

We have to say many thanks to reviewer for useful and constructive comments which push us to improve the manuscript.

Here is the list of comments and corrections.

1. Reviewer does not agree that "the model is free from the narrowband approximation for surface waves and relatively weak adverse current" (lines 4-5). The frequency (or wavenumber) detuning of their three-modes model has an order ε . Thus, we have a sort of narrow band approximation. The strength of the current should be consistent with the problem scaling when effects of nonlinearity and inhomogeneity are of the same order of magnitude and, thus, cannot be free from the approximation;

Of course, any modeling of Benjamin-Feir instability by its main property considers the narrow band approximation of the order ε . Absolute frequencies difference (see equation 7) for the stationary modulation has the same order for the whole zone of wave-current interaction. But wave number and intrinsic frequencies of waves are no more constant and can change significantly in accordance with current dependant dispersion relations and effects of nonlinearity. Variations can be essential and even of order unit (see Fig.1h) on a long current scale, and so indeed we have the narrow band property, but only locally in space. (Here appear another important question from the 2nd referee: may be due to interaction with current and nonlinearity effects resonance conditions are totally destroyed due to the large detuning?). To clarify this property we add to manuscript the figure describing the typical behavior of phase-shift difference function $\varphi[X] = 2\theta_1 - \theta_0 - \theta_2$ (Figure 1d). Intensity of quasi-resonance energy exchange mostly defined by this function together with wave amplitudes. (see equations (17), (20)). Result looks rather surprising - several strong phases' jumps take a place with corresponding changing of the wave energy fluxes direction. But in any case we see an intensive quasi-resonant energy exchange in the entire interaction zone.

The effects of current and nonlinearity are assumed to have the same order and in this sense, of course, we have some restrictions to current strength. The model considers values of current speed up to the initial group velocity of surface waves and "long enough" space scale to go with the acceptable accuracy and to catch the strongest effects of wave blocking.

Corresponding corrections and explanations are made in the text of paper

"the model is free from the narrowband approximation for surface waves and relatively weak adverse current"

- > the model considers essential variations of the wave numbers and frequencies of interacting waves and wave blocking adverse current

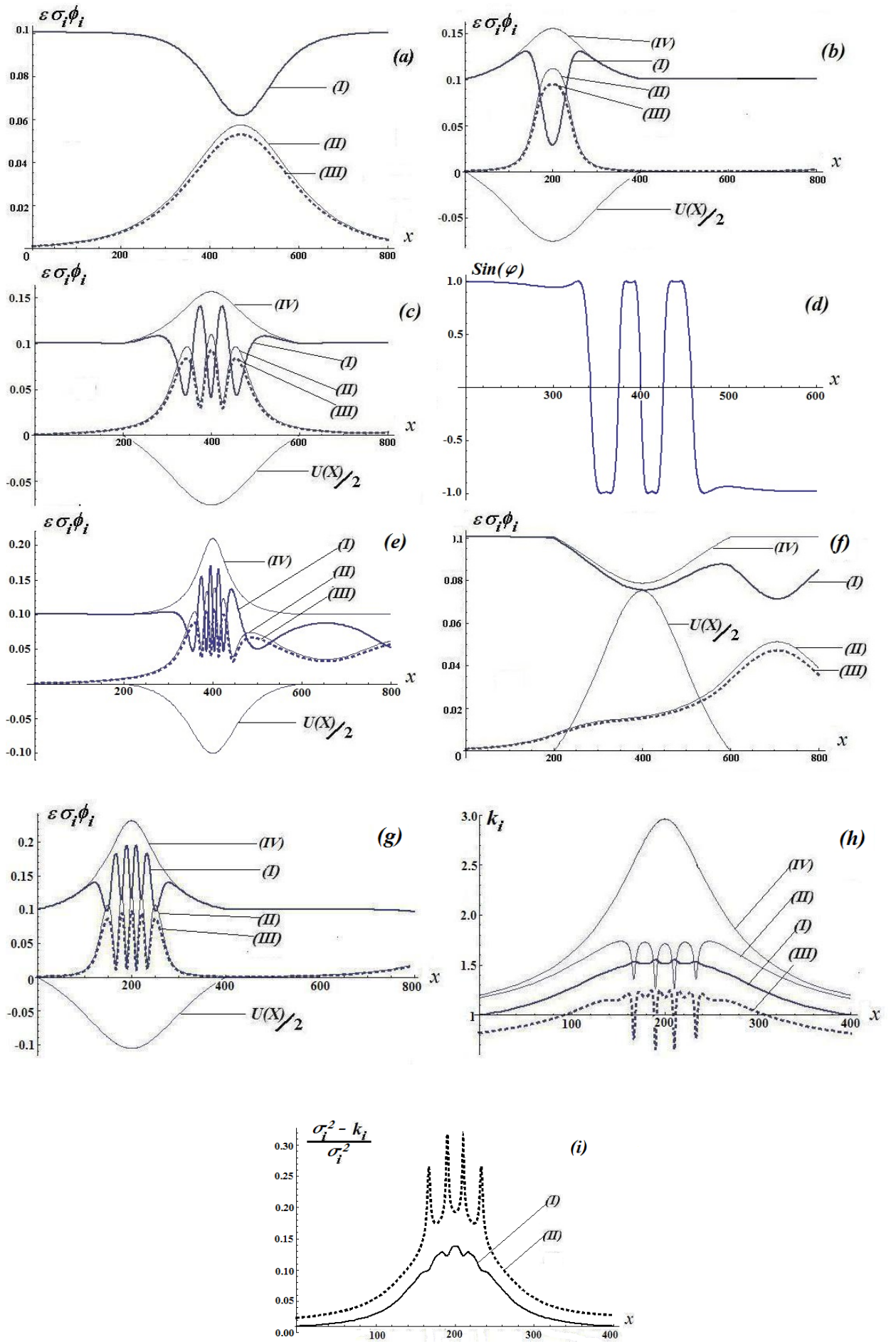


FIG. 1. (a) BF instability without current. (b), (c) Modulation of surface waves by adverse current

$U = U_0 \text{Sech}[\varepsilon^2(x - x_0)]$, ($U_0 = -0.15$); (b) $x_0 = 200$, (c) $x_0 = 400$. (d) phase difference function $\varphi[X] = 2\theta_1[X] - \theta_0[X] - \theta_2[X]$, $\theta_1[0] = 0; \theta_0[0] = \theta_2[0] = -\pi/4$, (e) Modulation of surface waves by adverse current $U = U_0 \text{Sech}[2\varepsilon^2(x - x_0)]$, ($U_0 = -0.2$), (f) Modulation instability for following current ($U_0 = 0.16, x_0 = 400$). (g), (h) Functions of wave amplitude and wave number respectively for $U_0 = -0.2$. (I), (II), (III) Amplitude envelopes of the carrier, superharmonic and subharmonic waves, respectively. (IV) Linear solution for the carrier envelope. The initial steepness of the carrier wave is $\varepsilon = 0.1$. (i) Relative distortion of the linear dispersion relation for the case (g), (I) – carrier, (II) – higher side band.

2. Authors develop their asymptotic approach in primitive variables that leads to rather cumbersome expressions. At the same time, the resulting equations should have evident properties of symmetry in indices of satellites 0; 2. In absence of current ($U = 0$) it has to give, in particular, the Manley-Rowe relations that are equivalent to the momentum conservation. These relations give a good basis for qualitative analysis of the effect of current inhomogeneity. Eqs. (16,17,20), unfortunately, do not emphasize this key feature of the system.

Following reviewer recommendations we rewrite equations (17), (20) in a more clear and compact form of wave action law:

$$\left\{ \begin{aligned} [\phi_0^2 \sigma_0]_T + [(U(X) + \frac{1}{2\sigma_0})\phi_0^2 \sigma_0]_X &= \varepsilon \phi_1^2 \phi_2 \phi_0 \sigma_1^3 \sigma_2^2 (2\sigma_1^3 - 2\sigma_1^2 \sigma_2 + 2\sigma_1 \sigma_2^2 - \sigma_2^3) \text{Sin}[\varphi]; \\ [\phi_2^2 \sigma_2]_T + [(U(X) + \frac{1}{2\sigma_2})\phi_2^2 \sigma_2]_X &= \varepsilon \phi_1^2 \phi_2 \phi_0 \sigma_0^2 \sigma_1^3 (2\sigma_1^3 - 2\sigma_0 \sigma_1^2 + 2\sigma_0^2 \sigma_1 - \sigma_0^3) \text{Sin}[\varphi]; \\ [\phi_1^2 \sigma_1]_T + [(U(X) + \frac{1}{2\sigma_1})\phi_1^2 \sigma_1]_X &= -\varepsilon \phi_1^2 \phi_2 \phi_0 \sigma_0 \sigma_1^2 \sigma_2 (\sigma_0^4 - \sigma_0^3 \sigma_1 - \sigma_0 \sigma_1 (\sigma_1 - \sigma_2)^2 + \\ &+ \sigma_0^2 (\sigma_1^2 - \sigma_1 \sigma_2 + 2\sigma_2^2) - \sigma_2 (\sigma_1^3 - \sigma_1^2 \sigma_2 + \sigma_1 \sigma_2^2 - \sigma_2^3)) \text{Sin}[\varphi]; \end{aligned} \right. \quad (17)$$

$$\left\{ \begin{aligned} [(U + \frac{1}{2\sigma_0})\phi_0^2 \sigma_0]_X &= \varepsilon \phi_1^2 \phi_2 \phi_0 \sigma_1^3 \sigma_2^2 (2\sigma_1^3 - 2\sigma_1^2 \sigma_2 + 2\sigma_1 \sigma_2^2 - \sigma_2^3) \text{Sin}[\varphi] \\ [(U + \frac{1}{2\sigma_2})\phi_2^2 \sigma_2]_X &= \varepsilon \phi_1^2 \phi_2 \phi_0 \sigma_1^3 \sigma_0^2 (2\sigma_1^3 - 2\sigma_0 \sigma_1^2 + 2\sigma_0^2 \sigma_1 - \sigma_0^3) \text{Sin}[\varphi] \\ [(U + \frac{1}{2\sigma_1})\phi_1^2 \sigma_1]_X &= -\varepsilon \phi_1^2 \phi_2 \phi_0 \sigma_0 \sigma_1^2 \sigma_2 (\sigma_0^4 - \sigma_0^3 \sigma_1 - \sigma_0 \sigma_1 (\sigma_1 - \sigma_2)^2 + \\ &+ \sigma_0^2 (\sigma_1^2 - \sigma_1 \sigma_2 + 2\sigma_2^2) - \sigma_2 (\sigma_1^3 - \sigma_1^2 \sigma_2 + \sigma_1 \sigma_2^2 - \sigma_2^3)) \text{Sin}[\varphi] \end{aligned} \right. \quad (20)$$

Derived system has a strong symmetry with respect to indexes 0 and 2 (it was one of the checking procedures for the validity of the final equations)

To perform the qualitative analysis of the problem, we suggest the law of wave action conservation flux in a slowly moving media as analogue of the three Manley-Rowe dependent integrals:

$$\begin{cases} \left(U + \frac{1}{2\sigma_2} \right) \phi_2^2 \sigma_2 + \left(U + \frac{1}{2\sigma_0} \right) \phi_0^2 \sigma_0 + \left(U + \frac{1}{2\sigma_1} \right) \phi_1^2 \sigma_1 = const; \\ \frac{1}{2} \left(U + \frac{1}{2\sigma_1} \right) \phi_1^2 \sigma_1 + \left(U + \frac{1}{2\sigma_0} \right) \phi_0^2 \sigma_0 = const; \\ \frac{1}{2} \left(U + \frac{1}{2\sigma_1} \right) \phi_1^2 \sigma_1 + \left(U + \frac{1}{2\sigma_2} \right) \phi_2^2 \sigma_2 = const; \\ \left(U + \frac{1}{2\sigma_0} \right) \phi_0^2 \sigma_0 - \left(U + \frac{1}{2\sigma_2} \right) \phi_2^2 \sigma_2 = const. \end{cases}$$

These integrals follow from the system (20) with acceptable accuracy $O(\varepsilon^4)$ for the stationary regime of modulation. The second and third relations here clearly show that the wave action flux of the side bands can grow up at the expense of the main carrier wave flux. The last relationship manifests the almost identical behavior of the main sidebands for the problem of their generation due to Benjamin-Feir instability.

The obtained system of equations (16), (20) in the absence of current is similar to classical Zakharov equations for discrete wave interactions (Mei, Stiassnie and You, 2009 . Mei et al. Theory and Applications of Ocean Surface Waves. World Scientific, 2009, 14.9.1-14.9.3). Corresponding references are added to the text. The main property in the presence of current is the variability of interaction coefficients. It is also answer to some of the similar comments of the referees 2nd and 4th.

3. In sect.4 authors introduce semi-empirical functions of wave dissipation and breaking. The extension of the conservative system (16-18) to the non-conservative counterpart (21-22) looks strange. Additional terms contain terms of different orders in ε . Additionally, denominators of these terms contain small differences of wavenumbers (order of ε). This extension requires to be re-written in more consistent way or additional comments.

We employ the adjusted dissipative model of Tulin and Li (1996) and Huang et al. (2011) to describe the effect of breaking on the dynamics of the water wave. The sinks of energy and momentum terms for each of the waves are calculated in accordance with the dissipative Schrodinger model for the complex amplitude $A \sim \sum \phi_i e^{i\theta_i} ::$

$$A_T + C_g A_x + i \frac{C_g}{4k} A_{xx} + \frac{i}{2} \omega k^2 |A|^2 A = \left(-\frac{DA}{g|A|^2} - 4i\gamma A \int \frac{\omega^2 D dX}{g|A|^2} \right) H \left[\frac{|A_x|}{A_s} - 1 \right]$$

where $D \sim gD_b |A|^4$, $D_b = O(10^{-1})$, $\gamma = O(10^{-1})$ - constants of proportionality taken from the field observations, g – gravity acceleration, H is the Heaviside unit step function, and A_s is the threshold value of the characteristic steepness $A_x = \varepsilon \sum \sigma_i \phi_i k_i$. Right side part leads to additional terms in the governing modulation equations (16), (17), (20). The terms $\varepsilon / (k_i - k_j)$ appear due to integration procedure and have an order of unit. The singularity was not detected in numerical simulations.

Wave breaking leads to permanent (not temporal) frequency downshifting at a rate controlled by breaking process. A crucial aspect here is the cooperation of dissipation and near-neighbor energy transfer in the discretized spectrum acting together.

Authors present simulations for parameters of previous experimental studies and show strong effects of current on modulational instability for very special cases. An important question is a root to these special and, somewhat, extreme cases. A dependence of the effect on parameters of the current inhomogeneity can be presented to show a gradual transition from weak inhomogeneity to the extreme cases. May be additional figures can make the presentation of the results more clear and attractive.

We add some more figures with additional examples of wave interactions to clarify its properties.

The numerical simulations for initially high steepness waves ($\varepsilon = 0.25$) propagation with wave breaking dissipation is presented in Fig. 3(a-c). We calculate the amplitudes of surface waves on linearly increasing opposing current $U(x) = -U_0 x$ with different strength U_0 . Most unstable regime was tested for frequency space $\Delta\omega_{\pm} / \omega_1 \sim \varepsilon$ and most effective initial phases $\theta_1(0) = 0, \theta_0(0) = \theta_2(0) = -\pi/4$

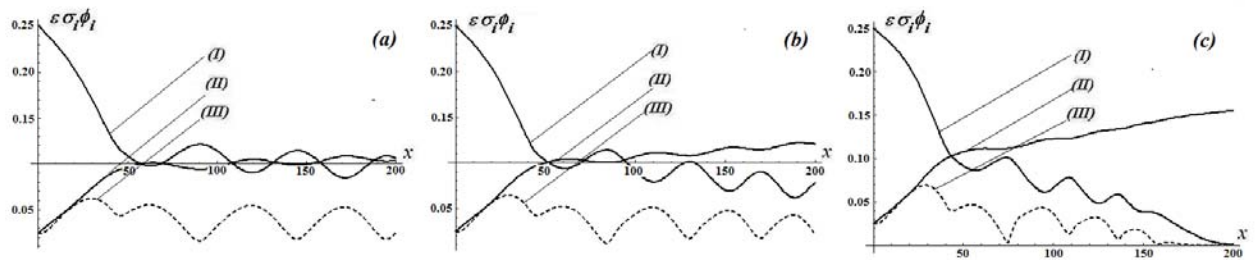


FIG. 3. Modulation of surface waves by the adverse current $U = U_0 x$. (a) $U_0 = -2.5 \cdot 10^{-4}$; (b) $U_0 = -5 \cdot 10^{-4}$, (c) $U_0 = -10^{-3}$. (I), (II), (III) - amplitude envelopes of the carrier, subharmonic and superharmonic waves, respectively. Initial wave steepness $\varepsilon = 0.25$. Dissipation parameters $D_b = 0.1, \gamma = 0.5$

A very weak opposite current $U_0 = 2.5 \cdot 10^{-4}$ (Fig.3(a)) has a pure impact on wave behavior: it is finally results in almost bichromatic wave train with two dominant waves: carrier and lower side band. Frequency downshift here is not clearly seen. Two times stronger current case with $U_0 = 5 \cdot 10^{-4}$ is presented in Fig. 3(b). We note some tendency to final energy downshift to the lower side band. Really strong permanent downshift with total domination of the lower side band is seen for two times more strong current $U_0 = 10^{-3}$.

One more example of wave modulation with not symmetrical behavior we present at Figure 1e. (Here is also answer to one of the comments of the 2nd referee) The modulation equations permit symmetrical solutions for the symmetrical current function, but, of course, here is no any special obligations. Outside of interaction zone we can have different kinds of nonlinear periodic waves depending from the boundary conditions and constant Stokes wave is only one of the possibilities. The symmetrical behavior is typical for a sufficiently long scale current. We add one more Figure 1.e with the same wave initial characteristics as for Figure 1c and two time's shorter space scale of the current. After interaction zone we see three wave system with comparable amplitudes and periodic energy transfers.

To give an idea about the strength of nonlinearity we present one more Figure (1i) with distortion of the linear dispersion relation for different modes. As one can see the effect of nonlinearity for the carrier (at maximum is about 10%) is much less compare to side bands (at peak is more than 30 %). The main impact of nonlinearity comes from amplitude Stokes dispersion.

The answers to comments of 2nd, 3rd and 4th referees are coming soon.

