Thanks for the comments presented by referee #1 and #2. In the comments of referee #1, the referee concerned about the contribution of long offset and low frequency in recovering the long-wavelength component of the subsurface model. The referee #2 kindly suggested us to do more research on the limitations of the envelope method. And thanks to their good advice, we have done a series of numerical tests on the limitations of offset and threshold low frequency for our two-step inversion method. Although all these discussions are model-dependent, they are still meaningful to our inversion problem. And I also hope the referees can give us more constructive advice on our tests to help us improve our paper.

And in the following part, we will present two type of tests to discuss the limitations of our two-step method on the offset and threshold low frequency issues. First, we will vary the maximum offset in the synthetic data and discuss how long the maximum offset is needed to well invert the subsurface model. Then secondly, we will fix the maximum offset at the threshold offset, and apply different high-pass filter to the synthetic data. By filtering out the data below different low frequency, we do our two-step inversion to find the threshold low frequency can be missed in the data.

In the numerical tests, we use a real well log velocity model and expend it into 2D model. The reason we choose such simple model is that the layer model is convenient for us to consider about the diffraction or aperture angle. The velocity model is shown in figure 1, and still we assume the density is known. To simulate synthetic data on the simple model, 80 explosive sources along the surface are inspired with the interval of 100m. A Ricker wavelet with the dominant frequency of 7Hz is used as the source function. Here we apply a roll in/out survey system. For every shot, fixed number of two component receivers are spaced every 10m along the top surface and the source is in the center of the receiver line. A homogenous models with Vp 1.8km/s and Vs 1.03Km/s are taken as the starting models in our tests. We apply two types of numerical experiments in our test.

Test one:

In the first type of test, we discuss the relationship between the effectiveness of inversion method and different maximum offset data. In this test, we test the maximum offset of 3.0km, 2.5km, 2.0km, 1.5km, 1km and 0.5km respectively.

Using the data with different maximum offset, we do both conventional EFWI and our two-step inversion. And to compare with the results clearly, we extract Vp and Vs vertical slices at position of 4km from both inversion results. Figure 2 to 7 show the inversion results with different maximum offset. We calculate the RMS velocity error for each inversion with different offset and plot it in figure 8.

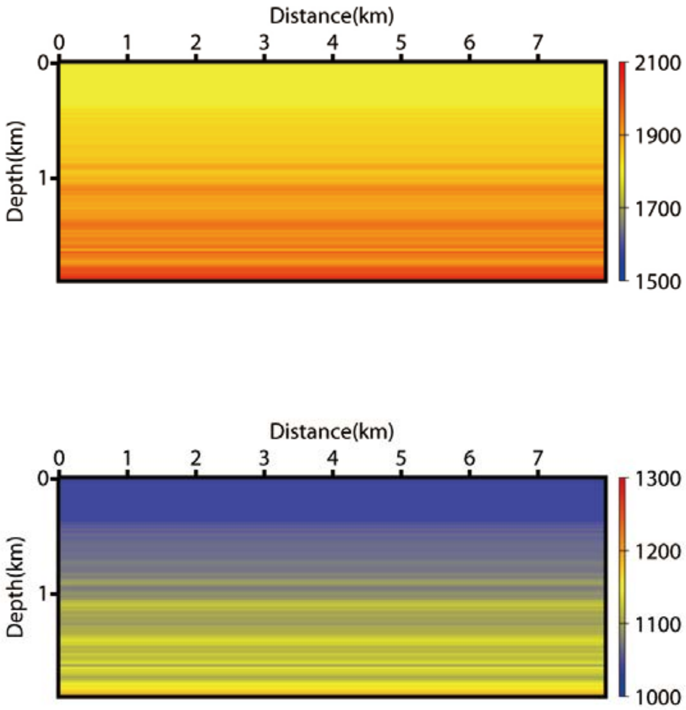

Figure 1 VP (top) and VS (bottom) model

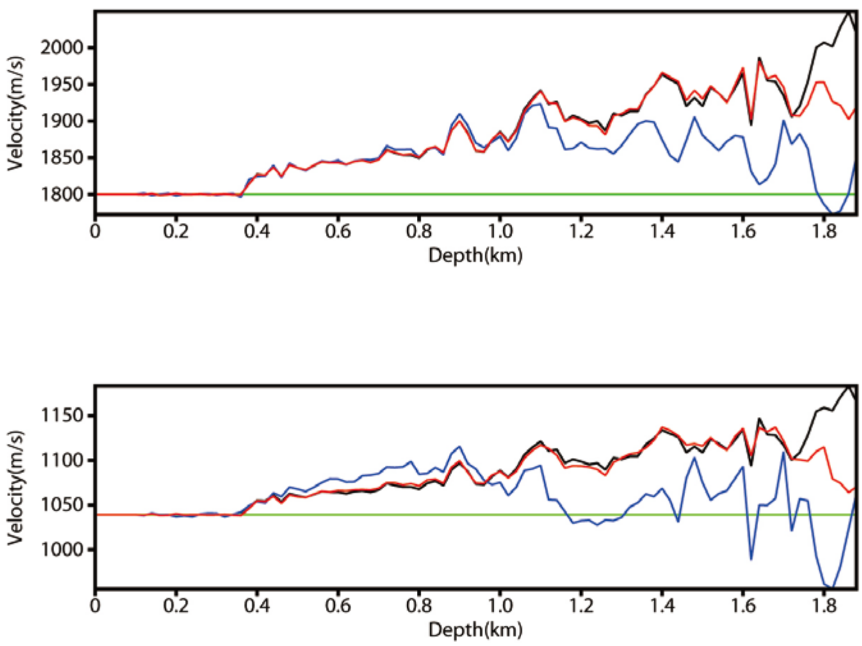

Figure 2 Inversion results of Vp (top) and Vs (bottom) with the maximum offset of 3.0km. (Black line： true model, green line: initial model, red line: two-step inversion results, blue line: conventional EFWI results)

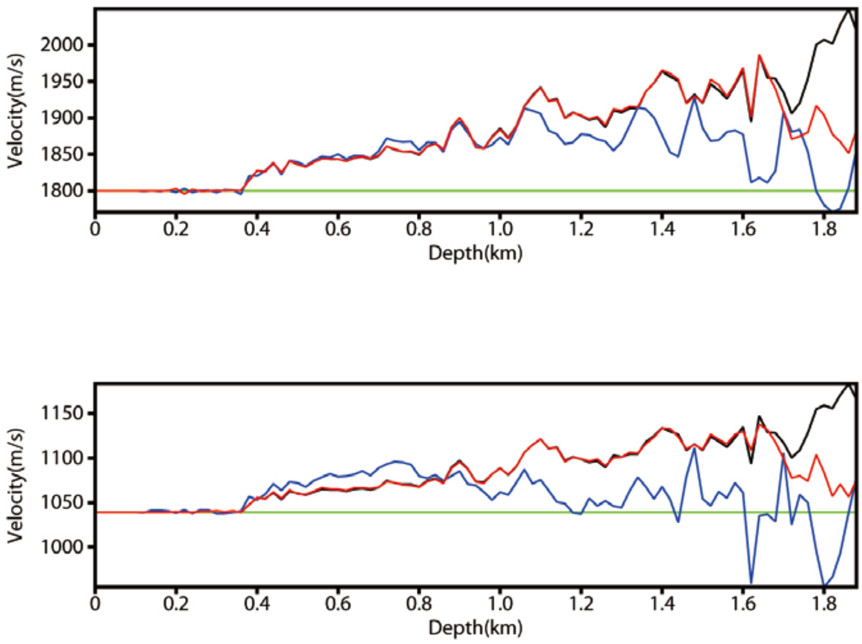

Figure 3 Inversion results of Vp (top) and Vs (bottom) with the maximum offset of 2.5km. (Black line： true model, green line: initial model, red line: two-step inversion results, blue line: conventional EFWI results)

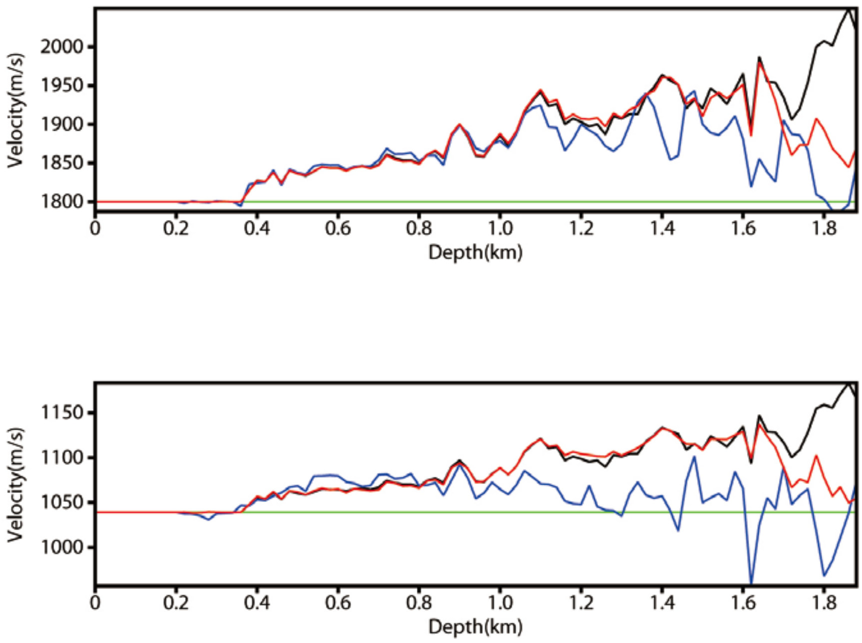

Figure 4 Inversion results of Vp (top) and Vs (bottom) with the maximum offset of 2.0km. (Black line： true model, green line: initial model, red line: two-step inversion results, blue line: conventional EFWI results)

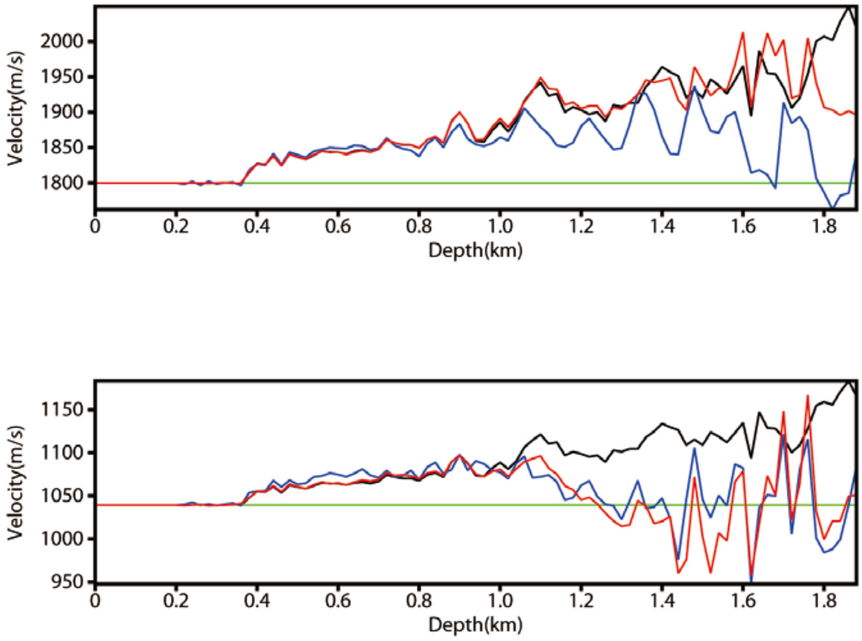

Figure 5 Inversion results of Vp (top) and Vs (bottom) with the maximum offset of 1.5km. (Black line： true model, green line: initial model, red line: two-step inversion results, blue line: conventional EFWI results)

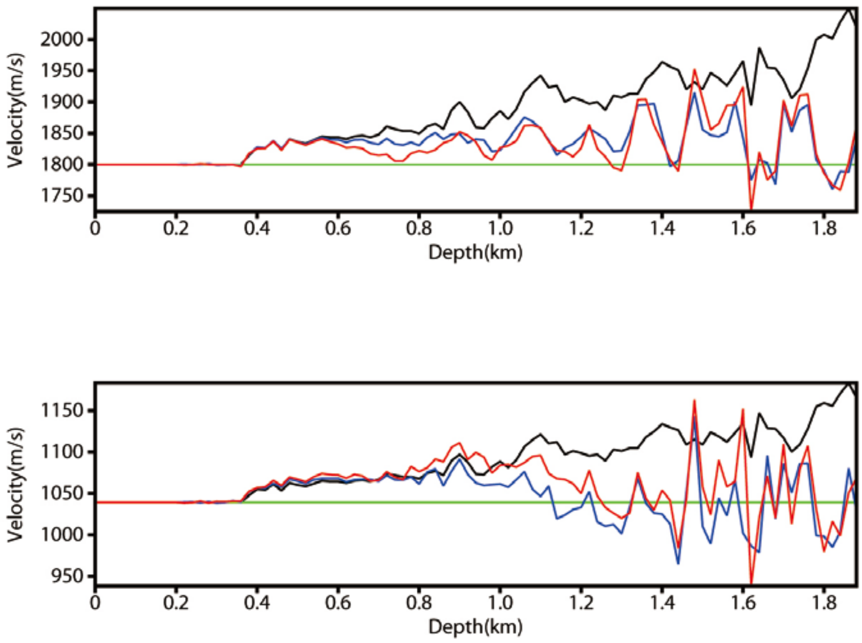

Figure 6 Inversion results of Vp (top) and Vs (bottom) with the maximum offset of 1.0km. (Black line： true model, green line: initial model, red line: two-step inversion results, blue line: conventional EFWI results)

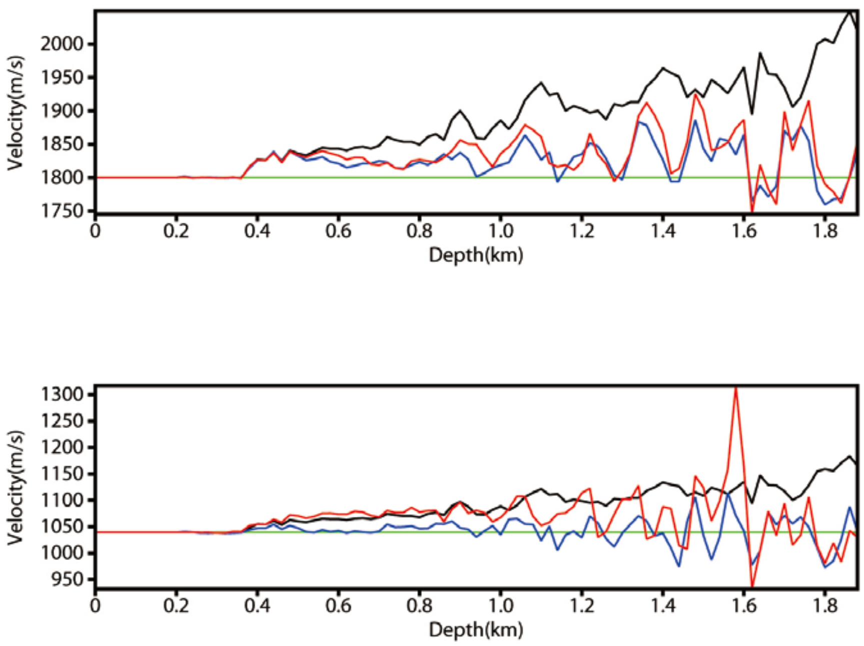

Figure 7 Inversion results of Vp (top) and Vs (bottom) with the maximum offset of 0.5km. (Black line： true model, green line: initial model, red line: two-step inversion results, blue line: conventional EFWI results)

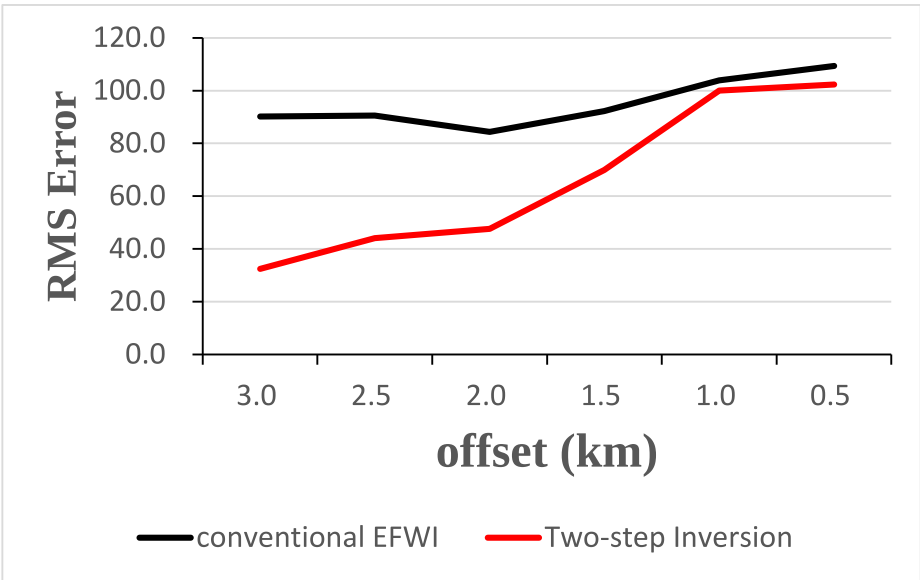

Figure 8 RMS velocity error with different maximum offset data

From the analysis of these inversion results, we see that, to such velocity model, the variability of maximum offset data can affect the conventional EFWI. The shorter the offset, the worse the inversion results. But to our two-step inversion, when the offset changing from 3.0km to 2.0 km, the final results remain almost the same. When we shorten the offset to 1.5km, it obviously affects the Vs inversion, but the result of Vp remains acceptable. If we further shorten the offset, the inversion results are getting worse. The reason for such phenomenon is that the shorter the offset, the less reflection single we can acquire. And due to low Vs velocity, it requires longer offset to collect the reflected transmitted PS wave. However, comparing the RMS velocity error, our two-step inversion behaves always well than conventional EFWI in our test.

Using a homogenous media as the starting model, the conventional EFWI suffered from cycle skipping problem more easily, no matter what offset data is used. If we add some priori information into the starting model, for example, using a gradient model as the stating model and using the data with maximum offset of 3.0km, then the inversion results can be improved obviously, as shown in Figure 9. It further proves that with our two-step method, the starting model dependence can be reduced.

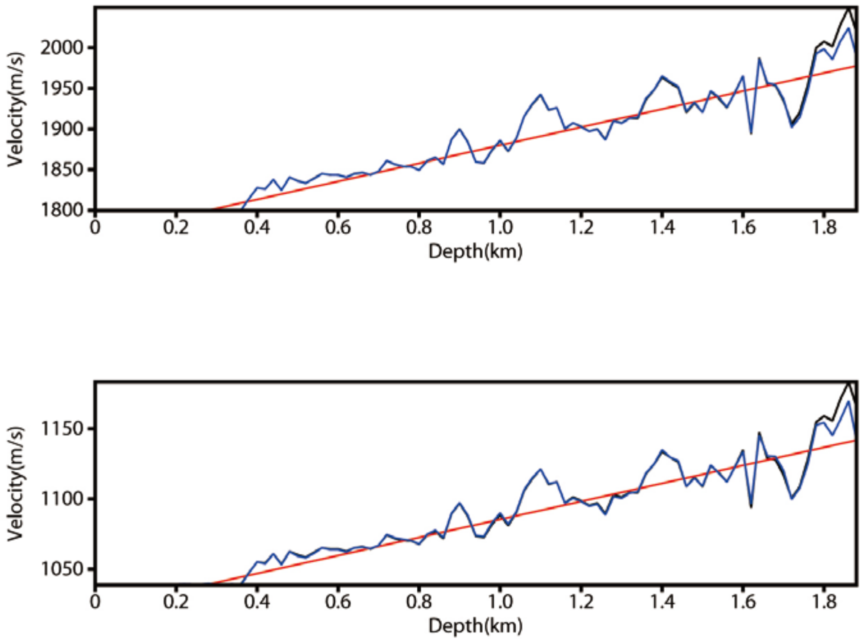

Figure 9 Conventional EFWI results of Vp (top) and Vs (bottom) using gradient starting model and the maximum offset of 3km (Black line： true model, red line: initial model, blue line: conventional EFWI result)

Test two:

In the second type of test, we aim to test the threshold low frequency that can be filtered out from the data. In this test, we fix our maximum offset with 2.0km and filtered out the data below 2HZ, 3Hz, 4Hz, 5Hz, 6Hz, 7Hz, 8Hz, 9Hz, respectively. The starting model is a homogenous model the same as used in first type of test.

Using the data with different low frequency filtered out, we apply our two-step method to do the inversion. And to clearly show how the low frequencies affect the inversion results, we compare with the results of filtered data and original data. We extract Vp and Vs vertical slices at position of 4km from both inversion results, shown in figure 10 to 17.

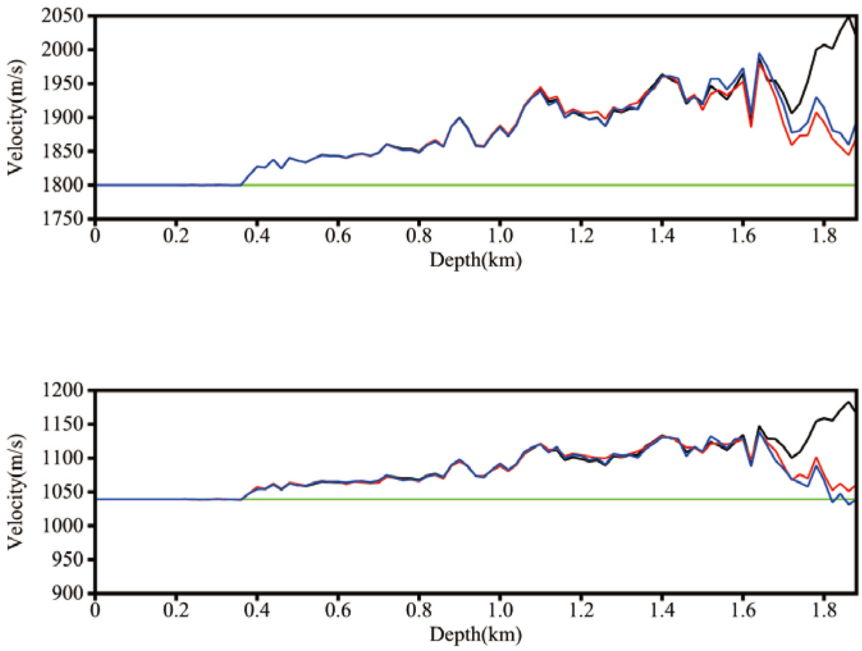

Figure 10 Two-step inversion results of Vp (top) and Vs (bottom) with original data and data filtered out below 2Hz (Black line： true model, green line: initial model, red line: result with original data, blue line: result with filtered data)

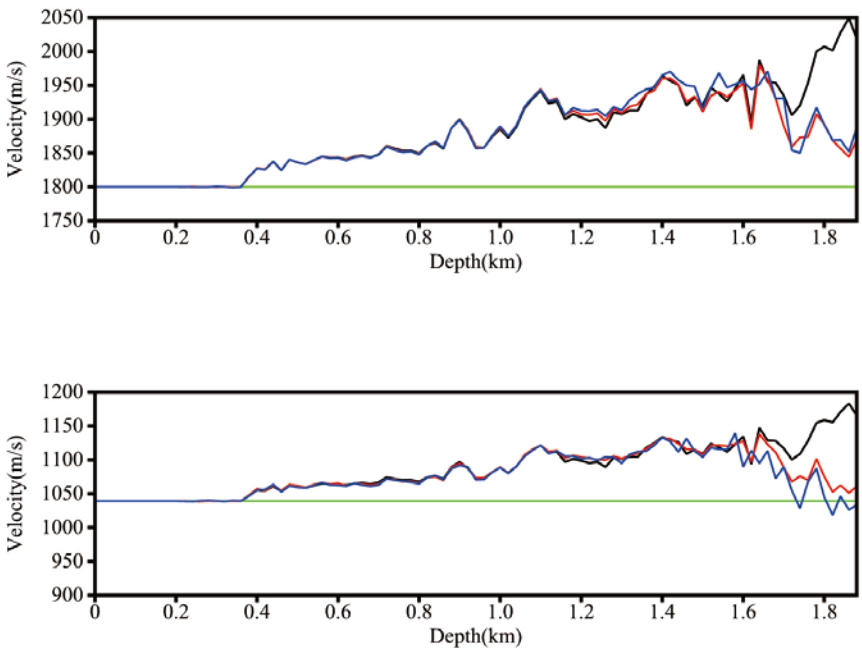

Figure 11 Two-step inversion results of Vp (top) and Vs (bottom) with original data and data filtered out below 3Hz (Black line： true model, green line: initial model, red line: result with original data, blue line: result with filtered data)

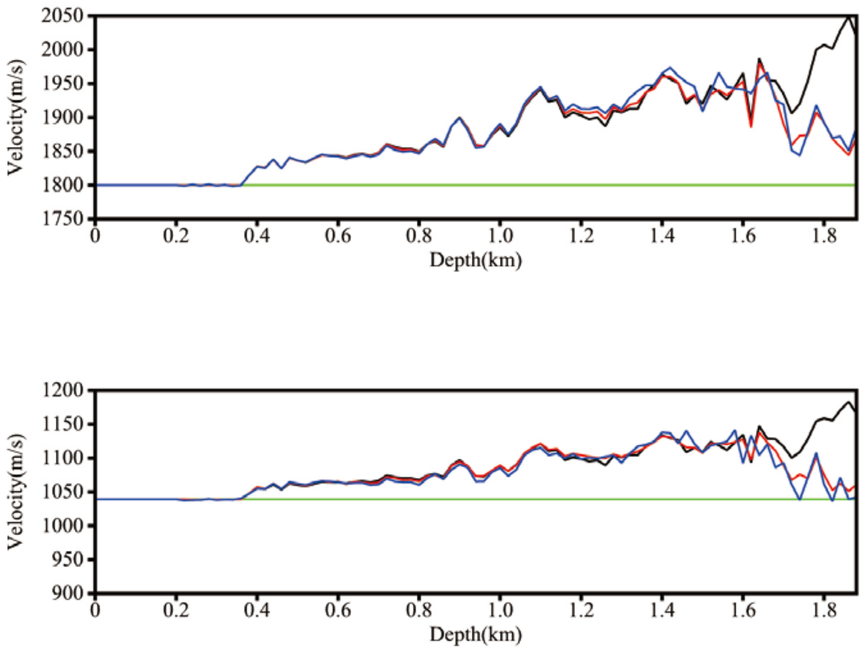

Figure 12 Two-step inversion results of Vp (top) and Vs (bottom) with original data and data filtered out below **4Hz** (Black line： true model, green line: initial model, red line: result with original data, blue line: result with filtered data)

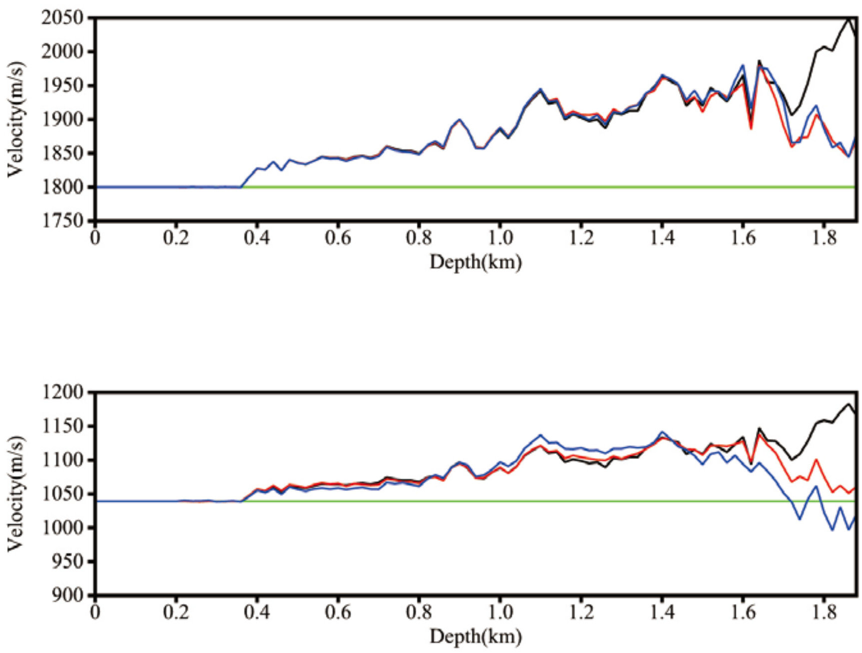

Figure 13 Two-step inversion results of Vp (top) and Vs (bottom) with original data and data filtered out below **5Hz** (Black line： true model, green line: initial model, red line: result with original data, blue line: result with filtered data)

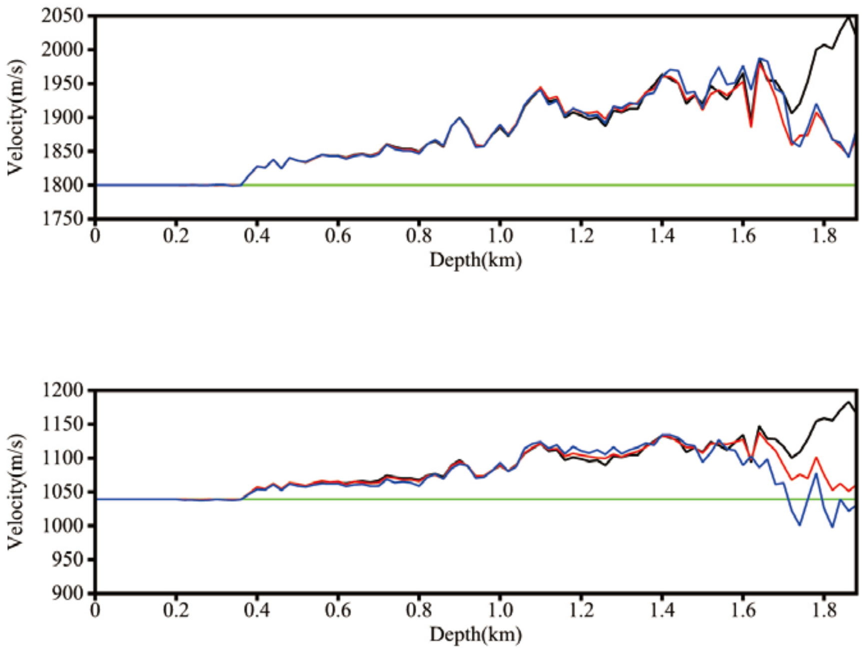

Figure 14 Two-step inversion results of Vp (top) and Vs (bottom) with original data and data filtered out below **6Hz** (Black line： true model, green line: initial model, red line: result with original data, blue line: result with filtered data)

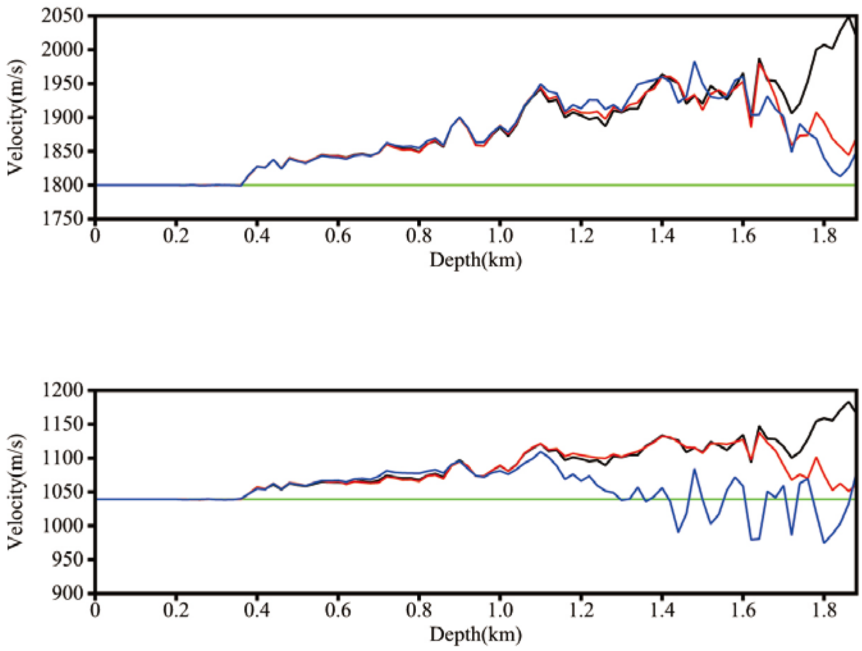

Figure 15 Two-step inversion results of Vp (top) and Vs (bottom) with original data and data filtered out below **7Hz** (Black line： true model, green line: initial model, red line: result with original data, blue line: result with filtered data)

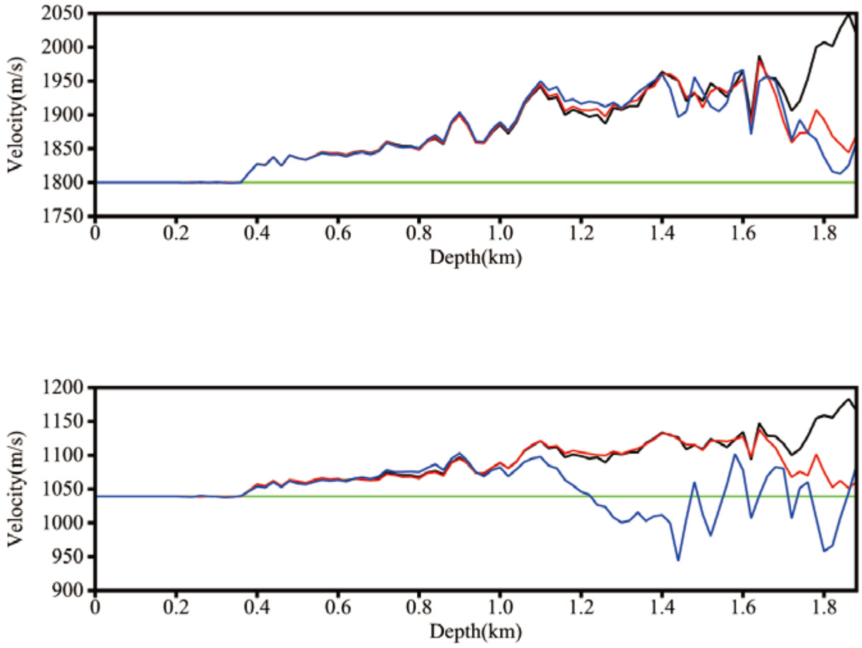

Figure 16 Two-step inversion results of Vp (top) and Vs (bottom) with original data and data filtered out below **8Hz** (Black line： true model, green line: initial model, red line: result with original data, blue line: result with filtered data)

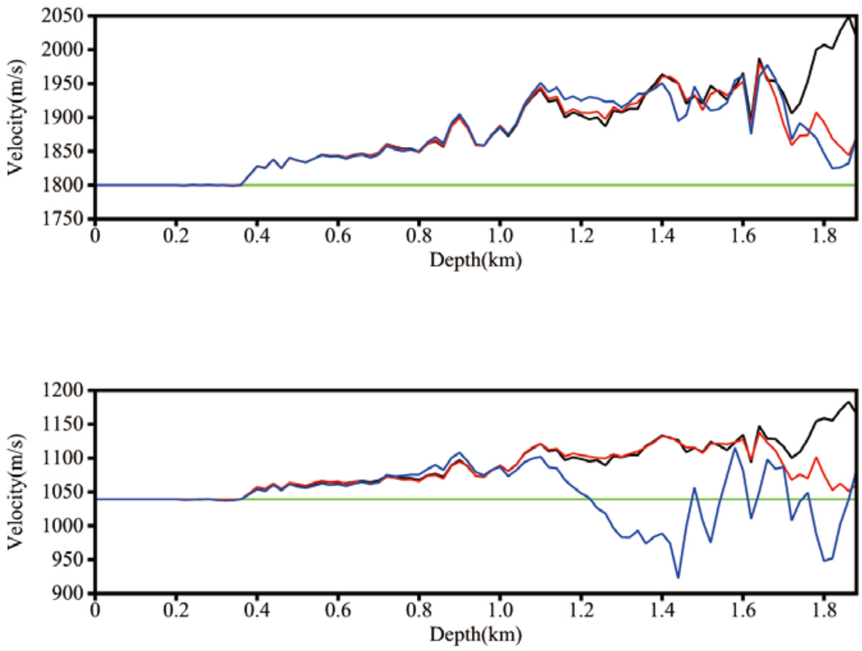

Figure 17 Two-step inversion results of Vp (top) and Vs (bottom) with original data and data filtered out below **9Hz** (Black line： true model, green line: initial model, red line: result with original data, blue line: result with filtered data)

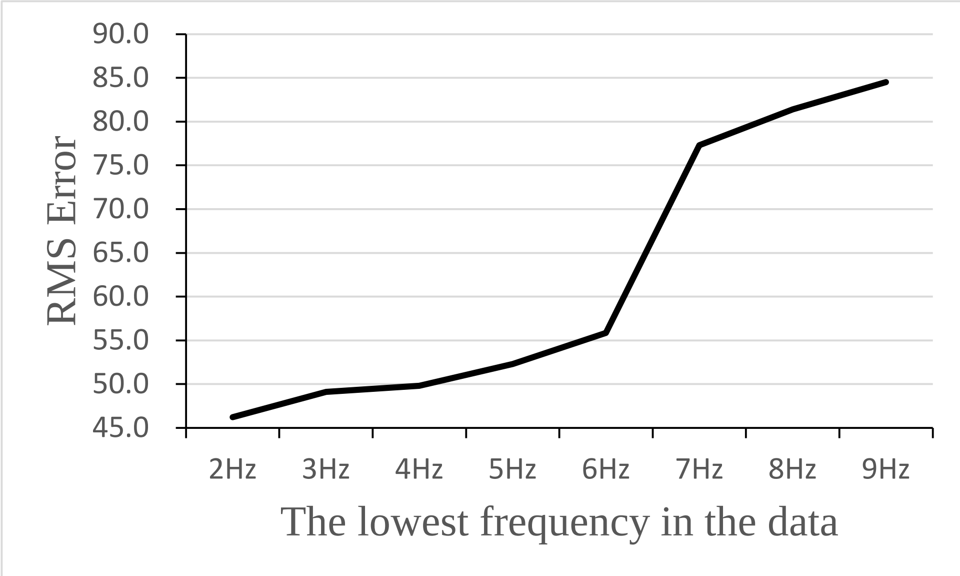

Figure 18 RMS velocity error with different filtered data

By comparing with the results, we notice that, to this testing model, the elastic envelope inversion has a threshold frequency of 7Hz. If the data contain low frequency lower than 7Hz, our two-step inversion can achieve acceptable results. And if the low frequency below 7Hz of the data are missing, the inversion result of Vs becomes worse. However, even the data below 9Hz is filtered out, the inversion result of Vp is still acceptable, which means that Vs inversion is more sensitive to the low frequency contained in the data. But for conventional EFWI, if we filtered out the low frequency, the inversion results become worse than those with original data, and we did not show them here. In this type of test, the threshold low frequency reveals the limitation of our two-step inversion method. But compared with conventional EFWI, it still behaves better than conventional EFWI when the missing low frequency did not reach the threshold frequency.

In the two type of numerical tests, we show the threshold offset and threshold low frequency of our two-step method. But comparing with the conventional EFWI, the threshold offset is largely reduced and the threshold low frequency is significantly increased by our two-step inversion method.