

Reply to npgd-1-C772

C1: ISSA improves SSAM by reformulating the calculation of PCs (equation 7) to incorporate RCs for missing values (equations 8 to 14). The improvement is small for mostly complete time series and increases as the quantity of missing data increases. I encourage the authors to post ISSA code for others to use.

R1: Thanks for your kindly suggestion. We will modify our code and post it soon.

C2: It appears that ISSA Eigenvectors v are calculated as they are in SSAM from the Toeplitz matrix formed from equation 5. This ISSA step should be added to the manuscript.

R2: We add the sentence in page 1951, line 9 **“Then we compute the eigenvalues and eigenvectors from the lagged correlation matrix C ”**.

C3: The eigenvectors are then used to create matrix G . It appears that matrix G must be created and equation 14 solved for each time step i . This is a large increase in computational effort compared to SSAM, which should be stated in the manuscript.

R3: We add the sentence in page 1953, line 23 **“The only disadvantage of our method is that it will cost more computational effort.”**

C4: In equation 11, the sums are for all times in the window with a missing value. The values of the eigenvector do not change with time, so the sum can be replaced with N_m , the number of missing values in the window (e.g. $\sum v_{1,j}v_{2,j} = N_m v_{1,j}v_{2,j}$). If $N_m=0$, equation 10 reduces to equation 3.

R4: The values of the eigenvector vary with the subscript j , so the $\sum v_{1,j}v_{2,j} \neq N_m v_{1,j}v_{2,j}$.

C5: p1953, line 8-13: Equation 15 is used to compare SSAM and ISSA which is good to include but the approach contains a contradiction that should be explained. To compare their results to SSAM, the authors set non-diagonal element in equation 11 to zero but also assume $v_{k,i} = L^{-1/2}$, in which case the diagonal elements would equal N_m/L where N_m is the number of missing data points in the window. The authors should explain this contradiction. For the case where $N_m/L \ll 1$, this contradiction would be minor. Is this contradiction inherently assumed in the formulation of SSAM, and if so, does it explain the relatively improving performance of ISSA as N_m/L (% missing data in table 1) increases? SSAM performance declines when $N_m/L > 0.5$ which is roughly when the diagonal elements of equation 11 become less than the non-diagonal elements—could this be the cause? Or does the ISSA assumption that missing values can be represented by an RC expression create this contradiction? Missing values are ignored when calculating the eigenvectors in both methods, but ISSA does not ignore missing values when calculating PCs.

R5: Thanks for your comment. The Schoellhamer (2001) did not tell us the reason to choose the scale factor L/L_i . And, we find when $v_{k,i} = L^{-1/2}$ and non-diagonal elements equal to zero are both satisfied, we can get the same formula as in Schoellhamer (2001). Thus, we assume it is he ignored this contradiction that makes his method poorer than ours.

C6: p1948 abstract: Add that the improvement is small for mostly complete time series and

increases as the quantity of missing data increases. Because of this, I suggest changing 'much smaller' to 'smaller'.

R6: We have changed 'much smaller' to "**smaller**".

C7: p1948, line 16: define SD

R7: SD means "**standard deviation**".

C8: p1948, line 17: A difference of 1.2 mg/L (~10%) is within typical measurement error.

R8: Although the percentage of missing data reaches 61%, but the distribution of observed data are very concentrated, thus the non-diagonal elements of matrix \mathbf{G}_i is very small. Then the improvement is also very small.

C9: p1948, line 25-26: use 'wide' only once in the sentence.

R9: We have changed the sentence into "**SSA has been widely used in geosciences to analyze a variety of time series**".

C10: p1949, line 9: Define GNSS

R10: GNSS represents "**Global Navigation Satellite System**".

C11: p1951, line 14: Insert paragraph break where SSAM ends and ISSA starts.

R11: We have revised as above.

C12: p1953, line 8: Insert paragraph break where ISSA ends and comparison to SSAM begins.

R12: We have revised as above.

C13: p1954, line 2-3: This section is about synthetic time series, not the real time series, so delete this sentence.

R13: We have delete this sentence.

C14: p1954, line 20: Delete 'even'.

R14: We have delete the word "**even**".

C15: Equation 18: define T (transpose?).

R15: T represents "**transpose**".

C16: p1955, line 8: delete the word 'clear'.

R16: We have delete the word "**clear**".

C17: p1956, line 19: the mean residual is not represented in table 2.

R17: We have added the mean residual in table 2.

C18: p1956, line 22: the difference of r^2 of 0.9178 and 0.9046 seems to be minor- is this statistically significant? Autocorrelation would probably have to be considered.

R18: The reason is almost the same with C8.

C19: Delete 'As' in last row, replace with 'SF'

R19: We have replaced "As" with "**SF**".

C20: p1957, line 7-8: Change 'With the missing data gets more, the improvements of the relative errors becomes more evident.' to 'As the fraction of missing data increases, the improvement of the relative error becomes greater'.

R20: We have change the sentence into "**As the fraction of missing data increases, the improvement of the relative error becomes greater**".

C21: p1957, line 12: The SSC improvements are minor and within measurement error.

R21: The reason is almost the same with C8.