Reply to referee #1

Thanks for your quick reply and advice.

About your concern on the reason why the envelope method can be a better performer than standard EFWI, we have the explanation in our paper. In the paper, we derive an approximate formula to express the multi-component data (Page 1773, Eq. (A10)).

$$u_i\left(t,\boldsymbol{x}_r;\boldsymbol{x}_s\right)=G_{ik}^{f}\left(t;\boldsymbol{x}_r,\boldsymbol{x}_s\right)s_k\left(t\right)+\mathbf{G}_{ik}^{sr}\left(t;\boldsymbol{x}_r,\boldsymbol{x}_s\right)\boldsymbol{\gamma}_k\left(t\right) \tag{A10}$$

From the formula, we can see that the multi-component data actually contains the low frequency information related to long-wavelength of background model (the two Green functions). But the multi-component data is approximately equal to the modulated result in which the modulation signal are the two Green functions and the carrier signal are the source or reflector coefficients series. And the spectrum of multi-component data is mainly determined by the wavelet or reflector coefficients series (illustrated by figure 2~7 in Page 1778~1783 in our paper). So the low frequency information being relevant to long-wavelength component of model cannot be recognized directly from the waveform data according to the modulation theory. **Hence, we use envelope operator to demodulate the data (see Eq. (10) in Page 1767) and retrieve those low frequency information to recover the background model.** We do not mean that the envelope operator can **create** long-wavelength component information of the subsurface model which do not exist in the data. **What the envelope operator does is to retrieve these low frequency information modulated in the multi-component data.** With the benefit of such low frequency information related to long-wavelength component of subsurface model, the envelope method may avoid the cycle skipping and hence to reduce the nonlinearity of the inversion (Figure 11 and 12 of our paper in page 1787 and 1788 show the nonlinearity of waveform and envelope misfit functions). And that also explains the phenomenon appearing in our numerical tests that the success of envelope method also requires the seismic data containing threshold offset and threshold low frequency. If the multi-component data do not contain the relevant information or these information are damaged, the envelope operator cannot create such information to satisfy the requirement of a successful inversion.

So why the envelope method can behave better than conventional EFWI? The reason is that the envelope operator can demodulate the multi-component data and retrieve the modulated low frequency information hidden in the waveform data.

A simple example can illustrate how the modulation procedure works. We assume a modulation signal as $(1+\cos\pi t)$, and a carrier signal as $\cos 8\pi t$, then the modulated signal is $(1+\cos\pi t)\cos 8\pi t$ and their shape and spectrum are shown in figure 1 below. In figure 1, we can see that though the frequency of modulation signal is 0.5Hz, after modulated with carrier signal whose frequency is 4Hz, such low frequency information of 0.5Hz does not appear in the spectrum of the modulated signal. The spectrum of the modulated signal is around 4Hz. But the envelope of the modulated signal extracts the frequency information of 0.5Hz, which means that envelope operator can demodulate the modulated signal.

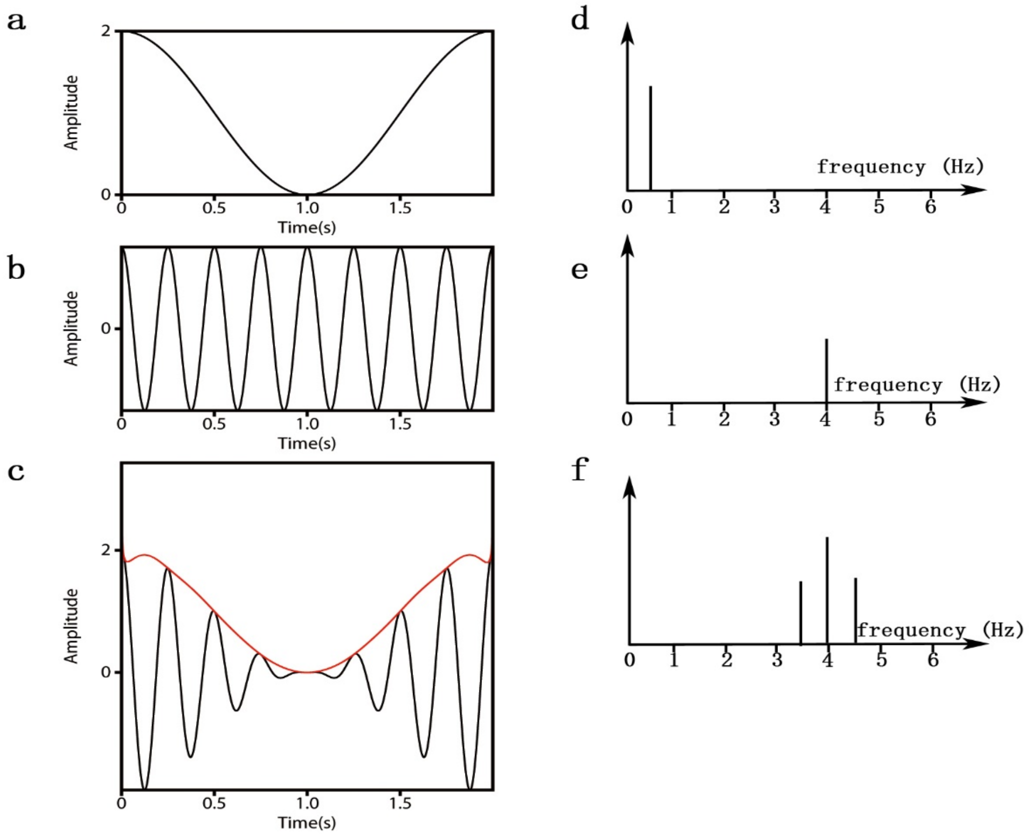

Figure 1 (a) modulation signal, (b) carrier signal, (c) modulated signal (black line) and its envelope (red line), (d) the spectrum of modulation signal, (e) the spectrum of carrier signal, (f) the spectrum of modulated signal

You also mentioned about the multiscale strategy in your reply. It is true that our two-step EFWI is a kind of multiscale method (Page 1771 Line 10). Multiscale strategy is a natural idea to handle the strong nonlinearity of seismic inversion. And so far, most of full waveform inversion strategy and methods also belong to multiscale strategy. For example, the first arrival FWI (Luo and Schuster,1991; Sheng and Schuster,1996), the Laplace FWI (Shin and Cha,2008), temporal windowing method (Shipp and Singh,2002; Freudenreich et al., 2001), layer-stripping method (Wang and Rao, 2009), multiscale strategy in frequency domain (Bunks et al., 1995; Pratt et al., 1996, 1998; Sirgue and Pratt, 2004; Fichtner and Tranpert, 2011; Baeten et al., 2013), integration method (Chauris et al., 2012) and so on. The basic idea of multiscale is to separate the complex inversion problem into different sub-parts to reduce the nonlinearity. And most of the separation is linear. But envelope method is a nonlinear multiscale method. Envelope EFWI is different from early arrival waveform inversion which only uses the early arrivals. The first difference between envelope method and early arrival inversion is that the envelope transform doesn't change the amplitude ratio of different arrivals and all arrivals are used simultaneously during the inversion. (Shown in figure 2 below). The second one is that without long offset data, the early arrival inversion cannot invert the deep part of the subsurface model. However, envelope method can recover the deep part of background model to a certain degree (see figure 16 and figure 22 of Page 1792 and 1798 in our manuscript). So our paper presents a solution to such situation when the long offset is not available and the spectrum of waveform data is lack of low frequency information. According to your suggestions in the comments, we do some numerical tests to compare the early arrival inversion method, the frequency domain multiscale method and our two-step method. And the results further prove that our two-step method is superior to both multiscale methods we used in these tests.

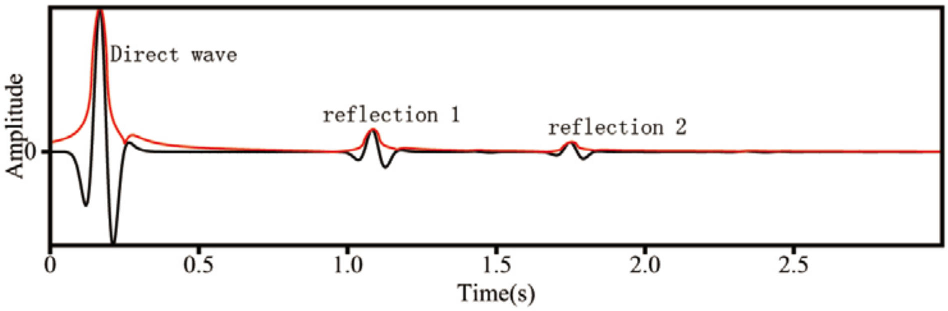

Figure 2 A seismic single (black line) and its envelope (red line)

In our numerical tests, we still use the well log velocity model we used in the last numerical tests. And all the parameters about the survey system remain the same as the last tests.

(The velocity model is shown in figure 3, and still we assume the density is known. To simulate synthetic data on the simple model, 80 explosive sources along the surface are inspired with the interval of 100m. A Ricker wavelet with the dominant frequency of 7Hz is used as the source function. Here we apply a roll in/out survey system. For every shot, 401 two component receivers are spaced every 10m along the top surface and the source is in the center of the receiver line. A homogenous models with Vp 1.8km/s and Vs 1.03Km/s are taken as the starting models in our tests.)

Firstly, we compare the inversion results of frequency domain multiscale method. For the frequency domain multiscale method test, we adopt the strategy presented by Brossier et al. (2009) and begin our inversion from the lowest frequency of 1.7Hz, then gradually increase to 10Hz, the results of Vp and Vs are shown in figure 4. From the comparison we clearly see that the frequency domain multiscale method failed to invert acceptable models in our test while two-step behaves much better.

Secondly, we compare the early arrival waveform inversion method with our two-step method. We use the temporal windowing strategy presented by Shipp and Singh (2002) to isolate the early arrivals from the data and do the inversion with these early arrivals firstly, then gradually open up the time window to use more arrivals. The final inversion results are shown in figure 5. With the comparison, we can clearly find that the early arrival waveform inversion method is different from our two-step method and the latter performances much better in this test. The reason why the above two multiscale methods do not work very well may be that the two methods are linear multiscale and they cannot retrieve the long-wavelength component information modulated in the data, while the envelope method can.

Thanks again!

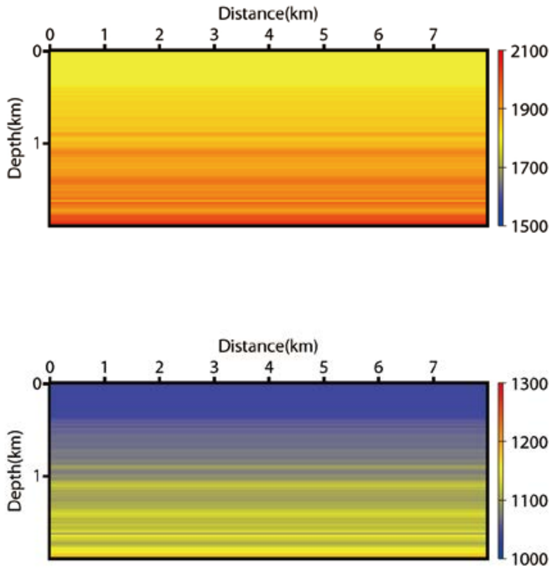

Figure 3 VP (top) and VS (bottom) model

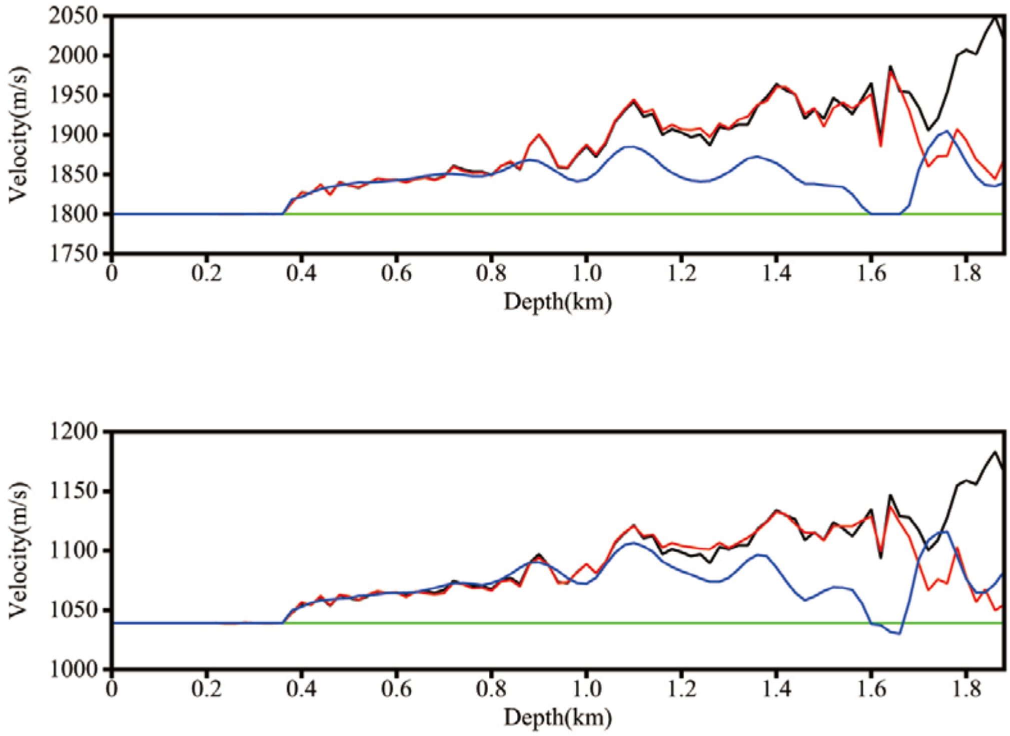

Figure 4 Inversion results of Vp (top) and Vs (bottom) of frequency domain multiscale method and two-step method. (Black line： true model; green line: initial model; red line: our two-step inversion results; blue line: frequency domain multiscale EFWI results)

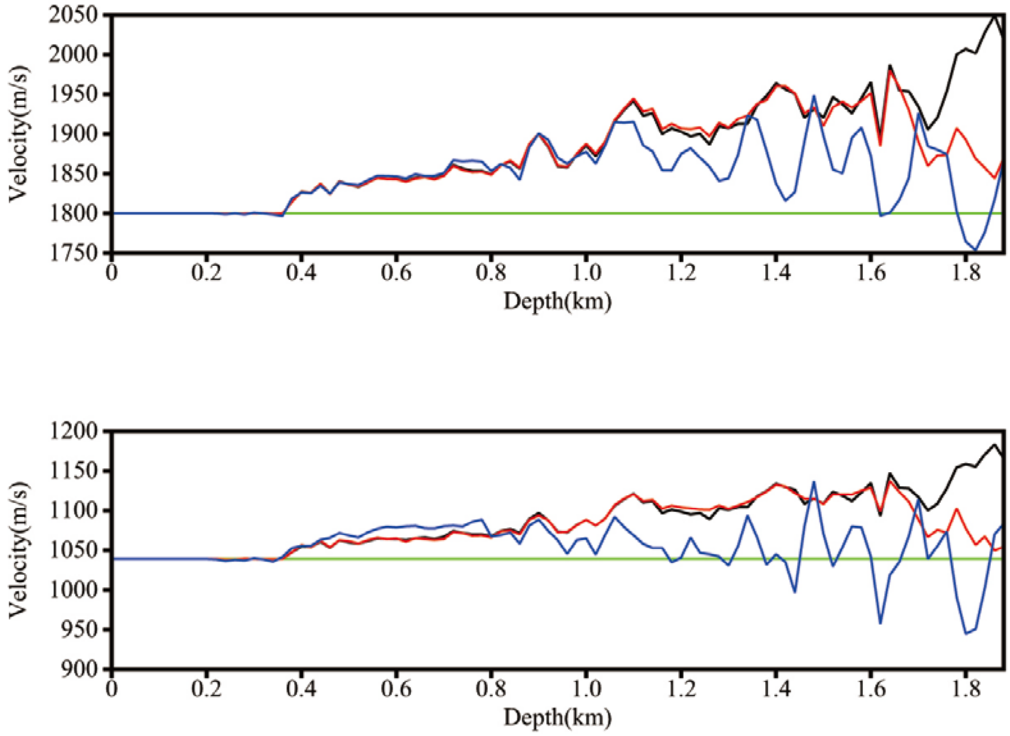

Figure 5 Inversion results of Vp (top) and Vs (bottom) of early arrival inversion method and two-step method. (Black line： true model; green line: initial model; red line: two-step inversion results; blue line: early arrival EFWI results)

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