Interactive comment on "Estimation of the total magnetization direction of approximately spherical bodies" *by* V. C. Oliveira Jr. et al.

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We would like to thank Referee J. Ebbing for his constructive and insightful comments. Below we present our comments and responses to his recommendations. We have performed several new tests on synthetic data that we hope will answer all of the points raised by the Referee. The results, figures, and source code for these tests are available online through the code hosting website Github at github.com/birocoles/Total-magnetization-of-spherical-bodies. Links to each specific synthetic test are provided in the relevant comments below.

General comments

Referee's comment: *"First, the magnetization direction of the spherical body is inverted and afterwards the magnetization of the prism to study the error introduced by a non-spherical geometry. But at the same time the inclination and declination are changed, so that no direct comparison with the inversion for the spherical body is possible. I would suggest inverting first for the same parameters, but by only changing geometry and in the second step changing inclination and declination more drastically compared to the applied inducing field. If the method is supposed to be able to resolve remanent magnetization, it would be interesting to see how the method performs for anomalies with reversed magnetization."*

Thank you very much for this suggestion. We have made a new test on synthetic data in which we applied our method to a sphere and a prism with the same magnetization. The results, figures, and numerical code used to produce these results can be found online in the IPython notebook (an interactive writing and programing environment) synthetic_tests_sphere_prism.ipynb.

The sphere has a radius $R = 2000\,m$ and the cube has a side length $R = 2000\,m$. The centers of these two bodies are located at the same Cartesian coordinates $x_0 = 0\,m$, $y_0 = 0\,m$ and $z_0 = 2000\,m$. They also have the same magnetization vector, with inclination $-9.5°$, declination $-167°$ and intensity $3.5\,A/m$. The simulated geomagnetic field has inclination $9.5°$ and declination $13°$. Note that both bodies have reversed magnetization, following the suggestion of the reviewer. The total-field anomaly produced by these bodies were calculated on the same regular grid with constant vertical coordinate $z = -150\,m$. These data were corrupted with a pseudo-random Gaussian noise of null mean and standard deviation $5\,nT$.

Applying our method to the sphere, we obtained the estimated inclinations $\hat{I} = -9.49770° \pm 0.00036°$ and $\tilde{I} = -9.50764° \pm 0.01022°$ (the caret "^" and tilde "~" denote the results computed by using, respectively, the least-squares and robust estimates) and declinations $\hat{D} = -167.01021° \pm 0.00069°$ and $\tilde{D} = -166.98518° \pm 0.07527°$. In

the case of the synthetic data produced by the cube, we obtained the estimated inclinations $\hat{I} = -9.58948° \pm 0.00026°$ and $\tilde{I} = -8.86599° \pm 0.00876°$ and declinations $\hat{D} = -164.57023° \pm 0.00049°$ and $\tilde{D} = -167.34047° \pm 0.01028°$. The direct comparison between these results shows the robustness of our method in estimating the magnetization of a non-spherical source, even in the presence of reversed remanent magnetization.

In this test, we calculated the noise-corrupted total-field anomalies close to the sources. The total-field anomaly produced by the cube exhibits non-dipolar features being very different from the one produced by the sphere. As shown in the section 3.3 (Robustness against non-spherical sources) of our manuscript, these non-dipolar features are attenuated if the data are calculated or measured far from the sources. This attenuation is more noticeable if the sources possess symmetry around three orthogonal axis (like the cube presented here). In the section 3.3 of our manuscript, we present the effects of these two factors: (1) the distance between the data (the magnetometer) and the source and (2) the symmetry of the source. These effects are analyzed by applying our method to 33 different synthetic-data sets.

Referee's comment: *"All the inversions presented consider that the location of the source body is known."*

Section 3.4 (Robustness against errors in the centre location) of our manuscript shows how the errors in the coordinates of the centre of the source affect the results obtained with our method. In this section, we assume different locations of the centre of the simulated spherical source along three orthogonal straight lines which are parallel to the x, y and z axis and cross the true centre of the source. Along each line, we applied our method by considering that the centre of the source is erroneously located at 21 regularly spaced points, totalling 63 inversions obtained with the least-squares approach and 63 inversion obtained with the robust approach. The results obtained in

all these 126 inversions are shown in Fig. 7. According to these results, our method is more sensitive to uncertainties in the prior information about the horizontal coordinates of the centre of the source along the horizontal directions than about the depth of the centre of the source.

Referee's comment: *"If the position of the source is known for example from Euler Deconvolution, the estimate of the inclination and declination is almost trivial even by forward modelling."*

We respectfully disagree. Even if the interpreter knew the centre of the source, the estimation of the magnetization direction by using interactive forward modelling would be a very difficult task involving an exhaustive and time-consuming trial-and-error procedure to yield an acceptable data fit. On the other hand, any inversion method has the advantage, compared with forward modelling, of automatically fitting observations. The estimation of the magnetization direction of a 3D source might be easy if the source is symmetrical, with known shape and if there is no interfering anomalies. This is shown in the section 3.1 (Validation test) of our manuscript. On the other hand, as shown in the section 3.2 (Robustness against interfering anomalies), the presence of interfering anomalies can mislead the estimation of the magnetization direction even if the magnetic data are produced by simple sources with known centres. The estimation of the magnetization direction of 3D sources can also be difficult if the total-field anomaly displays strongly non-dipolar features, as illustrated by the Figures 5a-c of our manuscript.

Referee's comment: *"More interesting would be an example, where a regional field superposes the local anomaly or to some degree two anomalies overlap. Euler Deconvolution will provide results in both cases, but with less confidence in the horizontal position, which will affect the magnetization directions."*

Thank you for this very good suggestion. We have created a new test with synthetic

data from an igneous intrusion formed by a sill which is fed by a vertical pipe. This intrusion is embedded in weakly magnetized sediments that are overlaying a basement which is magnetized by induction, generating a regional anomaly. The results, figures, and numerical code used to produce these results can be found online in the IPython notebook complex_test.ipynb. The simulated geomagnetic field has inclination $-39.8°$ and declination $-22.5°$. The synthetic intrusion has a reversed magnetization with inclination $I = 39.8°$ and declination $D = 157.5°$. In this example, the total-field anomaly predicted by the intrusion overlaps the one produced by the basement. Our method is applied to the noise-corrupted total-field anomaly produced by both the intrusion and the basement on a regular grid with constant vertical coordinate. The position of the synthetic intrusion is estimated by Euler Deconvolution. The synthetic intrusion is not an ideal source and does not have a characteristic structural index. In this case, we presume that the noise-corrupted total-field anomaly is produced by an spherical body and use a structural index equal to 3. The estimated location of the body obtained by Euler Deconvolution is placed outside the synthetic intrusion. Even using this poor estimation of the location of the source, our method obtained the estimated inclinations $\hat{I} = 37.50377° \pm 0.00035°$ and $\tilde{I} = 40.25973° \pm 0.04392°$ and declinations $\hat{D} = 167.61518° \pm 0.00060°$ and $\tilde{D} = 164.58968° \pm 0.09092°$. The caret ($^\wedge$) and tilde ($^\sim$) denote the results computed by using, respectively, the least-squares and robust estimates. This numerical test shows the robustness of our method when applied to retrieve the magnetization direction of a complex source whose centre is poorly estimated by Euler Deconvolution. We also illustrate the use of the reduction to the pole to verify the quality of the estimated magnetization direction. The reduction to the pole calculated with the magnetization direction obtained by our method leads to a predominantly positive field, which is very close to the true pole field.

We have also run several additional tests showing the application of our method to estimate the magnetization direction of different synthetic sources with known and estimated centres (by using Euler Deconvolution). The figures, results, and source code of the additional tests obtained with the least-squares approach can be found in the

IPython notebook synthetic_tests-L2.ipynb and the results obtained with the robust approach can be found in synthetic_tests-L1.ipynb. One of these tests show the influence of a superposed constant-regional field (50 nT) on the estimated magnetization direction. The regional field does not lead to wrong estimates of the centres of the sources by Euler Deconvolution because, in this case, this technique estimates a non-null base level. On the other hand, this regional-constant field misleads the magnetization direction obtained by our method. To overcome this problem, a regional-residual separation should be done prior to estimation. Finally, these additional tests also show the performance of our method in estimating the magnetization direction of synthetic models similar to the ones presented by Lelièvre and Oldenburg (2009) and Ellis, Wet and Macleod (2012).

References

Lelièvre, P. G. and D. Oldenburg, 2009, A 3D total magnetization inversion applicable when significant, complicated remanence is present. Geophysics, 74(3), L21–L30, doi: 10.1190/1.3103249

Ellis, R. G., B. Wet and I. N. Macleod, 2012, Inversion of magnetic data from remanent and induced sources: 22nd International geophysical conferenceand exhibition, ASEG, Expanded Abrastracts, 1-4.

Specific comments

Referee's comment: *"Page 2 Sentence starting Line 19: These results show that the non-outcropping sources near from the alkaline complex of Diorama have almost the same magnetization direction of that as the ones in the alkaline complex of Montes Claros de Goiás, strongly suggesting that these sources have been emplaced in the crust almost within the same geological time interval."*

Thank you. We have modified the manuscript.

Referee's comment: *"Line 26: mineral and petroleum exploration"*

Thank you. We have modified the manuscript.

Referee's comment: *"Page 3 Line 4 Rephrase as: of the most important data acquisition techniques due to the ability to cover large areas in a relative short period of time"*

Thank you. We have rephrased the manuscript.

Referee's comment: *"Line 23: Delete this sentence: The total-field anomaly represents the Euclidean norm of the magnetic induction produced by the magnetic sources in the subsurface. Repetition from before."*

Thank you. We have removed this sentence.

Referee's comment: *"Paragraph starting in Line 28: This you can delete as it is not relevant here."*

We agree with you. We have rewritten this paragraph. Thank you.

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