance of this model. Our results show that the estimated linear model is $|(d_T)_{max}|=10.794+1.732\ L_T$. The analysis of variance, which tests the statistical significance of the fitted model, indicates that a significant relationship of the form specified exists between $|(d_T)_{max}|$ and L_T (because the p-value is less than 0.05). In the daytime sample data, the linear model is significant but the *R-squared* –or determination coefficient- which represents the percentage of the variability in $|(d_T)_{max}|$ which has been explained by the fitted regression model is 84.3%. The regression has accounted for about 84% of the variability in the maximum Thorpe displacements measurements. The remaining 16% is attributable to deviations around the line, which may be due to other factors, for example, to a failure of the linear model to fit the data adequately.

The same statistical analysis was made using a power law fit and we got that the *R*-squared coefficient is 96.76%, that is the power law fit accounts for about 98% of the variability in the maximum Thorpe displacement $|(d_T)_{max}|$ as a function of the Thorpe scale, L_T. We conclude that both models, the power law fit and the linear one, are statistically significant but the power law fit has a better determination coefficient, that, is, it accounts better for the variability in the maximum Thorpe displacements measurements. Therefore, we consider that the power law fit is the best model for the daytime data set. We also present the graph of the daytime data and the linear fitted model. The plot includes the line of best fit. This plot clearly shows that the daytime data does not follow a linear model.



Second, we analyze the behaviour of the nighttime data set (figure 5). The statistical analysis was repeated: the linear model was fit using least squares and tests were run to determine the statistical significance of this model. Our results show that the estimated linear model is $|(d_T)_{max}|=-5.571+1.947\ L_T$. Again, the p-value of the analysis of variance is less than 0.05 and indicates that a significant relationship of the form specified exists between $|(d_T)_{max}|$ and L_T for the nighttime data set. The *R-squared* coefficient is 90.11%. The regression has accounted for about 90% of the variability in the maximum Thorpe displacements measurements. The remaining 10% is attributable to other factors (may be the linear model does not fit the data adequately). The same statistical analysis was made using a power law fit and we got that the *R*-squared coefficient is 95.89%, that is the power law fit accounts for about 96% of the variability in the maximum Thorpe displacement $|(d_T)_{max}|$ as a function of the Thorpe scale, L_T.

In the same way as in the previous case, the power law fit accounts better for the variability in the maximum Thorpe displacements data and we consider it is also the best model for the nighttime data set. We also present the graph of the nighttime data and the line of best fit. This plot clearly shows that the nighttime data does not follow a linear model.



For example, most (although not all) data in figure 3 appear to fit rather well to a linear dependence.

The figure 3 represents the data in logarithmic scale, and it is clear that they fit well to a linear relation. The following figure shows the same data of figure 3 (of the paper), but on a linear scale. We observe that the data do not fit so well to a linear relation (mainly due to the behaviour of the greatest values of the maximum Thorpe displacement and the Thorpe scale).

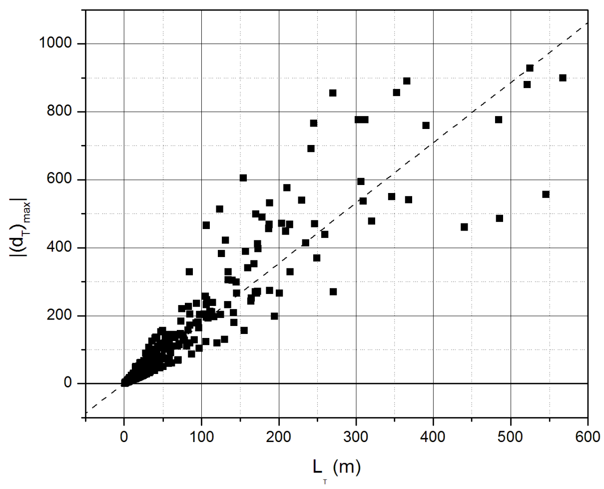
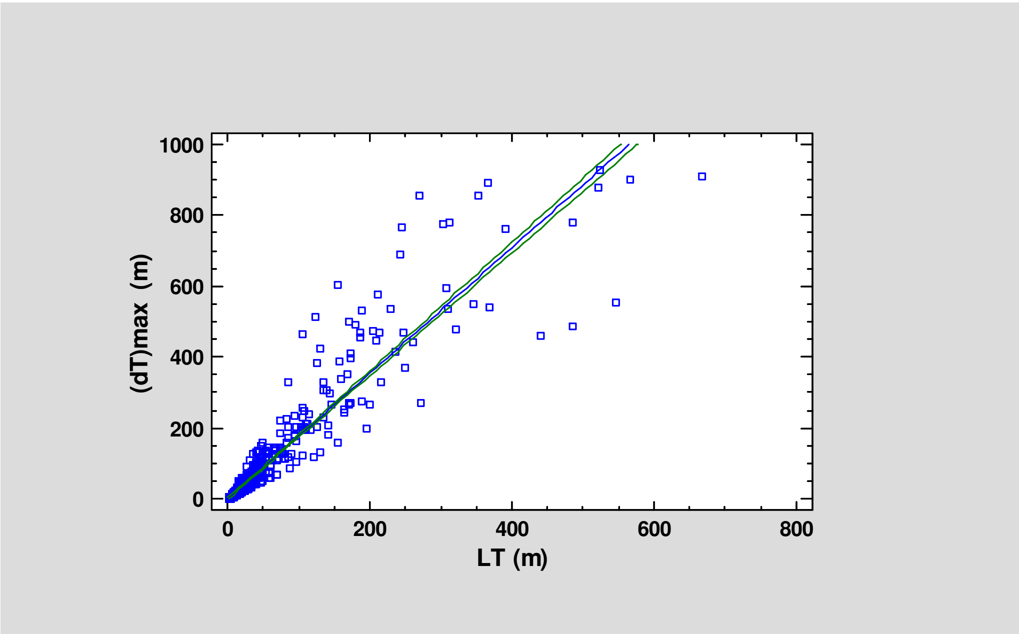


Figure 2. Absolut value of the maximum Thorpe displacement vs. Thorpe scale for all field campaigns. The representation is in linear scale.

As mentioned in the previous comment, we have realized a new statistical study to know if the linear fit is better or not than the power law fit. We made a simple regression procedure to construct a statistical model describing the dependence of $|(d_T)_{max}|$ on L_T. The linear model was fit using least squares and tests were run to determine the statistical significance of the model. Our results show that the estimated linear model is $|(d_T)_{max}|=0.218+1.771\ L_T$. The analysis of variance indicates that a significant relationship of the form specified exists between $|(d_T)_{max}|$ and L_T (because the p-value is less than 0.05). The percentage of the variability in $|(d_T)_{max}|$ which has been explained by the fitted regression model is 87.9% which is the value of the determination coefficient. For the campaign data, the regression has accounted for about 88% of the variability in the maximum Thorpe displacements measurements. The remaining 12% is attributable to deviations around the line, which may be due to other factors, for example, to a failure of the linear model to fit the data adequately.

The same statistical analysis was made using a power law fit and we got that the *R*-squared coefficient is 96.95%, that is the power law fit accounts for about 97% of the variability in the maximum Thorpe displacement $|(d_T)_{max}|$ as a function of the Thorpe scale, L_T.

We also present the graph of the data (as before) and the fitted model plotted with confidence limits. The plot includes the line of best fit and the confidence intervals for the mean response which describe how well the location of the line has been estimated given the available data sample.



As a conclusion, we conclude that both models, the linear fit and the power law one, are statistically significant but the power law fit has a better determination coefficient, that, is, it accounts better for the variability in the maximum Thorpe displacements measurements. Therefore, we consider that the linear regression is not the best model.

The authors need to make sure that all statistical concepts they use (P value, F test, etc) are properly defined.

We agree (we have used the typical statistical notation) and we are going to describe these terms properly in the revised version of the paper.

The *p*-value helps us to determine the significance of the results when we perform a hypothesis test which is used to test the validity of a claim that is made about a population. This claim that's on trial, in essence, is called the null hypothesis. The alternative hypothesis is the one we would believe if the null hypothesis is concluded to be untrue. The *p*-value is defined as the probability of obtaining a result equal to or "more extreme" than what was actually observed, assuming that the null hypothesis is true. We use a *p*-value (always between 0 and 1) to weigh the strength of the evidence. A small *p*-value (typically ≤ 0.05) indicates strong evidence against the null hypothesis, so you reject the null hypothesis. A small *p*-value (typically ≤ 0.05) indicates strong evidence against the null hypothesis.

The *R* coefficient or linear correlation coefficient is a normalized measurement of how two variables are linearly related. It represents the correlation coefficient of two variables. If the correlation coefficient is close to 1, it would indicate that the variables are positively linearly related. The R-squared coefficient is called the determination coefficient which represents the proportion of the variance (fluctuation) of one variable that is predictable from the other variable. It is a measure that allows us to determine how certain one can be in making predictions from a certain model/graph. The coefficient of determination is a measure of how well the regression line represents the data.

As it was mentioned at the paper, it is necessary to do a multiple regression analysis. The comparison of regression lines procedure is designed to compare the regression lines relating *y* and *x* at two or more levels of a categorical factor. Tests are performed to determine whether there are significant differences between the intercepts and the slopes at the different levels of that factor.

Comparing two regression lines is the simplest model of covariance analysis. It uses the independent variable *x* as covariate and dependent variable *y* as outcome in a 2 group analysis of variance (decomposition of the variability of the dependent variable *y* into a model sum of squares and a residual or error sum of squares). Of particular interest is the F-test on the model line which tests the statistical significance of the fitted model. A small p-value (less than 0.05) indicates that a significant relationship of the form specified exists between *y* and *x*. The F-test is any statistical test in which the statistic has an F-distribution under the null hypothesis. It is most often used when comparing statistical models that have been fitted to a data set, in order to identify the model that best fits the population from which the data were sampled.

Also, the use of the term "linear" needs to be more consistent in the paper. [….] I guess the authors use the term "linearity" interchangeably in algebraic and statistical sense, which is rather confusing.

We agree and we have revised the text to clarify the statistical sense of the "linear" term when itis mentioned.

The authors write "We observe that the linear relationship $|(d_T)_{max}| = L_T$ ***proposed by other authors... "; but the other authors have proposed, in particular, a linear relationship with a ratio*** $|(d_T)_{max}=L_T$ ***which is different from 1.***

Yes, it is true. It is an unfortunate phrase that has no relation to the context of the paragraph and creates confusion. It is also a bad explanation because it seems that the relation $|(d_T)_{max}|=L_T$ is that others authors have deduced and this is not true. We used $|(d_T)_{max}=L_T$ as the perfect relationship, a pattern or reference but it is not true nor necessary. Therefore, we have decided to remove this phrase and all the comments related to the relationship $|(d_T)_{max}=L_T$ in the text, in the figures and figure captions.

It is not clear why the light grey line in figure 5 represents the linear fit, as stated in the caption, while it is clearly a powerlike function, in logarithmic coordinates.

We have revised the figure and we have redone the calculations. Furthermore, the new figure 5 is more understandable because we use other shades of gray and we have eliminated the relationship $|(d_T)_{max}=L_T$, which is not essential.

Study of the overturning length scales at the Spanish planetary boundary layer

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Abstract

The focus of this paper is to analyze the behaviour of the maximum Thorpe displacement $(d_T)_{max}$ and the Thorpe scale L_T at the atmospheric boundary layer (ABL), extending previous research with new data and improving our studies related to the novel use of the Thorpe method applied to ABL. The maximum Thorpe displacements varyes between -900 m and 950 m for the different field campaigns. The Thorpe scale L_T ranges between 0.2 m and 680 m for the different data sets which cover different stratified mixing conditions (turbulence sher-driven and convective regions). We analyze the relation between $(d_T)_{max}$ and the Thorpe scale L_T and we deduce that they verify a power law. We also deduce that there is a difference in exponents of the power laws for convective conditions and shear-driven conditions. This different power laws could identify overturns created under different mechanisms.

1 Introduction

Atmospheric boundary layer (or ABL) is almost always turbulent. In the absence of turbulence, atmospheric temperature profiles become increasingly monotonic, due to the smoothing effect of molecular diffusion. Turbulence gives rise to an effective eddy diffusivity and as well as other causes (as fluid instabilities or internal wave breaking) makes vertical overturns appear as inversions in measured temperature profiles. These overturns produce small-scale turbulent mixing which is of great relevance for many processes ranging from medium to a local scale. Unfortunately, measuring at small scales is very difficult. To overcome this disadvantage it is interesting to use theories and parameterizations which are based on larger scales. For example, the theories of turbulent stirring which often depend on hypotheses about the length scales of turbulent eddies. Vertical overturns, produced by turbulence in density stratified fluids as lakes or the ABL, can often be quantified by the Thorpe displacements d_T and the Thorpe scale L_T (Thorpe, 1977).
Next we present the atmospheric data used for the analysis. In section 3 we present the Thorpe method and the definitions of the scale descriptors used. In section 4, the results of Thorpe displacements, the maximum Thorpe displacement and the Thorpe scale L_T at ABL are presented and discussed.

2 Atmospheric data sets and meteorological instrumentation

The results presented in this paper are based on three ABL field campaigns made at Spain and called Almaraz94-95, Sables98 and Sables2006. ABL data from 98 zeppelin-shaped tethered balloon soundings ranging from 150 m to 1000 m were carried out in Almaraz94-95 field campaign made in Almaraz (Cáceres, Spain). The ABL profiles were obtained from 25 to 29 September 1995 in the time intervals 06:00-12:00 and 15:00-00:00 GMT. And from 5 to 10 June 1994 in the time intervals 05:00-12:00 and 17:00-00:00 GMT. Almaraz94-95 experiment collects data over a whole day and, therefore, covers different stratified conditions and mixing conditions – from shear-driven turbulence to convective regions. For more details see López et al. (2008). Sables98 (Stable Atmospheric Boundary Layer Experiment in Spain) took place over the northern Spanish plateau in the period 10–28 September 1998. The campaign site was the CIBA (Research Centre for the Lower Atmosphere). Two meteorological masts (10 m and 100 m) were available at CIBA with high precision meteorological instruments (Cuxart et al., 2000). Additionally, a triangular array of cup anemometers was installed for the purpose of detecting wave events and a tethered balloon was operated at nighttime. A detailed description can be consulted in (Cuxart et al., 2000). Sables98 field campaign only collects data over the night and, therefore, under neutral to stable conditions. Sables2006 field campaign took place from 19 June to 5 July 2006 at the CIBA. As in Sables98, different instrumentation was available on a tower of 100 m, a surface triangular array of microbarometers was also deployed and a tethered balloon was used to get vertical profiles up to 1000 m. As in Sables98, Sables2006 field campaign also collects data over the night. Therefore, Sables98 and Sables2006 experiments let us to analyze the behaviour of overturns under stable conditions while Almaraz94-95 under unstable conditions (and also stable ones). These three sets of data were selected for this analysis because they cover different mixing conditions (turbulence shear-driven and convective regions).

3 Thorpe method and overturn length scales

Thorpe devised an objective technique for evaluating a vertical length scale associated with overturns in a stratified flow (Thorpe, 1977; Itsweire, 1984; Gavrilov et al., 2005). Thorpe's technique consists of rearranging a density profile (which contains gravitationally unstable inversions) so that each fluid particle is statically stable. If the sample at depth z_n must be moved to depth z_m to generate the stable profile, the Thorpe displacement d_T is z_m- z_n (Thorpe, 1977; López et al., 2008; López et al., 2015). The Thorpe displacement d_T is not necessarily the real space actually travelled by the fluid sample, is an estimate of the vertical distance from the given vertical profile to the statically stable one that each fluid particle has to move up- or downward to its position in the stable monotonic profile (Thorpe, 1977, Dillon, 1982). Over most of a typical profile, the local stratification will be stable and the Thorpe displacement zero. A turbulent event is, therefore, defined as a region of continuously nonzero d_T, i.e, overturns are defined as a profile section for which $\sum_i d_{T_i} = 0$ while $d_{Ti} \neq 0$ for most i (Dillon, 1982; Peters et al., 1995).

The maximum of the Thorpe displacements scale $(d_T)_{\max} = \max\left[d_T(z)\right]$ represents the larger overturns which might have ocurred at earlier time when buoyancy effects were negligible ((Thorpe, 1977; Dillon, 1982; Itsweire, 1984) and it could be considered as an appropiate measure of the overturning scale.

The Thorpe scale L_T is the root mean square *(rms)* of the Thorpe displacements $(L_T)_{rms} = L_T = \langle d_T^2(z) \rangle^{1/2}$. Therefore, it is a statistical measure of the vertical size of overturning eddies (Thorpe, 1977; Dillon, 1982; Itsweire, 1984; Fer et al., 2004) and is proportional to the mean eddy size as long as the mean horizontal potential temperature gradient is much smaller than the vertical gradient. For our field ABL measurements, we can consider that the ABL is horizontally homogenous because the average horizontal temperature gradient $(4 \cdot 10^{-4}\,(K/m))$ is smaller than the average vertical temperature gradient $(2 \cdot 10^{-2}\,(K/m))$ (López et al., 2015).

Because of the expensive nature of collecting data at microscale resolution, there is a great interest to use parameterizations for small-scale dynamics which are based on larger scales –as L_T or $(d_T)_{max}$-. Therefore, it is very important to analyze the relation between L_T and $(d_T)_{max}$ for selecting the most aproppiate overturning scale.

4 Quantitative results

Our methodology is based on reordering 111 measured potential temperature profiles, which may contain inversions, to the corresponding stable monotonic profiles. Then, the vertical profiles of the displacement length scales $d_T(z)$ or Thorpe displacements profiles can be calculated by using a bubble sort algorithm with ordering beginning at the shallowest depth (Thorpe, 1977; Dillon, 1982; Itsweire, 1984; López et al., 2008; López et al., 2015). This simple sorting algorithm works by repeatedly stepping through the data list to be sorted, comparing each pair of adjacent items and swapping them if they are in the wrong order (López et al., 2015).

4.1 Thorpe displacement profiles at ABL

Usually, the signature that might be expected for a large overturning eddy is: sharp upper and lower boundaries with intense mixing inside - displacement fluctuations of a size comparable to the size of the disturbance itself are found in the interior -. While common in surface layers strongly forced by the wind, these large features are not always found as in our ABL case (López et al., 2008; López et al., 2015). For our ABL studies, Thorpe displacements observed at profiles could be qualitative classified in two groups as figure 1 shows. The two graphs of figure 1 correspond to a campaign made 25 September 1995. The left graph of figure 1 is at 07:00 GMT (stable conditions) and the right graph is at 17:00 GMT (convective conditions). The two kind of behaviours are as follows. First, the Thorpe displacements under neutral and stable stratification conditions are usually zero except in a region with isolated *Z* patterns which would correspond to discrete patches (figure 1, left curve). These isolated overturns are very few well-defined sharp overturns which appear at sunset, night and sunrise profiles. Secondly, we find other features that are smaller, some having an eddylike shape similar to the larger disturbances, some a random mix of small scale fluctuations without sharp boundaries (figure 1, right curve). These are the second group or non-zero Thorpe displacement regions with indistinct and distributed features which appear under convective and/or neutral conditions (at noon, afternoon and evening profiles). These Thorpe displacements are rarely zero for the whole profile. To verify this behaviour see López et al. (2008) and López et al. (2015).

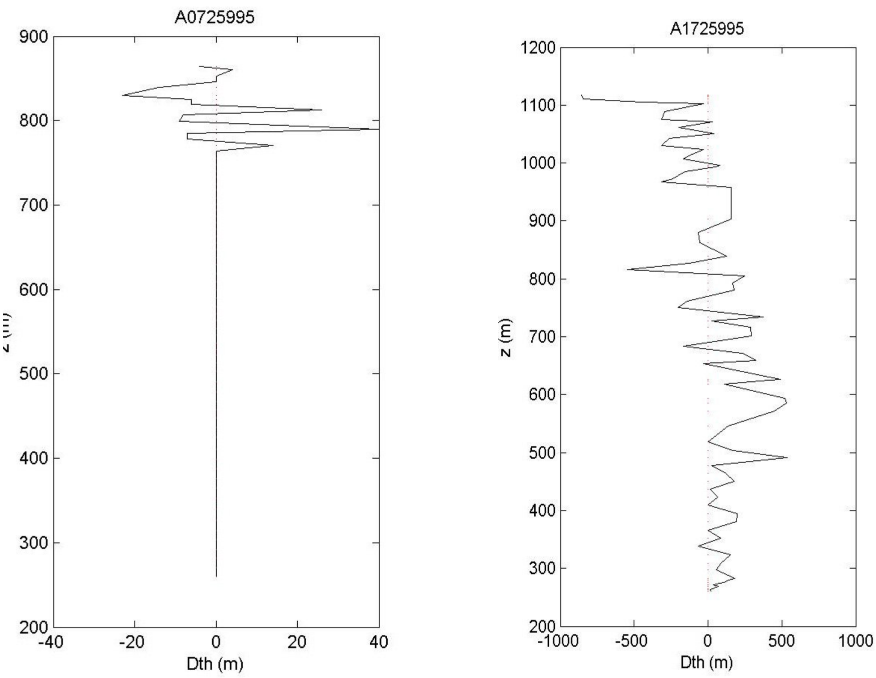

Figure 1. Left curve, Thorpe displacements profile with an isolated patch corresponding to 07:00 GMT. Right curve, Thorpe displacements profile with a random mix of fluctuations corresponding to 17:00 GMT.

4.2 Time evolution of maximum Thorpe displacements and Thorpe scale

Figure 2 shows the time evolution of the maximum Thorpe displacement $(d_T)_{max}$ along a day for the three field campaigns. The scale $(d_T)_{max}$ is very small (approximately zero) under stable conditions from 23:00 to 06:00 GMT (between sunset and sunrise) for all the experiments. From 19:00 GMT, it is observed that scale $(d_T)_{max}$ decreases. The greatest values of $(d_T)_{max}$ appears under convective conditions from 09:00 to 19:00 GMT being positive and negative. But the positive values of $(d_T)_{max}$ are greater than the negative ones. The positive $(d_T)_{max}$ has its greatest values about 950 m and the greatest negative $(d_T)_{max}$ are about 600 m (absolut value). These results mean as follows. Thorpe displacements were defined as the difference between the final height and the initial height of the fluid particle., i.e., $d_T=(z_m)_{final}-(z_n)_{initial}$. If $d_T>0$ $((z_m)_{final}>(z_n)_{initial})$, the fluid particle has to go up to reach its stable position, and if $d_T<0$ $((z_m)_{final}<(z_n)_{initial})$, it has to go down to reach its stable point. From figure 2 we can deduce that fluid particles go up and downwards with a great the vertical distance travelled under convective stratification conditions. Under stable stratification conditions –at night-, the fluid particles also move up and downwards but with small values for the vertical distance travelled. Hence, it is clear that the maximum Thorpe displacement is always greater under convective conditions than under stable ones, independently of its sign. Therefore, the maximum Thorpe displacements is a parameter which could represent the dynamical behaviour of air particles and its relation with the stratification conditions. Finally, there is a gap in figure 2 due to non registered data between 13:00 and 14:00 GMT.

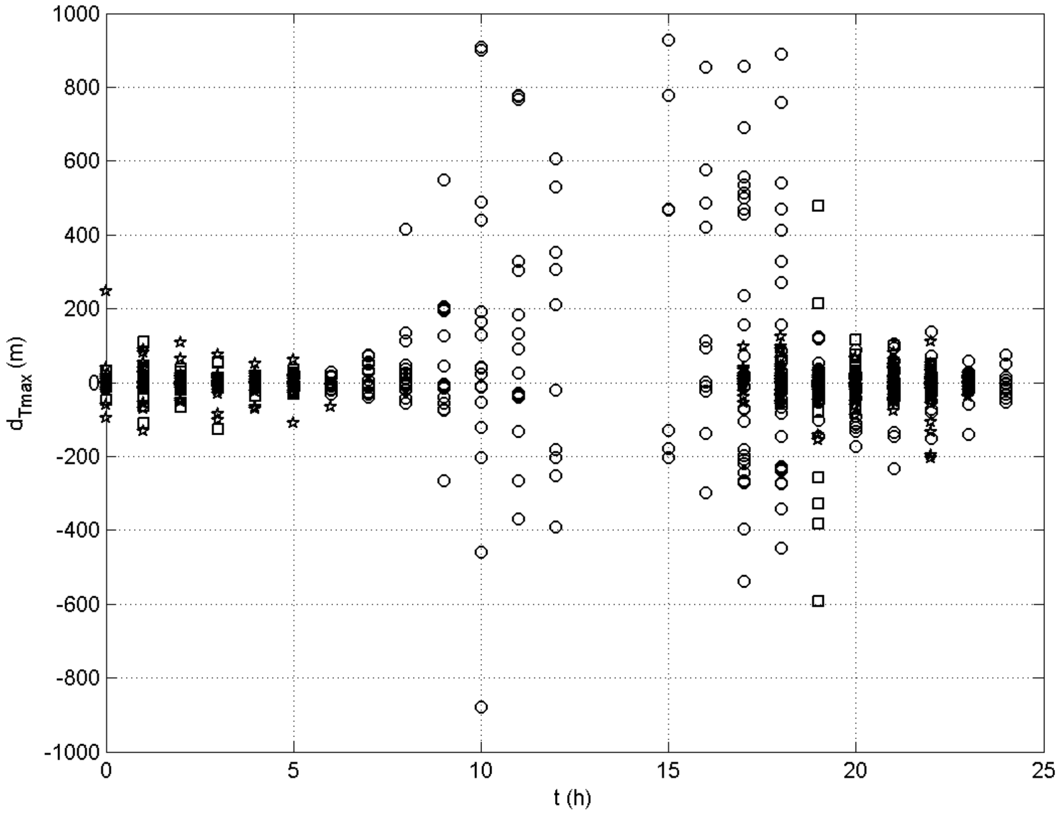

Figure 2. Time evolution of the maximum Thorpe displacements during a day cycle. The symbols are as follows: o is for Almaraz94-95 data, ☆ is for Sables98 data and □ is for Sables2006 data. The error of Thorpe displacements is ±1 m.

Figure 3 shows the time evolution of the Thorpe scale, L_T during a day for the three field campaigns. The Thorpe scale L_T has small values (close to zero) under neutral and stable conditions from 20:00 to 09:00 GMT (between sunset and sunrise). This scale reaches its greatest values under convective conditions from 09:00 to 19:00 GMT. There are two distinct behaviours with high (L_T>100 m) and low (L_T<100 m) Thorpe scales. In most of the turbulent patches, the Thorpe scale does not exceed several tens of meters and they appear under stable and neutral stratification conditions when the Thorpe displacements are also small and related to instantaneous density gradients. In contrast, under convective conditions, Thorpe scales are relatively large. They exceed hundreds of meters and they may be related to convective bursts. Hence, the Thorpe scale L_T is always greater under convective conditions than under stable ones and it is a parameter which could also represent the dynamical behaviour of air particles. As in figure 2, there is a gap in figure 3 due to the not registered data between 13:00 and 14:00 GMT. Both scales, the Thorpe scale L_T and the maximum Thorpe displacement $(d_T)_{max}$, have small values (close to zero) under neutral and stable conditions, and their greatest values appear under convective conditions. Therefore, it is reasonable to think which of the two scales could represent better the dynamical behaviour of turbulent overturns.

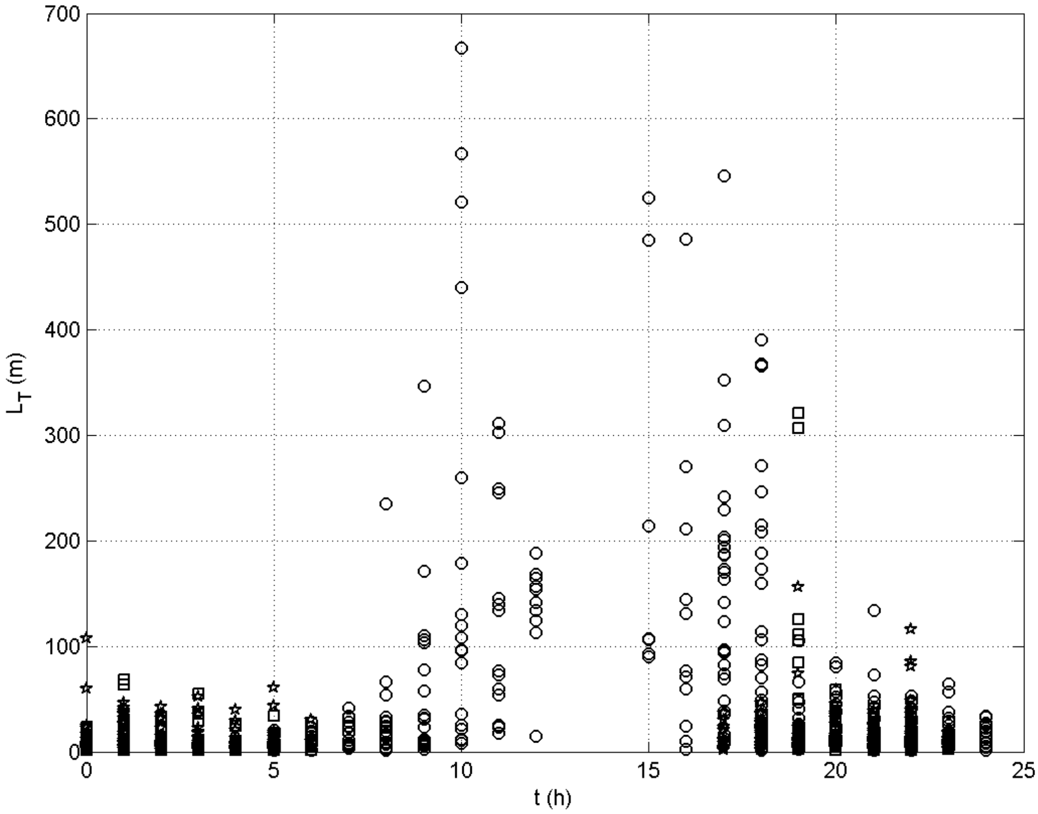

Figure 3. Time evolution of the Thorpe scale during a day cycle. The symbols are as follows: o is for Almaraz94-95 data, ☆ is for Sables98 data and □ is for Sables2006 data. The error of Thorpe displacements is ±1 m.

Moreover, it is neccessary to choose an appropiate overturning scale to characterize instabilities leading to turbulent mixing, the turbulent overturning motions themselves and to look for a relation with the Ozmidov scale at ABL data (Dillon, 1982; Lorke and Wüest, 2002; Fer et al., 2004). We could choose the Thorpe scale rather than the maximum Thorpe displacement because we only sample vertically while the turbulence is three dimensional and, therefore, the Thorpe scale is more likely to be a statistically stable representation of the entire feature (Dillon, 1982). But the maximum of the Thorpe displacements is also considered as an appropiate measure of the overturning scale and it is always greater than L_T (better detectable by a limited resolution instrument). Different researchers have found a linear model between L_T and $(d_T)_{max}$ for profiles from the equatorial undercurrent (Moum, 1996; Peters et al., 1995) and a high linear correlation computed from the Banyoles99 field data where the ratio $(d_T)_{max}/L_T$ is approximately equal to 3 (Piera Fernández, 2004). It must exist a correlation between L_T and $(d_T)_{max}$ because when computing the rms of a set of Thorpe displacements with high kurtosis distributions, the final result depends on the largest values (Piera Fernández, 2004; Stansfield et al., 2001). A similar linear correlation between L_T and $(d_T)_{max}$ has been found by other researchers: a ratio $(d_T)_{max}/ L_T \approx 3.3$ is obtained in the oceanic thermocline (Moum, 1996), a ratio $(d_T)_{max}/ L_T \approx 2.4$ is obtained from laboratory experiments (Itsweire et al., 1993) and, finally, the ratio $(d_T)_{max}/ L_T$ is nearly 3 in numerical simulations (Smyth and Moum, 2000). But for microstructure profiles from strongly stratified lakes, a power law –as $(d_T)_{max} \sim (L_T)^{0.85}$- is found (Lorke and Wüest,

2002). This relation also holds for profiles from other lakes under very different conditions of mixing and stratification with a strong correlation that holds over four orders of magnitude (Lorke and Wüest, 2002).
Hence, we analyze the relation between L_T and $(d_T)_{max}$ scales for our ABL data. Figure 4 shows the maximum Thorpe displacement versus the Thorpe scale at log scale, using the data of the three field campaigns. We observe that the linear model proposes by other authors (Moum, 1996; Peters et al., 1995; Piera Fernández, 2004; Itsweire et al., 1993; Smyth and Moum, 2000) does not verify for our ABL data.

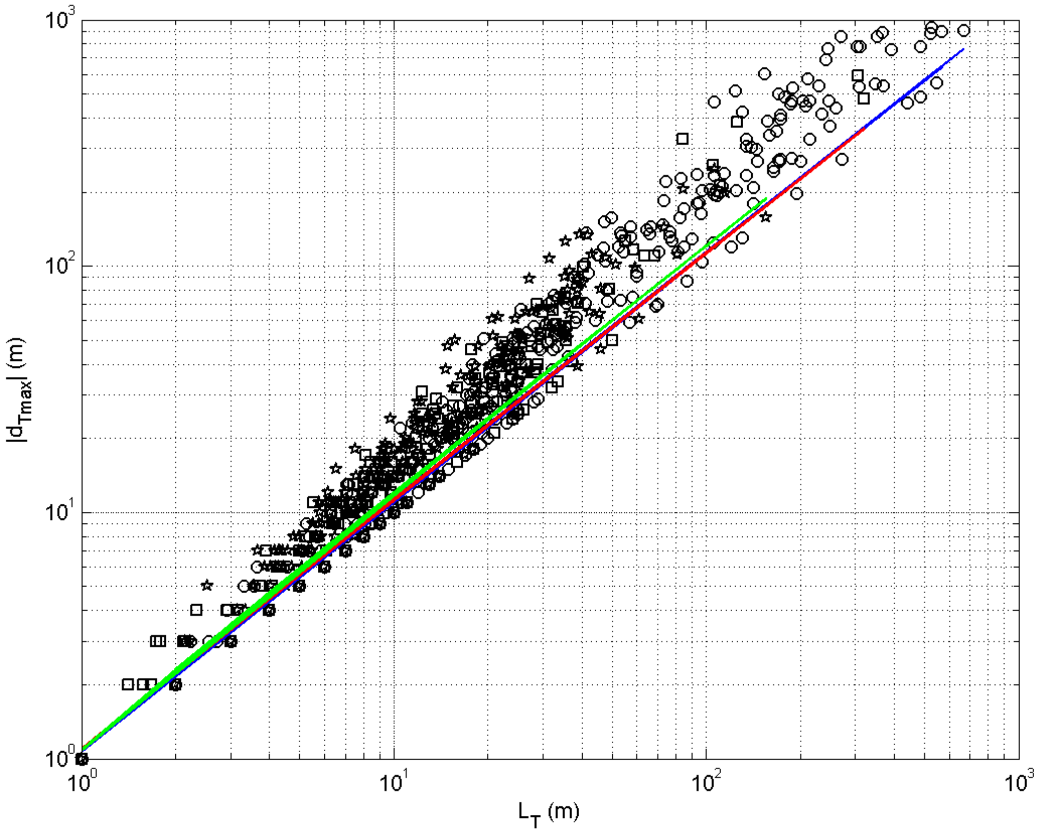

Figure 4. Absolut value of the maximum Thorpe displacement versus Thorpe scale. The symbols are as follows: o is for Almaraz94-95 data, ☆ is for Sables98 data and □ is for Sables2006 data. The linear fit for Almaraz94-95 data is indicated by the blue line, the linear fit for Sables2006 data is indicated by the red line and, finally, the linear fit for Sables98 data is indicated by the green line.

Therefore, we could think that the nearly constant ratio $(d_T)_{max}/ L_T$ obtained in a wide range of field and laboratory experiments, does not verify in our ABL data (figure 4). And, hence, the shape of Thorpe displacements distribution could change at ABL. We also observe a strong correlation which holds over three orders of magnitude as in other researches from profiles in lakes (Lorke and Wüest, 2002). It is the first time that such a relation between this two overturning length scales is found for ABL data (figure 4).
As other authors, we could state that this high correlation indicates that the Thorpe scale is determined by the overturns near to the maximum Thorpe displacement. We find the following power law:

$$\left|(d_T)_{\max}\right| \sim (L_T)^{1.14}, \qquad (1)$$

which is similar to the one deduced by Lorke (Lorke and Wüest, 2002) from profiles in strongly stratified lakes. We realize a simple linear regression analysis. Of particular interest is the P-value[1] associated to the analysis of variance[2], which tests the statistical significance of the fitted model. For our case the P-Value is less than 0.05 (operating at the 95% confidence level) which indicates that the linear model between $|(d_T)_{max}|$ and L_T is statistical significant. Moreover, the R-squared coefficient[3] is 96.95% which represents that the linear simple regression accounts for about 97% of the variability in the maximum Thorpe displacement $|(d_T)_{max}|$ as a function of the Thorpe scale, L_T.

This relation between the maximum Thorpe displacement and the Thorpe scale by a power law has been deduced for the overall data (not separating the data from the three field campaigns). But we have used three different experiments data set with different mixing conditions. SABLES98 and SABLES2006 experiments have been realized at night (turbulence by shear-driven) and ALMARAZ94-95 during a day cycle and, therefore, convective regions have not been excluded. Hence, we consider to analyze if this power law is different from night to day. The objective is to study if it is possible to distinguish between the shear-driven overturns and the convective ones. First, we separate the data from the three experiments in two set: data obtained overnight (from Sables98, Sables2006 and Almaraz94-95 field campaigns) or night data set, and data which have been obtained during the day (only from Almaraz experiment) or day data set. Then we realize a linear simple regression analysis with an adjustment by least squares for the two data sets. And, finally, we realize a comparison of the regression lines relating $|(d_T)_{max}|$ and L_T at the two levels of our categorical factor (daytime and nighttime).

Figure 5 represents the maximum Thorpe displacement versus the Thorpe scale only for the daytime data set (from 07:00 to 19:00 GMT). We observe a strong correlation which holds over three orders of magnitude as it was deduced for the whole data set and other researches (Lorke and Wüest, 2002).

We realize the linear simple regression analysis. The P-value associated to the analysis of variance is less than 0.05 (operating at the 95% confidence level[4]) which indicates that the linear fit between $|(d_T)_{max}|$ and L_T is statiscally significant as before. The R-squared coefficient represents the percentage of the variability in $|(d_T)_{max}|$which has been explained by the fitted linear regression model and is about 97%.

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[1] The *p*-value helps us to determine the significance of the results when we perform a hypothesis test which is used to test the validity of a claim that is made about a population. The *p*-value is defined as the probability of obtaining a result equal to or "more extreme" than what was actually observed. We use a *p*-value (always between 0 and 1) to weigh the strength of the evidence. A small *p*-value (typically ≤ 0.05) indicates strong evidence against the initial claim (null hypothesis).

[2] The analysis of variance (ANOVA) is a statistical tool that separates the total variability of a data set into two components: random (which do not have any statistical influence on the given data set) and systematic factors (which have some statistical effect on the data). The Anova test is used to determine the impact independent variables have on the dependent variable in a regression analysis.

[3] The R-squared coefficient is called the determination coefficient which represents the proportion of the variance (fluctuation) of one variable that is predictable from the other variable. It is a measure that allows us to determine how certain one can be in making predictions from a certain model. In our case, the coefficient of determination is a measure of how well the regression line represents the data.

[4] The confidence level is a measure of the reliability of a result. A confidence level of 95 per cent or 0.95 means that there is a probability of at least 95 per cent that theresult is reliable.

Figure 6 represents the maximum Thorpe displacement versus the Thorpe scale only for the nocturnal data set (from 20:00 to 06:00 GMT). We also observe a strong correlation which holds over three orders of magnitude as before (see Figure 4 and Figure 5).
Finally, we realize the linear simple regression analysis. The P-value associated to the analysis of variance is less than 0.05 (operating at the 95% confidence level) which indicates that the linear model is statistically significant as before. Moreover, the R-squared coefficient is 95.89 which represents that the linear regression accounts for about 96% of the variability in the maximum Thorpe displacement $|(d_T)_{max}|$.
Therefore, we have deduced that the relation between the maximum Thorpe desplacement $|(d_T)_{max}|$ and the Thorpe scale L_T by a power law is different from day to night. For the nighttime data set the power law is:

$$\left|(d_T)_{\max}\right| \sim (L_T)^{1.17}. \qquad (2)$$

And for the daytime data set the relation is the following:

$$\left|(d_T)_{\max}\right| \sim (L_T)^{1.12}. \qquad (3)$$

We observe that the kind of relation is the same (a power law) but the exponents are different. So we question if these coefficients are statistically different and if there is or not a different behaviour of the overturn length scales between day and night.

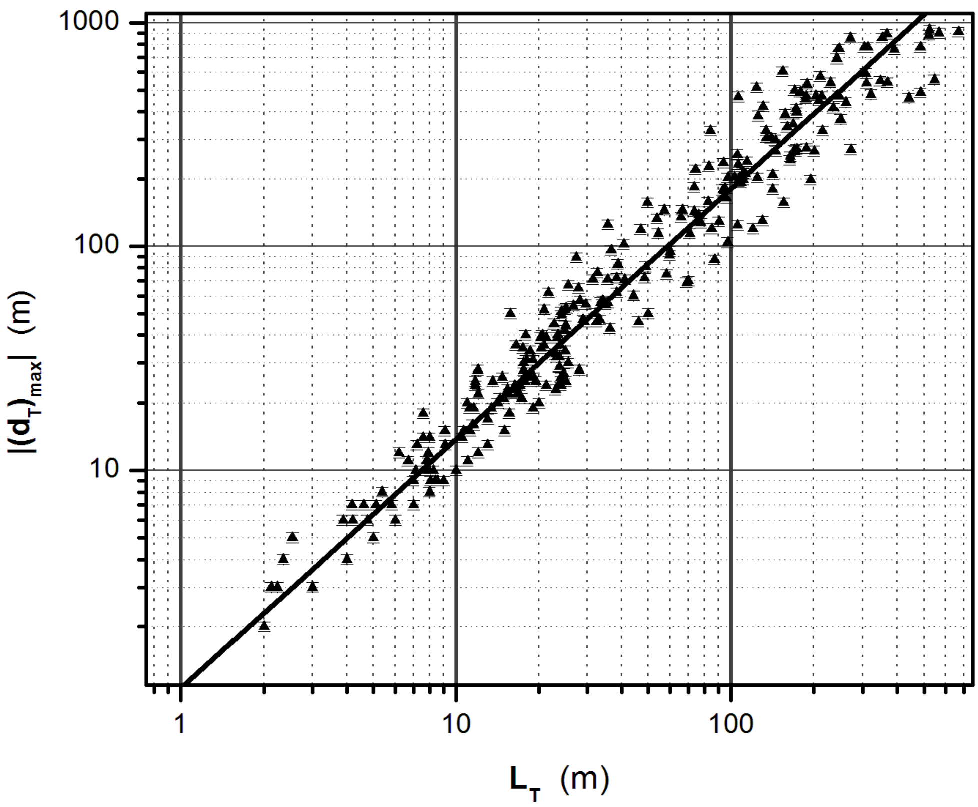

Figure 5. Absolut value of the maximum Thorpe displacement versus Thorpe scale for the daytime data set (▲). The linear fit is indicated by the continuous black line.

These exponents are the slopes of the regression lines fitted to daytime and nighttime data sets (see Figure 5 and Figure 6). To know if they are statistically different we need to realize a comparison of regression lines. This procedure is a test to determine whether there are significant differences between the intercepts and the slopes at the different levels of our factor (day and night). This test fits two different regression lines to the nighttime and daytime data sets and realizes two analysis of variance (one for each linear model and secondly for comparingg the two regression lines). For the first analysis, the P-Value is less than 0.05, if we operate at the 5% significance level, and indicates that the linear fit between $|(d_T)_{max}|$ and L_T is statistically significant for daytime and nighttime data sets (t-statistic tests[5] have also been made which P-Values are less than 0.05 indicating that the model coefficients are significantly different from 0). The second analysis of variance is performed to determine whether there are significant differences between the slopes of the daytime and nighttime fitted lines. The F-test[6] for slopes tests if the slopes of the lines are all equal. Operating at the 1% significance level[7], we find a P-value (for slopes) which is less than 0.01, and, therefore, there are significant differences between the slopes of the daytime and nighttime lines (we get the same result for the intercepts).

There is one more question, that is, to analyze if the power law fits the data better than a linear one in statistical terms. We have made a simple regression analysis to construct three statistical models describing the dependence of $|(d_T)_{max}|$ on L_T considering the three different situations, i. e., the whole data, the daytime data and the nighttime data sets. The linear models were fitted using least squares and tests (analysis of variance) were run to determine the statistical significance of the fitted model.

For all the three datasets, we got the same results. The analysis of variance indicated that a linear model between $|(d_T)_{max}|$ and L_T is statistically significant (because the p-value is less than 0.05). But the *R-squared* –or determination coefficient- which represents the percentage of the variability in $|(d_T)_{max}|$ which has been explained by the fitted regression model is less in the power law fit (87.9% for the whole data set, 84% for the daytime data set and 90.11% for the nighttime data set) than in the linear one (96.95% for the whole data set, 96.76% for the daytime data set and 95.89% for the nighttime data set). As a consequence, the remaining of the unexplained variability is attributable to deviations around the line, which may be due to other factors, for example, to a failure of the linear model to fit the data adequately. We conclude that both models, the power law fit and the linear one, are statistically significant but the power law fit has a better determination coefficient and it accounts better for the variability in the maximum Thorpe displacements measurements. Therefore, we consider that the power law fit is the best fitted model for the three data sets.

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[5] A two-sample t-test examines whether two samples are different and it is a statistical analysis of two population means.

[6] The F-test tests the statistical significance of the fitted model. A small p-value (less than 0.05) indicates that a significant relationship of the form specified exists between two variables, *y* and *x*. It is most often used when comparing statistical models that have been fitted to a data set, in order to identify the model that best fits.

[7] In hypothesis testing, the significance level is the criterion used for rejecting the null hypothesis (an hypothesis about a population parameter). The significance level is the probability of rejecting the null hypothesis given that it is true.

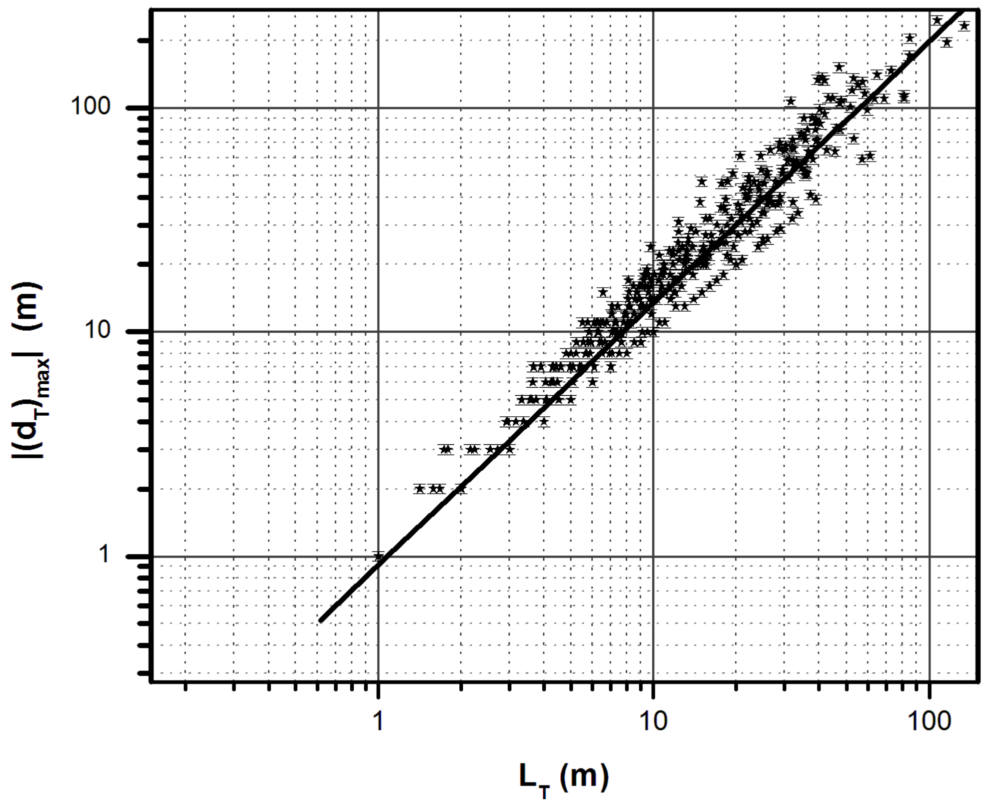

Figure 6. Absolut value of the maximum Thorpe displacement versus Thorpe scale for the nighttime data set (o).The linear fit is indicated by the continuous black line.

Finally, we deduce that the two power relation between the maximum Thorpe displacement $|(d_T)_{max}|$ and the Thorpe scale L_T for nighttime data (equation 2) and daytime data (equation 3) are significant different with a 99% confidence level. Therefore, we could classified overturns between day and night ones, i.e., we could distinguish between convective and shear-driven mechanism originating overturns.

As mentioned before, although both scales ($(d_T)_{max}$ and L_T are alternative length scales to characterize turbulent overturns, it is reasonable to choose one of the two scales to represent better overturns. If there is a high linear correlation between the maximum Thorpe displacement $(d_T)_{max}$ and the Thorpe scale L_T, the last one could be considered a better descriptor of the overturn properties although it depends mainly on the values of $(d_T)_{max}$ and the relative errors from both scales are approximately equal (Piera, J., 2004). But we have just deduced that the relation between the maximum Thorpe displacement $(d_T)_{max}$ and the Thorpe scale L_T does not follow a linear model at our ABL research, unless a power law as other authors (Lorke and Wüest, 2002). Consequently, there would not be a constant ratio $|(d_T)_{max}|/L_T$ which could suggest that the shape of Thorpe displacements distribution could change. Therefore, it is necessary to study the probability density functions (*pdf*) of the Thorpe displacements to understand better the relation between $(d_T)_{max}$ and L_T. Moreover, the Thorpe scale is mainly determined by larger overturns which are not very frequent (Stansfield et al., 2001) and it would be very useful to determine it based on the probability density function of the Thorpe displacements. This *pdf* study would allow us to decide which of the two overturn length scales is a more representative measure of turbulent overturns.

5 Conclusions

The paper presents results related to the time evolutions of the ABL turbulent parameters L_T and $(d_T)_{max}$ during a day with different levels of stability. Secondly, the paper adds insight to the problem of the relationship between these two overturning length scales at ABL.

The Thorpe scale L_T and the maximum Thorpe displacement $(d_T)_{max}$, have small values under neutral and stable conditions, and their greatest values appear under convective conditions. The values of the Thorpe scale ranges in $(1, 660)$ m that are greater than effective values in the stratosphere which are L_T~$1–1.1$ m (Gavrilov, 2013), values in mixing surface layers and seasonal thermoclines which are L_T~$0.03–1.90$ m (Dillon, 1982), values in vertical mixing process induced by internal tides which are L_T~$0.2–4.2$ m (Kitade et al., 2003) or values in dense overflow which are L_T~$1–17$ m (Fer et al., 2004). The greater values appear under convective conditions which could generate overturns of larger scale. Under shear-driven conditions, our Thorpe scales are smaller than convective ones, ranging in $(1, 100)$ m, but they are also greater if we compare them with the scales of other authors. Therefore, we deduce that there would be a relation between the ABL processes which generate mixing and the overturn size and behaviour (for example, the terrain shape interacts with the ability of the ABL to produce local mixing very near the ground and this could be affect to overturns). This theme will need further field work where different conditions are met (combination of the boundary condition effects and of stability combining the 3D and 2D characteristics of scale to scale direct and inverse cascades, intermittency of the forcing and scale to scale stratified turbulence cascade (Vindel et al., 2008; Yagüe et al., 2006)).

Eqs. (1) to (3) shows that the relationship between the Thorpe scale L_T and the maximum Thorpe displacement $(d_T)_{max}$ is a power law which has been statiscally demostrated. We must therefore conclude that the linear model proposes by other authors (Moum, 1996; Peters et al., 1995; Piera Fernández, 2004; Itsweire et al., 1993; Smyth and Moum, 2000) is not adequate for our ABL data. Research will continue on this interesting question which is related to the selection a length scale for characterizing turbulent overturns. This last problem would be better analyzed if we study the probability density function (pdf) of overturning length scales. The objective is to decide if L_T is or not statistically a more appropiate length scale than $(d_T)_{max}$. Moreover, it is interesting to verify the assumption that the Thorpe scales have a universal probability density function which could be used to verify how accurately the Thorpe scales were computed and also to determine if $(d_T)_{max}$ is statistically better than L_T as overturning length scale. It is very likely that the pdf parameters depend on the governing background conditions generating Thorpe displacements, which are different in the boundary layers from those in the interior layers with intermittent mixing, or in convective conditions from shear-driven conditions. We also would like to verify if the density probability function is decaying exponentially for increasing displacement length with a separate cut-off before $(d_T)_{max}$.

In the future, we will go on studying the power relationship between the maximum Thorpe displacement and the Thorpe scale corresponding to ABL data to verify the power law deduced at this paper. For this purpose, we will use new set of ABL data from new field campaigns. We will analyze the probability density function of overturning length scales to clarify better the relation between $(d_T)_{max}$ and L_T and as a tool to choose the more appropiate turbulent patch length scale. Moreover, we would like to study the following hypothesis if the Thorpe scale is greater than the integral

scale there would be a local convective process and if it is not, there would be stratification.
Finally, there is another subject which is important to mention. At future researches, we need to study better the overturn identification as Piera et al. (2002). They propose a new method based on wavelet denoising and the analysis of Thorpe displacements profiles for turbulent patch identification. Although their method is for microstructure profiles (that is not our case), it reduces most of the noise present in the measured profiles (increasing the resolution of the overturn identification) and it is very efficient even at very low-density gradients for turbulent patch identification. Another way to get overturn identification would be, for example, to use a 3 or 4 dimensional parameter space formed by (L_O, L_T, L_{MO}) to locate mixing events and also to study the evolution of the processes.

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