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Interactive comment on "Self-breeding: a new method to estimate local Lyapunov structures" *by* J. D. Keller and A. Hense

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We see from the reviewers comments that we should clarify on what we want to show in our manuscript with a general reply to both reviewers:

Our approach aims at estimating uncertainty structures optimized for a time interval τ similar to the SVs, however, in this case purely based on non-linear instead of linearized error growth using the full model $\mathcal{M}$. This emphasis on non-linearity may seem unneccessary for the simple L96 model but our goal is the generation of perturbations leading to reasonable error growth in highly non-linear regimes such as atmospheric convection for NWP. Growing errors at these scales might be transient and of small magnitude compared to error growing modes on longer timescale (e.g. atmospheric rossby waves) but have a strong impact on short-range forecast performance as well

as longer lasting effects through aggregation towards larger spatio-temporal scales. Therefore, our paper is designed to demonstrate the applicability of our approach and will is shortly to be followed by a paper on real scenario case studies using a high-dimensional ($N \sim 10^8$) NWP model.

Using the definition from Pazo et al. (2010) and references therein, we can distinguish between 3 different types of Lyapunov vectors (LV): Backward LVs (B-LVs) with $\lim_{t\to-\infty} \frac{1}{t} ln\|\mathbf{M}(t,0)\mathbf{b}_n(0)\| = \lambda_n$, Forward LVs (F-LVs) with $\lim_{t\to\infty} \frac{1}{t} ln\|\mathbf{M}(0,t)\mathbf{f}_n(0)\| = \lambda_n$ and Characteristic LVs (C-LVs). Further, (forward) singular vectors (SVs) as used in numerical weather prediction (NWP), estimate the eigenvector spectrum of $\mathbf{M}^*(\tau,0)\mathbf{M}(\tau,0)$ for a chosen optimization time τ, again by using the linear approximation.

What we are looking for are therefore structures $\mathbf{s}(0)$ which fulfill $\mathbf{s}(\tau) = \mathcal{M}(\tau,0)\mathbf{s}(0)$ with $\mathbf{s}(\tau)$ being the non-linear error growing modes at optimization time τ and the full non-linear propagator $\mathcal{M}(t_1,t_0)$ of our perturbations model state from time t_0 to time t_1. Using the aforementioned definition of Lyapunov vectors our approach will lead to a non-linear analog of the forward LVs but local in the phase space similar to the singular vector approach.

Our approach does not aim at a complete description of Lyapunov theory-based error growth, i.e. of backward, forward and characteristic LVs, but for a small subspace of the error growth phase space targeted using the optimization time τ.

Detailed reply to the concerns of the anonymous referee:

1) The authors discuss global vs local Lyapunov vectors (LVs). This classification, however intuitive it may seem, is misleading. LVs are mathematically well defined objects and precise mathematical definitions must be provided in order to make sense to what they are meaning by local/global LVs. For instance, the authors say global LVs determine the average growth at the system's attractor scale, while local LV depend on the point in the attractor. Well, there is no such a thing as a LV that does not depend *on the position in the attractor. There is no such a thing as a LV that is independent on the point of the trajectory!!*

One may "interpret" the authors have in mind backward (forward) LVs as global quantities because they are computed from the remote past (far future) and so they have information of the whole attractor. However, this does not make them independent of the trajectory point. These sets of vectors are always computed at some time t of the system's trajectory. By the same token, one may also interpret "local LVs" refer to singular (forward or backward) vectors. If this is the case, why not using the existing well accepted terminology. The citations used to refer to both sets of vectors do not help because they are often referring to different objects.

We agree with the referee's comments on Lyapunov vectors and their dependence on the attractor/trajectory and that our text is misleading in this point. We refer here to the clarification stated at the beginning of our reply.

*2) Singular vectors are not estimates for the forward LVs, they are eigenvectors of M*M operator and tend to the forward LVs in the far future limit. At any short time, they are not even estimates. This is not a minor point, see my comment 5 below.*

The self-breeding method is supposed to generate non-linear estimates of the error growing modes for a (short) specific time interval which may seem trivial in the case of the Lorenz 96 model but may be beneficial when applying it to a complex multi-scale model. We know that we do not estimate the LVs themselves but an analogue of them for a limited time interval, i.e. local in the sense of the phase space.

3) The authors do not seem to have grasped the full implications of the norm choice when constructing BVs. I do not understand the text at the end of section 2.1. They cite Pazo et. al., 2013, but they do not seem to understand that BV depend on the norm and that some norms produce more diversity (higher dimension) of the ensemble than others. BVs collapse into the dominant (backward) LV can be controlled by the use of the zero-norm. This avoids using for instance orthogonalization of the BV ensemble,

with all its artifacts and unwanted effects (like the reset of the all spatial correlation information contained in the BVs).

We fully agree that the choice of norm is important for the breeding method and did not state otherwise in the manuscript. To clarify the orthogonalizsation procedure: the orthogonalization method is not similar to a Gram-Schmidt approach(e.g. Keller et al. 2010). The method aims at the maximization of the variance of the ensemble and does not produce artifacts or unwanted effects.

4) Orthogonalization as explained in Sec 2.4. Why the whole time cycle summation enters in the formula (6)? I would expect one needs to orthogonalize at the end of the interval. In any case, this step is not well explained.

A detailed description of the orthogonalization can be found in Keller et al. (2010) as stated in the article. We use information from the whole time interval to also include the temporal correlations of the different realizations in the orthogonalization process.

*5) Sec. 4, Local Lyapunov estimates. I have real troubles with this section. 5.a) Here, the authors want to compute the forward LVs so they can compare with their forward BVs. Forward LVs are the eigenvectors of M*M and not those of MM* (see Legras and Vautard, for instance).*

We agree with the comment.

5.b) The authors say they compute 50 random perturbed runs and average M_2. This I do not understand at all. One does not need perturbed runs to get M because it is the operator of the linearized evolution equation. It just depends on the system state at time t and time horizon t_end. All in all, I am uncertain what set of vectors are the authors comparing to singular forward, backward, or something else.

As explained above, we seek to estimate the spectrum of error growing modes for the (short) time interval from t_0 to t_1 in a non-linear fashion.

5.c) The authors then say: "In theory, the global Lyapunov exponents and vectors could *be obtained by repeating this procedure for an infinite number of target time steps and averaging over the resulting structures'. This is plain incorrect. The forward LVs and Lyapunov exponents will come from taking the infinite time limit (very long time limit in simulations).*

We agree with the criticism. It should read "The global Lyapunov exponents and vectors could be approximated by repeating this procedure for an infinite number of target time steps and averaging over the resulting structures".

6) I do not fully understand the proposed self-breeding method. They seem to breed a vector for a time cycle, from t up to time t_bred, then adding the result (rescaled) back to the initial state at t and repeat for a number of times. However, this does not make much sense to me. After breeding one cycle, one is left with an ensemble of perturbations that can be more or less close to the (forward) tangent space at time t_bred. These perturbations are not tangent at time t and, therefore, they would point out of the attractor when added at the state time t. They are somehow incompatible with the linear dynamics at time t. Note that in the usual (backward) breeding, one resets the amplitude and adds the BV to the present state, not the initial or any other state. This gives an ensemble progressively projected into the tangent space. Just as described in Sec 2.1 of the paper.

We understand the principle of the usual breeding algorithm. We are also aware that the proposed method might seem to make inappropriate use of the perturbations (at time t_1) as they are "inconsistent" with the current system state at t_0 and might well point out of the attractor. However, we did this intentionally in order to find 2-dimensional (model space and time - we erroneously wrote 4-dimensional in the manuscript) states reaching from t_0 to t_1 which describe the error growth characteristics for this specific part of the attractor.

7) In my opinion, the orthogonalization of the BVs and comparison with the singular vectors makes little sense when used to validate the algorithm. Given any ensemble

of perturbations they tend to collapse into the leading LV direction. Depending on the norm used, this collapse can be complete or partial and so, the ensemble dimension would always be less than the number of members. Orthogonalizing just removes all the information about other unstable directions. Obviously, the orthogonalized set would cover better the subspace spanned by the singular LVs because themselves form a orthogonal set, but this does not mean the ET BVs represent better the dominant instabilities. Actually, the dominant unstable directions are not orthogonal to each other. Therefore, fig. 7b seems trivial.

We agree with the comment regarding the use of the Lorenz 96 model. However, for a real-world multi-scale application (e.g. numerical weather prediction), the orthogonalization of bred vectors has proven to be effective with respect to the verification of forecast originating from such a set of ET BVs (cf. Keller et al. 2010).

Interactive comment on Nonlin. Processes Geophys. Discuss., 1, 1509, 2014.