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Interactive comment on “Precision variational approximations in statistical data assimilation” *by* J. Ye et al.

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We appreciate the comments from the referee. We address them below.

First, regarding the novelty of the work: we are puzzled by the referee's suggestion that these results are contained in our Reference 1 (referred to on p. 1605, Lines 6-7). A reading of that reference will show no discussion of the annealing method for tracking a minimum of the action. Indeed, we were unaware of the annealing method when that reference was published in June, 2013, and became aware of it a year later on reading the PhD dissertation of the last author, Quinn.

We believe the confusion may have arisen in that in the first reference there is a similar Taylor expansion to evaluate the effective action. That expansion, as in statistical field

theory and quantum field theory, is derived from a Legendre transform of the action with respect to the physical variables or fields. What results is an expansion that can be represented diagrammatically as loop corrections to the saddle path Xˆ0, with action value A_0(Xˆ0), to obtain a full effective action A. As in statistical field theory, knowledge of A presumes complete knowledge of the system. If model errors exist, whereby R_f is nonzero, then A is correct to all orders of R_f.

Here, on the other hand, we are not concerned with the full effective action A, and instead are investigating the fluctuations of the action of the stationary state of the path integral, A_0, with changes in the model error term R_f. The purpose is to systematically introduce model precision into the 4DVar method.

On the relationship to 4DVar the referee is quite right that we may not have explained that in enough detail. The first referee also pointed this out, and our response to that referee is on the discussion board, and we will rewrite the paper to expand on this. Basically at every stage of the annealing procedure we use 4DVar, implemented with whatever numerical optimization routine one prefers, to find the saddle points of the action A_0(X) as a function of R_f.

As one uses 4DVar at every stage of annealing, the question about the difference between the annealing method and 4DVar is a good one. The value of the annealing approach is that one can track with care the lowest action state and when that splits off from the other allowed saddle points via its value of A_0(Xˆ0) be certain it dominates the integral evaluated via the Laplace method. We note that in Quinn's dissertation, one is extremely unlikely to miss this path if one starts with large R_f.

The application to a PDE, such as the shallow water equations or multilayer Quasi-geostrophic models is possible, and in order not to bury the essential new tool of annealing, we have chosen to keep this paper focused on simple, yet instructive, models used routinely in testing new data assimilation methods.

On the matter of the chi-squared distribution in p. 1608 equation 6: this is totally standard statistics. The referee suggests a reference and we will include one in the edit. In this regard, as noted in the reply to Referee #1, we have found the use of this in a paper by Bennett and Chua from about a decade ago, and we will include that as well.

As a final matter, the action A_0(X) is derived in detail in the first reference. We will expand on the statement on page 1605. Just for clarity, and we will reemphasize this in the revised text: X is the whole path of the model states x(t) which is D-dimensional. Xˆ0 is our notation for the path we find with the lowest action value A_0(Xˆ0). Lower case x is the D-dimensional state. x(0) is the state at t_0 when the measurement period commences.

We appreciate the comments from Referee #2, and we trust this reply goes a long way in addressing the concerns expressed. All these comments will be accounted for the in the rewritten manuscript.

Jingxin Ye, Nirag Kadakia, Paul Rozdeba, Henry Abarbanel, and Jack Quinn December 7, 2014

Interactive comment on Nonlin. Processes Geophys. Discuss., 1, 1603, 2014.