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Interactive comment on "Toward the assimilation of images" *by* F.-X. Le Dimet et al.

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We thank the Anonymous Referee #3 for carefully reviewing our work and for his remarks and suggestions. Below are our responses and comments to the questions raised by the Referee:

1 Comment 1

1.1 Comment from Referee

I did not understand the order of the figures. Usually the figures have numbers that correspond to the appearance in the paper. This is not the case here. For instance

figure 6 appears very early in the text, before some figures that have smaller numbers. Why ? I think that it is better to change the numbers of the figures to respect the general rule of appearance in the text.

1.2 Author's response

This is an important point for the reader and we thank the Referee for his attention. We will address this problem of figure ordering in the revision.

1.3 Author's changes in manuscript

The figures are now reordered as follows:

- 6 becomes 3
- 3 becomes 4
- 7 becomes 5
- 4 becomes 6
- 5 becomes 7
- No change for other figures

2 Comment 2

2.1 Comment from Referee

At the end of paragraph 3.1, there is a comment on the two main types of data assimilation methods (Kalman methods and variational ones). Is it true that Kalman methods are not implemented in operational centers ? It seems to me that there is a kind of non-objectivity from the authors.

2.2 Author's response

This question is of crucial importance and we thank the Referee for raising it. Our assertion referred to the traditional Kalman Filter and should not be associated with all the Kalman Filter approaches as it is the case in this version of the paper. The Ensemble Kalman Filter (Evensen, 1994) method includes an elegant definition of the approximation of the covariance matrix and its practical use as long as the infrastructures allow to run the ensemble forecast.

We also thank the Editor for his comment on this question, as well as O. Talagrand for pointing to various sources to answer this comment.

2.3 Author's changes in manuscript

The revised text makes it clear that the problem with the evolution of the covariance matrix is relevant only for traditional Kalman Filter. It also makes it clear that the ensemble approach solves that problem. The point 1 of approaches of data assimilation in section 3.1 now reads

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1. Approaches derived from the Kalman filter. They are based on Bayesian estimation and are of great theoretical importance. **Having to deal with a huge covariance matrix, the traditional Kalman filters are not implemented in operational centers. The Ensemble Kalman filter (Evensen, 1994) overcomes that limitation by introducing an ensemble approximation of the covariance matrix.**

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3 Comment 3

3.1 Comment from Referee

The threshholding procedure in 6.1.5 could be more clearly explained for non specialists of curvelets.

3.2 Author's response

The Referee raises here an important question of the difficulty associated with the topic of curvelet in general. Paragraph 6.1.2 has a short description of the curvelet decomposition. To help the non specialist of curvelet understanding the thresholding procedure, and based on the parallel comment of Referee #2 we rewrote the paragraph.

3.3 Author's changes in manuscript

See our response to the comment of Referee #2.

4 Comment 4

4.1 Comment from Referee

The paragraph 6.2 is not written as well as the other ones. I suggest to rewrite it in a style that is consistent with the rest of the paper and that is easier to understand.

4.2 Author's response

The remark of the Referee is capital for this paragraph that focus on the topic of Lyapunov exponents as observation operator for images in Data Assimilation. We rewrote the paragraph to get more uniformity.

4.3 Author's changes in manuscript

The paragraph 6.2 is rewritten as follows:

6.2 Observation operators based on finite-time Lyapunov exponents and vectors computation

Ocean tracer images (Sea Surface Temperature and Ocean Color for instance) show patterns, like fronts and filaments, that characterize the flow dynamics. They are closely related to the underlying flow dynamics and are referred to as *Lagrangian Coherent Structures* (LCS). They are material curve which exhibits locally the strongest attraction, repulsion or shearing in the flow over a finite-time interval (Haller 2000b, Haller, 2011). Their location and shape are the signature of integrated dynamic in-

formation that should be exploited in a data assimilation scheme. LCSs are usually identified in a practical manner as maximizing ridges in Finite-Time Lyapunov Exponents (FTLE) fields (Haller, 2001a). FTLE is a scalar local notion that represents the rate of separation of initially neighboring particles over a finite-time window $[t_0, t_0 + T]$, $T \neq 0$. It is defined as the largest eigenvalue of the Cauchy–Green strain tensor of the flow map. The corresponding eigenvector is called the Finite-Time Lyapunov Vector (FTLV). Let $\vec{X}(t) = \vec{X}(t; \vec{X}_0, t_0)$ be the position of a Lagrangian particle at time t, which started at $\vec{X}_0$ at $t = t_0$ and was advected by the time-dependent fluid flow $U(\vec{X}, t)$, $t \in [t_0, t_0 + T]$. An infinitesimal perturbation $\delta\vec{X}(t)$ started at $t = t_0$ from $\delta_0 = \delta\vec{X}(t_0)$ around $\vec{X}_0$ then satisfies, for all $t \in [t_0, t_0 + T]$,

$$\frac{D\delta\vec{X}(t)}{Dt} = \nabla U(\vec{X}(t), t).\delta\vec{X}(t), \tag{1}$$

$$\delta\vec{X}(t_0) = \delta_0, \vec{X}(t_0) = \vec{X}_0. \tag{2}$$

Let $\lambda_{\max}$ be the largest eigenvalue of the Cauchy–Green strain tensor

$$\Delta = \left[\nabla\phi_{t_0}^{t_0+T}(\vec{X}_0)\right]^* \left[\nabla\phi_{t_0}^{t_0+T}(\vec{X}_0)\right], \tag{3}$$

where $\phi_{t_0}^t : \vec{X}_0 \mapsto \vec{X}(t; \vec{X}_0, t_0)$ represents the flow map of the system (it links the location $\vec{X}_0$ of a Lagrangian particle at $t = t_0$ to its position $\vec{X}(t; \vec{X}_0, t_0)$ at time $t \neq t_0$). The *forward* FTLE at the point $\vec{X}_0 \in \Omega$ and for an advection time T starting at $t = t_0$ is defined as

$$\sigma_{t_0}^{t_0+T}(\vec{X}_0) = \frac{1}{|T|} \ln \sqrt{\lambda_{\max}(\Delta)}. \tag{4}$$

FTLV is the eigenvector associated with $\lambda_{\max}$. The FTLE thus represents the growth factor of the norm of the perturbation $\delta\vec{X}_0$ started around $\vec{X}_0$ and advected by the flow after the finite advection time T. Maximal stretching occurs when $\delta\vec{X}_0$ is aligned with the FTLV. *Backward FTLE and* FTLV (BFTLE and BFTLV) are similarly defined, with the time direction being inverted, in Eq. (1).

BFTLE (BFTLV) is a scalar (vector) that is computed at a given location $\vec{X}_0$. Seeding a domain with particles initially located on a grid leads to the computation of discretized scalar (BFTLE) and vector (BFTLV) fields. Ridges of the BFTLE field approximate attracting LCSs (Haller, 2011). An example of a BFTLE and corresponding BFTLV orientation maps, computed from a mesoscale (1/4degree) time-dependent *surface* velocity field coming from a simulation of the North-Atlantic ocean, is given in Fig. 9. The BFTLE field shows contours that correspond reasonably well to the main structures such as filaments, fronts and spirals that appear in the Sea Surface Temperature (SST) field of the same simulation (see Fig. 10 left panel). Note that this field can be distinguished by spatial observations. Also the BFTLVs align with the gradients of this tracer field: Figs. 9 and 10 (right panels) show that BFTLVs and SST gradients have similar orientations. These similarities illustrate the strong link between the tracer field patterns and the underlying flow dynamics. BFTLE and BFTLV properties may thus be exploited to identify appropriate structure space to be used in a direct image assimilation framework (Titaud et al., 2011).

First, thanks to its almost-lagrangian nature (Lekien, 2005a) BFTLE field can be considered as an image of tracer field. Then $\mathcal{H}_{\mathcal{X}\to\mathcal{F}}\ :\ U \mapsto BFTLE(U)$ defines a model-to-image operator which is composed with an image-to-structure operator $\mathcal{H}_{\mathcal{F}\to\mathcal{S}}$:

$$\mathcal{H}_{\mathcal{X}\to\mathcal{S}}(U) = \mathcal{H}_{\mathcal{F}\to\mathcal{S}}(BFTLE(U)). \tag{5}$$

Note that BFTLE produces images with stronger discontinuities than operators based on passive tracer advection: in this later case, the numerical diffusion softens the discontinuities which makes the comparison with high-resolution satellite images less accurate.

Secondly, the alignment of the BFTLV with the tracer gradients (Lapeyre, 2002; d'Ovidio, 2009) allows the construction of a *strict* model-to-structure operator: structures are identified as the orientation of the gradient of the image and the observation

operator is simply defined as

$$\mathcal{H}_{\mathcal{X}\to\mathcal{S}}(U) = BFTLV(U). \tag{6}$$

Several studies showed that model-to-structure operators based on BFTLE-V are sensitive to perturbations of the velocity field in a direct image assimilation framework: Fig. 11 left panel (resp. right panel) shows a set of misfits in the image ridges space (resp. in the image gradient orientation space) bewteen BFTLE (resp. BFTLV) and SST fields with respect to the amplitude of random perturbations: each misfit admits a unique minimum close to the non-perturbed state. Moreover BFTLV shows a more robust behavior than BFTLE: misfts are smoother and minima are identical. These studies clearly illustrate the feasability of the use of such operators in direct image assimilation to control surface velocity fields. For more details about theoretical framework and experimental setup about the use of BFTLE-V as model-to-structure operator see (Gaultier, 2013; Titaud et al., 2011).

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