

General reply

We thank the reviewer for pointing out an amazing series of papers by Krauskopf, Osinga, Doedel and others, which we were unfamiliar with before. Indeed, the transient behavior we observed in the Lorenz model is apparently related to the beautiful Lorenz manifold (in addition to the reference [2] in the reviewer's comments, see below references [1] for the most recent qualitative paper and [2] for a more mathematical exposition of the global invariant manifolds' significance for the persistent transients in the Lorenz model). If anything, our present discussion of the transients in the Lorenz model (which the reviews and replies are an integral part of) can serve as an introduction to the subject of these papers, which have thus far remained in "the dark", with the purpose of exposing them to a more 'applied' nonlinear geosciences community. We still maintain that inasmuch as the Lorenz model is a metaphor of reality, it is also the one of potential transient complexity in the real world.

Reply to minor comments

- (1) We've chosen the region B from the ranges of a long numerical simulation of the Lorenz model. The trajectories are scarce in the vicinity of the origin (due to a strong flow expansion along the unstable eigenvector there); therefore, our results for the transient flow are insensitive to exclusion of the small additional volume containing such trajectories. To confirm this, we repeated all the numerical experiments by redefining the region B using the ranges from the simulation initialized very close to the origin, which extends the z -range to $[0, 48.5]$ and slightly expands x - and y -ranges too. The results obtained with the new definition of region B are essentially identical to the results reported in the original manuscript.
- (2) We used trajectory-averaged values of local Lyapunov exponents (perhaps not the best term to describe eigenvalues of the local stability matrix, but the one which has been used before) to simply tag trajectories that are likely to exhibit strong flow expansion in their vicinity. Using the finite-time Lyapunov exponents (by estimating eigenvalues of the strain tensor $J^T J$, where J is the finite-time Jacobian matrix whose evolution satisfies the tangent linear equations) produces essentially the same results (**Fig. R1**). **Figure R2** provides another example of trajectories initialized in the vicinity of a trajectory characterized by a large value of the average (transient) local (or finite-time) Lyapunov exponents, with the ghost of the Lorenz manifold lurking inside!

References

[1] https://www.math.auckland.ac.nz/~berndk/transfer/ko_bridges2014.pdf

[2] E. J. Doedel, B. Krauskopf and H. M. Osinga. Global invariant manifolds in the transition to preturbulence in the Lorenz system. *Indagationes Mathematicae*, 22(3-4): 223–241, 2011.

Figures

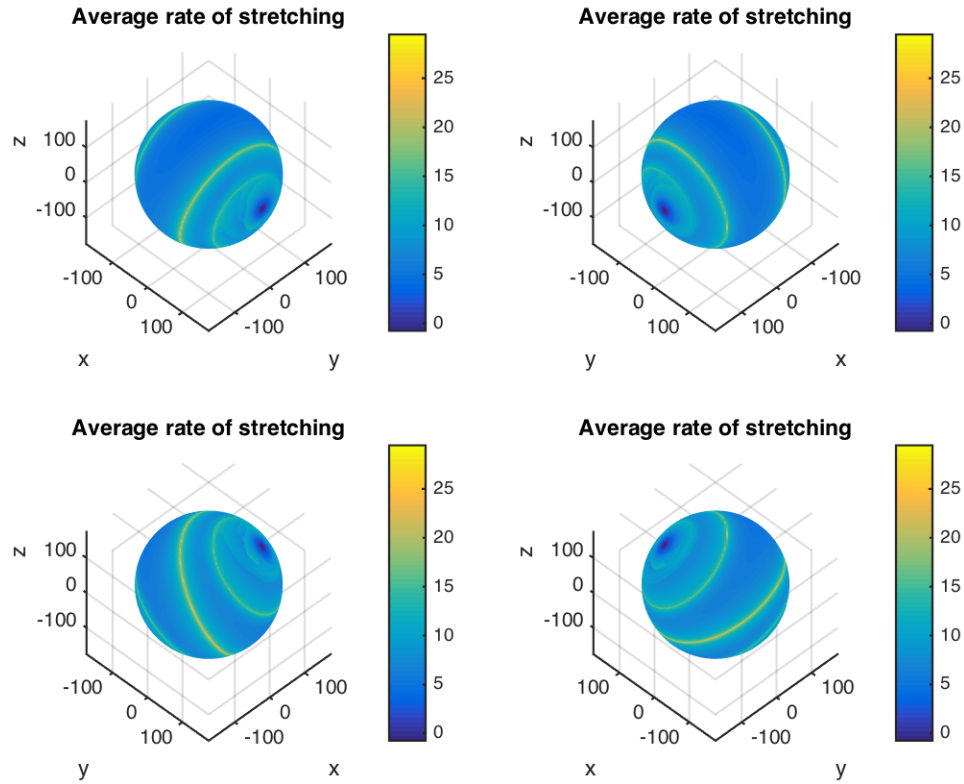


Figure R1: Distribution of the averaged finite-time Lyapunov exponents computed over the transient portion of trajectories (before first entry into the attractor region B) initialized on the same sphere S as in Fig. 1 of the main text. *Comment: Essentially the same patterns as for the trajectory-averaged local Lyapunov exponents (Fig. 2 of the main text).*

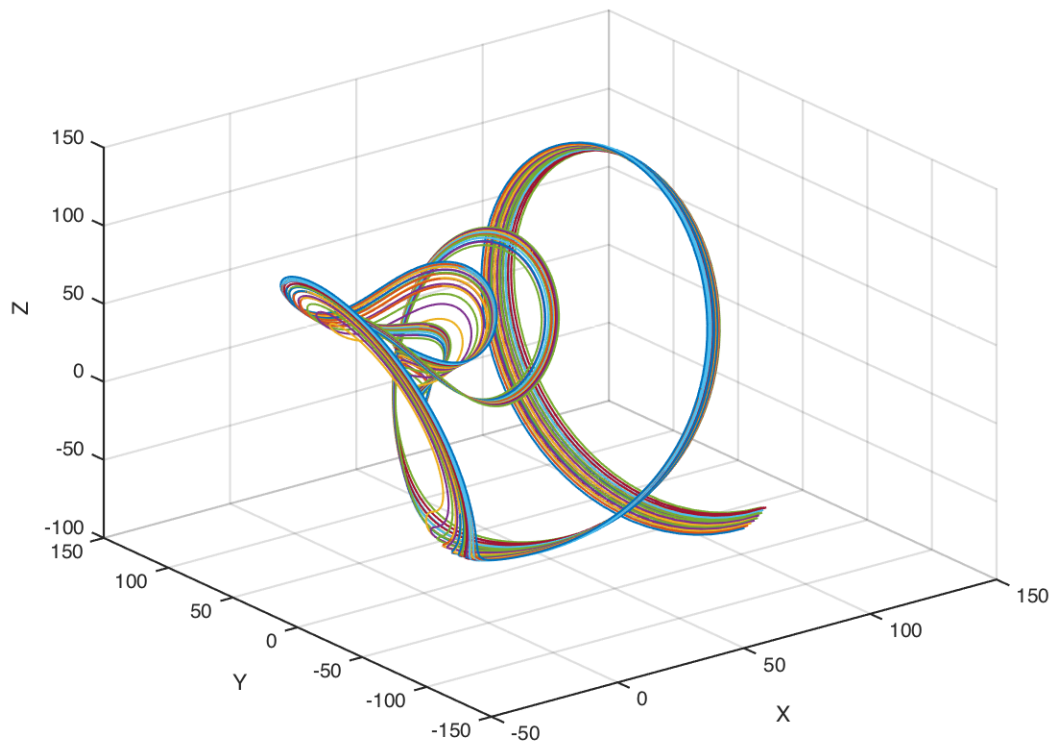


Figure R2: Another example of transient trajectories sensitive to initial conditions (see also Fig. 3 of the main text). Comment: *This is a typical situation for initial conditions taken in and around the spiraling belts of longest transient times (Fig. 1) and highest averaged Lyapunov exponents (Fig. 2, Fig. R1).*