Nonlin. Processes Geophys. Discuss., 1, C622–C626, 2014
www.nonlin-processes-geophys-discuss.net/1/C622/2014/

Nonlinear Processes in Geophysics
Discussions
Open Access

Interactive comment on "Self-breeding: a new method to estimate local Lyapunov structures" *by* J. D. Keller and A. Hense

J. D. Keller and A. Hense

jkeller@uni-bonn.de

Received and published: 21 November 2014

We see from the reviewers comments that we should clarify on what we want to show in our manuscript with a general reply to both reviewers:

Our approach aims at estimating uncertainty structures optimized for a time interval τ similar to the SVs, however, in this case purely based on non-linear instead of linearized error growth using the full model $\mathcal{M}$. This emphasis on non-linearity may seem unneccessary for the simple L96 model but our goal is the generation of perturbations leading to reasonable error growth in highly non-linear regimes such as atmospheric convection for NWP. Growing errors at these scales might be transient and of small magnitude compared to error growing modes on longer timescale (e.g. atmospheric rossby waves) but have a strong impact on short-range forecast performance as well

Full Screen / Esc

Printer-friendly Version

Interactive Discussion

Discussion Paper

as longer lasting effects through aggregation towards larger spatio-temporal scales. Therefore, our paper is designed to demonstrate the applicability of our approach and will is shortly to be followed by a paper on real scenario case studies using a high-dimensional ($N \sim 10^8$) NWP model.

Using the definition from Pazo et al. (2010) and references therein, we can distinguish between 3 different types of Lyapunov vectors (LV): Backward LVs (B-LVs) with $\lim_{t\to-\infty} \frac{1}{t} ln\|\mathbf{M}(t,0)\mathbf{b}_n(0)\| = \lambda_n$, Forward LVs (F-LVs) with $\lim_{t\to\infty} \frac{1}{t} ln\|\mathbf{M}(0,t)\mathbf{f}_n(0)\| = \lambda_n$ and Characteristic LVs (C-LVs). Further, (forward) singular vectors (SVs) as used in numerical weather prediction (NWP), estimate the eigenvector spectrum of $\mathbf{M}^*(\tau,0)\mathbf{M}(\tau,0)$ for a chosen optimization time τ, again by using the linear approximation.

What we are looking for are therefore structures $\mathbf{s}(0)$ which fulfill $\mathbf{s}(\tau) = \mathcal{M}(\tau,0)\mathbf{s}(0)$ with $\mathbf{s}(\tau)$ being the non-linear error growing modes at optimization time τ and the full non-linear propagator $\mathcal{M}(t_1,t_0)$ of our perturbations model state from time t_0 to time t_1. Using the aforementioned definition of Lyapunov vectors our approach will lead to a non-linear analog of the forward LVs but local in the phase space similar to the singular vector approach.

Our approach does not aim at a complete description of Lyapunov theory-based error growth, i.e. of backward, forward and characteristic LVs, but for a small subspace of the error growth phase space targeted using the optimization time τ.

Detailed reply to the concerns of reviewer Diego Pazo:

(1) *The manuscript is very confusing and misleading concerning the mathematical definitions. Local LVs are not well defined, with contradicting definitions (more details below). Another example, the adjective "forward" is used for referring to objects that in Legras and Vautard paper (the main theoretical citation of the manuscript) are termed backward.*

NPGD

1, C622–C626, 2014

We define local LVs as the growing mode directions for a specific point in phase space along a small part of the corresponding trajectory of the attractor, i.e. $\mathbf{s}(\tau) = \mathcal{M}(\tau, \prime)\mathbf{s}(\prime)$ with τ the length of the trajectory used and $\mathbf{x}(0)$ being the phase space location.

(2) *The local Lyapunov structures the authors pursue to estimate are not well explained and motivated.*

We intend to use our self-breeding approach to identify these local Lyapunov structures in high-dimensional models for real world problems (e.g. NWP). These models exhibit error growth at multiple scales and amplitudes. We believe that with this method we are able to determine error growth characteristics for a specific scale in such a scenario and use them to enhance probabilistic forecasts especially for small scale phenomena.

(3) *This research and the results are more understandable in a linear framework. Even if the manuscript deals with finite vectors, the amplitudes adopted (0.005-0.1) are small enough to be well described by the linear theory. In sum, the proposed self-breeding method is equivalent to obtain the leading eigenvector of a certain matrix (see below).*

This may be true for the Lorenz96 model. However, models for real-world problems may contain considerable non-linearities even on small tempo-spatial scales (e.g. convection, turbulence for NWP models). As this manuscript should only serve as a conceptual implementation and show its basic functionality. For the Lorenz96 model, we therefore agree with you on the equivalency you described.

(4) *There is a number of points where the manuscript is confusing:*

(4a) *The explanation of the self-breding is too short and no rationale for the selfbreeding is provided.*

We agree that we have not been clear enough in the current version of our manuscript: The idea behind our self-breeding method is to make use of the established breeding technique and its non-linear approach with the goal of obtaining non-linear estimates for the optimized short range error growth in highly non-linear regimes of a chaotic

NPGD

1, C622–C626, 2014

Interactive Comment

system. The application of the method to the Lorenz96 model serves as a conceptual implementation in a low order model. See reply to (2) for further motivation.

(4b) *In Sec. 5, I assume four-dimensional state means the vector has the largest components at four sites. The authors are basically computing the eigenvector of M with the largest eigenvalue; Note that all solutions in Fig. 3 are very similar (up to a sign flip) indicating approximately linear dynamics.*

With the real-world application in mind, we erroneously wrote four-dimensional while meaning only two-dimensional for the Lorenz96 model, i.e. one dimension being the model space an one dimension for time.

(4c) *The problem of comparing the operators M and $M^T M$ (or $M M^T$) has been discussed, for instance, in (Yoden & Nomura, 1992), see also Pazo (2009). The decreasing localization strength of the self-breeding BV with rescaling interval, observed in Fig.4, is perfectly consistent with the result in Pazo (2009), where the Lorenz96 model was also studied.*

We take note of the comment.

(4d) *The orthogonalization in Sec. 2.4. is said to be performed only in the $N_B V$ dimensional subspace of the BVs. Is it similar to a Gram-Schmidt orthogonalization? The details are not provided, but in any case it is not understandable why information from the whole interval $t = 1, .., N_t$ is needed in Eq. (6).*

The method is not similar to the Gram-Schmidt orthogonalization. A detailed description can be found in Keller et al. (2010) as stated in the article. We use information from the whole interval $t = 1, .., N_t$ to also include the temporal correlations of the different realizations in the orthogonalization process.

(4e) *Roughly speaking Sec. 5.2 intends to compare the orthogonal eigenvectors of M MËĘT, and the set of self-breeding BVs as described in Sec 2.4). It is difficult to understand what the authors are doing in Fig. 5 since no formula is included. What*

NPGD
1, C622–C626, 2014

Interactive Comment

Full Screen / Esc

Printer-friendly Version

Interactive Discussion

Discussion Paper

I can infer is that the 16 dimensions the untransformed vectors may project at most on the Lyapunov vectors, Fig. 7(a), is probably related with the 16 positive LEs of the system. Not surprisingly, BVs can only capture unstable directions.

Still, the orthogonalization (ensemble transform) procedure allows us to estimate the full Lyapunov spectrum using the non-linear approach.

Regarding the method: As we obtain two different subspaces, we would like to objectively compare these two. Therefore we try to find two orthogonal transformations which transform both respective subspaces into another subspace. When put into formulas this is identical to the method of canonical correlations, cf von Storch and Zwiers (2001).

(4f) *The citation to (Pazó, 2013) does not correspond to what is said in the text.*

At the respective part of the manuscript we state that the breeding technique has many characteristics which can be tuned and that the norm is one way to do it, as it is done in Pazo (2013). We therefore do not see where the citation does not correspond to the text.

(4g) *The ensemble transform in Sec. 5.3 is not sufficiently explained. The final result, Fig. 7(b), is not very surprising (at least as it is explained). At the light of the previous criticisms, these comments are nonetheless superfluous.*

The ensemble transform is a well known method for which we also provided references.

Interactive comment on Nonlin. Processes Geophys. Discuss., 1, 1509, 2014.