Response to the Referee 1 (Pelinovsky E.)

Some technical comments to improve the text:
1. I would recommend making the figures in color scale and of a larger size. Now it is difficult to follow the lines on them

We have pasted the figures 1, 3, 4, 7, 8, 9, 10 in color scale and of a larger size (as far as possible) into the paper (p. 32, 34, 35, 38, 39, 40, 41 in pdf).

2. I may recommend that all analytical expressions for the coefficients will be collected and published in supplement materials to this paper (it is now usual practice for many journals) instead of information that they are available on request from Oxana Kurkina.

We have added all analytical expressions for the coefficients in supplement materials to this paper (p. 26 in pdf):

$$\gamma_1^{\pm}=\frac{1}{12c^{\pm}H_3l_1(1-l_2)(1-4rq^{\pm 2}l_1(1-l_2)(l_2-l_1))}\times((144rq^{\pm}H_3l_1^2(1-l_2)^2(l_1-l_2)-12H_3l_1(1-l_2))\beta+$$

$$+((8H_3^3l_1^2c^{\pm}(1-l_2)^2(l_1^3-l_2^3)+24H_3^3l_1^3c^{\pm}(1-l_2)^2(l_1-l_2))rq^{\pm}-2H_3^3l_1c^{\pm}(1-l_2)(l_1^2-l_2^2))\alpha+$$

$$+(384l_1^2c^{\pm}q^{\pm 3}r(1-l_2)^2(l_1-l_2)-240l_1c^{\pm}q^{\pm 2}r(1-l_2)^2(l_1-l_2)+48l_1^2c^{\pm}q^{\pm 2}(l_1^2-l_1(1+l_2)+l_2)+$$

$$+24c^{\pm}(q^{\pm}l_1(l_2-l_1)+1+2l_2))\beta+(40H_3^2c^{\pm 2}(1-l_2)^2(q^{\pm 3}l_1^2(l_1^3-l_2^3)-q^{\pm 2}l_1(l_1^3-l_2^3))+120H_3^2c^{\pm 2}(1-l_2)^2\times$$

$$\times q^{\pm 3}l_1^3(l_2^2-l_2l_1)-96H_3^2c^{\pm 2}q^{\pm 2}(1-l_2)^2l_1^2(l_2^2-l_2l_1)+4H_3^2c^{\pm 2}q^{\pm}(1-l_2)^2(l_2l_1^2-l_1^3))r+(12l_1^4-$$

$$-4l_1^3(5+7l_2)+4l_1^2(11(1-l_2)^2-3-21l_2)+4l_1(7(1-l_2)^3-22(1-l_2)^2+21(1-l_2)-12)-$$

$$-4l_2(7(1-l_2)^2-4(1-l_2)+3))H_3^2c^{\pm 2}q^{\pm 2}l_1^2+(20l_1^3(1-l_2)-12l_1^2(3-2l_2)-4l_1(5(1-l_2)^3-$$

$$-11(1-l_2)^2+6(1-l_2)-6)H_3^2c^{\pm 2}q^{\pm}l_1+(l_1^2(1-4l_2)-l_2^2(1-4l_2))H_3^2c^{\pm 2})$$

$$\gamma_2^{\pm}=\frac{1}{12c^{\pm}H_3l_1(1-l_2)(1-4rq^{\pm 2}l_1(1-l_2)(l_2-l_1))}\times((216rq^{\pm}H_3l_1^2(1-l_2)^2(l_1-l_2)-18H_3l_1(1-l_2))\beta+$$

$$+((24H_3^3l_1^2c^{\pm}(1-l_2)^2(l_1^3-l_2^3)+72H_3^3l_1^3c^{\pm}(1-l_2)^2(l_1-l_2))rq^{\pm}-6H_3^3l_1c^{\pm}(1-l_2)(l_1^2-l_2^2))\alpha+$$

$$+(192l_1^2c^{\pm}q^{\pm 3}r(1-l_2)^2(l_1-l_2)-48l_1c^{\pm}q^{\pm 2}r(1-l_2)^2(l_1-l_2)+96l_1^2c^{\pm}q^{\pm 2}(l_1^2-l_1(1+l_2)+l_2)+$$

$$+12c^{\pm}(2q^{\pm}l_1(l_2-l_1)+1+2l_2))\beta+(56H_3^2c^{\pm 2}(1-l_2)^2\left(q^{\pm 3}l_1^2(l_1^3-l_2^3)-q^{\pm 2}l_1\left(\frac{13}{7}l_1^3-l_2^3\right)\right)+168H_3^2c^{\pm 2}(1-l_2)^2\times$$

$$\times q^{\pm 3}l_1^3(l_2^2-l_2l_1)-96H_3^2c^{\pm 2}q^{\pm 2}(1-l_2)^2l_1^2\left(\frac{5}{2}l_2^2-l_2l_1\right)-4H_3^2c^{\pm 2}q^{\pm}(1-l_2)^2(l_2l_1^2-l_1^3))r+(12l_1^4-$$

$$-4l_1^3(7+5l_2)+4l_1^2(19(1-l_2)^2+3+15l_2)+4l_1(17(1-l_2)^3-38(1-l_2)^2+15(1-l_2)-12)-$$

$$-4l_2(17(1-l_2)^2-2(1-l_2)+3))H_3^2c^{\pm 2}q^{\pm 2}l_1^2+(52l_1^3(1-l_2)-12l_1^2(9-8l_2)-4l_1(13(1-l_2)^3-$$

$$-19(1-l_2)^2-6(1-l_2)-6)H_3^2c^{\pm 2}q^{\pm}l_1+(l_1^2(5-8l_2)-l_2^2(5-8l_2))H_3^2c^{\pm 2})$$

$$\beta_1^{\pm} = \frac{1}{720c^{\pm}\left(1-4rq^{\pm 2}l_1(1-l_2)(l_2-l_1)\right)} \times \left(\left(-360+4320l_1(1-l_2)(l_1-l_2)rq^{\pm 2}\right)\beta^2 + \right.$$

$$+\left(480H_3^2c^{\pm}rq^{\pm 2}l_1(1-l_2)(l_1^3-l_2^3)+1440H_3^2c^{\pm}rq^{\pm 2}l_1^2(1-l_2)(l_2^2-l_2l_1)-120H_3^2c^{\pm}(l_1^2-2l_2+l_2^2)\right)\beta +$$

$$+\left(\left(12l_1(l_1^5-l_2^5)+60l_1^2(l_2^4-l_2l_1^3)+120l_1^3(l_1l_2^2-l_2^3)\right)q^{\pm 2}+\left(14l_2l_1^4-20l_2^3l_1^2+6l_2^5\right)q^{\pm}\right)H_3^4c^{\pm 2}(1-l_2)r +$$

$$+\left(-6l_1^6+30l_1^5+20l_1^4\left(-3+(1-l_2)^2\right)+60l_1^3l_2(2-l_2)-2l_1^2\left(7(1-l_2)^4-30(1-l_2)^2+15\right)+\right.$$

$$\left.-2l_1l_2(2-l_2)\left(7(1-l_2)^2-3\right)\right)H_3^4c^{\pm 2}q^{\pm}l_1+\left(-7l_1^4+10l_1^2l_2(2-l_2)+l_2(2-l_2)\left(7(1-l_2)^2-3\right)\right)H_3^4c^{\pm 2}\Big)$$

$$\tilde{\alpha}^{\pm}(1-l_2, 1-l_1, \frac{1}{r}) = \alpha^{\pm}(l_1, l_2, r),$$

$$\tilde{\alpha}_1^{\pm}(1-l_2, 1-l_1, \frac{1}{r}) = \alpha_1^{\pm}(l_1, l_2, r),$$

$$\tilde{\gamma}_1^{\pm}(1-l_2, 1-l_1, \frac{1}{r}) = \gamma_1^{\pm}(l_1, l_2, r),$$

$$\tilde{\gamma}_2^{\pm}(1-l_2, 1-l_1, \frac{1}{r}) = \gamma_2^{\pm}(l_1, l_2, r)$$

p. 25 line 11
This sentence has been corrected: Coefficients of Gardner equations and equations (26) for interfacial waves of both modes propagating on both interfaces.

p. 26 line 10
This sentence has been corrected: The coefficients at terms describing linear dispersion $\beta^{\pm}$, $\tilde{\beta}^{\pm}$ as well as coefficients $\beta_1^{\pm}$ and $\tilde{\beta}_1^{\pm}$ are equal for the equations for different interfaces within the same mode.

p. 26 line 16
This sentence has been deleted: The analytical expressions for the coefficients are available on request from Oxana Kurkina (oksana.kurkina@mail.ru)

3. Formula (30) can be omitted because it coincides with (28)
Formula (30) has been deleted.
4. On p. 12 after Eq. (26). The second-order Eq. (26), not Eq. 28.
It has been corrected.
5. In references papers Oceanology and JETP Letters are accompanied by +. Why?
It has been corrected.

Response to the Referee 2

1. I don't think that the words "Propagation regimes ..." correctly reflect the paper content. The paper is devoted rather to the derivation of the model equations and analysis of soliton structures than to the discussion of possible regimes of wave propagation. Perhaps, the title "Soliton structures in three-layer fluid" is more relevant to the paper content?
We'd rather prefer it didn't change, because we wish to focus on possible regimes of wave propagation depending of nonlinear coefficients' values and the present title of the manuscript implicitly reflects existence of not only solitonic structures, but also localized non-radiating solutions (breathers), possible when cubic nonlinearity coefficient is positive.

2. The authors write in the Abstract that they consider immiscible fluid layers. But they do not include surface tension between the layers into consideration. Therefore their results pertain in fact to miscible fluid layers.

p. 2 line 3

This sentence has been corrected: Long weakly nonlinear finite-amplitude internal waves in a fluid consisting of three inviscid layers of arbitrary thickness and constant densities (stable configuration, Boussinesq approximation) bounded by a horizontal rigid bottom from below and by a rigid lid at the surface are described up to the second order of perturbation theory in small parameters of nonlinearity and dispersion.

3. It can be mentioned that some coefficients at the nonlinear terms may vanish not only in a three-layer fluid, but even in a two-layer fluid with the surface tension between the layers too. The authors can refer to the papers:

1. *Giniyatullin A.R., Kurkin A.A., Kurkina O.E., Stepanyants Y.A. Generalised Korteweg–de Vries equation for internal waves in two-layer fluid. Fundamental and Applied Hydrophysics, 2014, v. 7, n. 4 (in Russian).*
2. *Kurkina O., Singh N., Stepanyants Y. Structure of internal solitary waves in two-layer fluid at near-critical situation. Comm. Nonlin. Sci. Num. Simulation, 2015, v. 22, n. 5, 1235–1242.*

p. 6 line 7

This sentence has been added: Some coefficients at the nonlinear terms may vanish even in a two-layer fluid with the surface tension between the layers too (Giniyatullin et al., 2014; Kurkina et al., 2015). After: This feature is common for several wave classes in stratified environments.

p. 27 line 19

This paper has been added to the References: Giniyatullin A.R., Kurkin A.A., Kurkina O.E., Stepanyants Y.A. Generalised Korteweg–de Vries equation for internal waves in two-layer fluid. Fundamental and Applied Hydrophysics, 2014, v. 7, n. 4 (in Russian).

p. 28 line 29

This paper has been added to the References: Kurkina O., Singh N., Stepanyants Y. Structure of internal solitary waves in two-layer fluid at near-critical situation. Comm. Nonlin. Sci. Num. Simulation, 2015, v. 22, n. 5, 1235–1242.

4. I am suggesting to present all coefficients of the derived equations explicitly in the Appendix.

We have added all analytical expressions for the coefficients in supplement materials to this paper (p. 26 in pdf).

p. 26 line 16

This sentence has been deleted: The analytical expressions for the coefficients are available on request from Oxana Kurkina (oksana.kurkina@mail.ru)

5. In the first sentence of Sect. 5 it is claimed that Eqs. (29) are integrable by means of the inverse scattering method only for ONE specific set of coefficients. It is not true, there are two sets of coefficients when the fifth-order KdV equation is integrable – the Sawada–Kotera equation and Kaup–Kupershmidt equation, see:

http://mathworld.wolfram.com/Sawada-KoteraEquation.html
http://en.wikipedia.org/wiki/Kaup%E2%80%93Kupershmidt_equation.

p. 15 line 13

These sentences have been corrected: Equations (29) are integrable using the inverse scattering method only for two specific sets of nontrivial values of their coefficients (Newell, 1985; Zwillinger, 1997; Weisstein).

These sets, however, are not applicable for the problem of internal wave motion in stratified environments, for which this equation apparently remains nonintegrable.

p. 30 line 33

These sources have been added in the References: Weisstein, Eric W. "Sawada-Kotera Equation." From MathWorld – A Wolfram Web Resource. http://mathworld.wolfram.com/Sawada-KoteraEquation.html

Weisstein, Eric W. "Kupershmidt Equation." From MathWorld – A Wolfram Web Resource. http://mathworld.wolfram.com/KupershmidtEquation.html

p. 31 line 7

This book has been added in the References: Zwillinger, D.: Handbook of Differential Equations, 3rd ed. Boston, MA: Academic Press, 834 pp., 1997.

6. As for the technical corrections, I would advise authors to enlarge sizes of Figs. 3, 7 and 9; they are poorly visible in the current format.

We have pasted the figures 3, 7, 9 of a larger size (as far as possible) into the paper (p. 34, 38, 40 in pdf).

We have also fixed the typo in the analytical expression for the coefficient of cubic nonlinearity, the correct expression is:

$$\alpha_1^{\pm} = \frac{1}{2(1-l_2)^2 H_3^2 c^{\pm} l_1^2 \left(1 - 4rq^{\pm 2}(1-l_2) l_1 (l_2 - l_1)\right)}$$

$$\times\Big(\alpha^{\pm 2} H_3^2 l_1^2 (1-l_2)^2 \left(-12rq^{\pm 2} l_1 (1-l_2)(l_2-l_1) - 1\right) + \alpha^{\pm} H_3 c^{\pm} (1-l_2) l_1 \Big(4rq^{\pm 2}(1-l_2)^2 (l_2-l_1) l_1 \left(-20 l_1 q^{\pm} + 11\right) +$$

$$+16q^{\pm 2} l_1^2 (1-l_1)(l_2-l_1) + 2q^{\pm} l_1 (l_2 - l_1) + 5(1-2l_2)\Big) + 12rq^{\pm 2} c^{\pm 2} (1-l_2)^3 (l_2-l_1) l_1 \left(-10q^{\pm 2} l_1^2 + 15q^{\pm} l_1 - 3\right) +$$

$$+12q^{\pm 3} c^{\pm 2} l_1^3 \left(-4l_1^3 + 3l_1^2(3+l_2) + l_1\left((1-l_2)^2 + 6(1-l_2) - 12\right) + (5-l_2) l_2\right) -$$

$$-18q^{\pm 2} c^{\pm 2} l_1^2 \left(2l_1^2(3-l_2) + l_1\left(2(1-l_2)^2 + (1-l_2) - 8\right) + l_2(5-l_2)\right) + 18q^{\pm} c^{\pm 2} l_1 (1+l_2)(l_2-l_1) -$$

$$-6c^{\pm 2}\left(3(1-l_2)^2 - 3(1-l_2) + 1\right)\Big)$$