We would like to thank the reviewer for her/his careful reading of the article and for the useful comments which helped us to improve and clarify the manuscript. We have addressed all the comments as explained below. Also, we have proposed major changes in the article to put into light the comparison DBFN-4Dvar and to improve its readability.

a) the main point regards Eq.(6) and (7). The authors only cite a personal communication that should explain why after an infinite number of iterations their algorithm should converge to a trajectory calculated without the diffusive and the nudging term. We think that this point is important in driving the reader in the comprehension of the results presented. So the authors should give some theoretical justifications and verify it in their results.

Thanks for raising this question. The personal communication concerns the statements that under convergence conditions and under the hypothesis that at convergence both forward and backward trajectories are equal, then Eqs. (6) and (7) hold. To see this we write the DBFN system as:

$$\begin{aligned} \frac{\partial \vec{x}_k}{\partial t} &= \mathcal{F}(\vec{x}_k) + \nu\Delta\vec{x}_k + \vec{K}(\vec{x}_{obs} - \mathcal{H}(\vec{x}_k)) \\ \vec{x}_k(0) &= \tilde{\vec{x}}_{k-1}(0), \qquad\qquad 0 < t < T, \end{aligned} \tag{1}$$

$$\begin{aligned} \frac{\partial \tilde{\vec{x}}_k}{\partial t} &= \mathcal{F}(\tilde{\vec{x}}_k) - \nu\Delta\tilde{\vec{x}}_k - \vec{K}'(\vec{x}_{obs} - \mathcal{H}(\tilde{\vec{x}}_k)) \\ \tilde{\vec{x}}_k(T) &= \vec{x}_k(T), \qquad\qquad T > t > 0. \end{aligned} \tag{2}$$

where $k \in N_{\geq 1}$ stands for iterations.
We see that if $\vec{K'} = \vec{K}$ and the forward and backward limit trajectory are equal, i.e $\tilde{\vec{x}}_\infty = \vec{x}_\infty$, then taking the sum between Eqs.(1) and (2) shows that $\vec{x}_\infty$ satisfies the model equations without the Nudging and diffusion:

$$\frac{\partial \vec{x}_\infty}{\partial t} = \mathcal{F}(\vec{x}_\infty) \tag{3}$$

while taking the difference between Eqs.(1) and (2) shows that $\vec{x}_\infty$ satisfies the Poisson equation:

$$\Delta\vec{x}_\infty = -\frac{\vec{K}}{\nu}(\vec{x}_{obs} - \mathcal{H}(\vec{x}_\infty)) \tag{4}$$

Concerning the BFN and DBFN convergence, we prefer to make references to past works in the introduction to avoid increasing the length of the article: Auroux and Blum (2005) for a ODE linear system ; Ramdani et al. (2010) for reversible linear PDE equations (Wave and Schrödinger equations); Auroux and Nodet (2012) for linear and non-linear transport equation under viscous and non viscous conditions.
Then, we added a paragraph to the article explaining how we obtain Eqs. (6) and (7) and the issues related to convergence.

b) it is not clear the behavior of the diffusive term in the backward integration. We understood that this term eliminates the small scale

structures both in forward and in backward integration. The sign indicated in (4) suggests this interpretation but some sentences at pag. 1080, line 15 and following let the reader quite confused.

We agree with the reviewer, the diffusion term as it is written in Eqs.(3) and (4) eliminates the small scale structure both in forward and backward mode. However, the point discussed in pag. 1080 is to clarify that ideally the true inverse model should not dissipate energy both in forward and backward integration. If analytically this makes the backward integration ill-posed, numerically and for finite Data Assimilation window it is the very small scales (high wavenumber) that pose the problem. That is why we suggest the use of the BFN (not the DBFN) followed by a digital filter which eliminates the necessary energy to keep the numerical solution stable.
Since this comment may be a source of confusion, we decided to take it out of the article and just say that for sake of stability we used the DBFN.

c) It is completely unclear what are the different kinds of K's used. At Pag. 1080 it seems (we use latex notation) that K =k H^T R^-1, then the authors speak about a "K based on the PLS regression model", somewhere else (e.g. Pag. 1083) it seems that after the DBFN the PLS regression is used. We strongly suggest the authors to make the technical details of the different experiments of their method clear.

We thank the reviewer for this useful remark. We have used two versions of K. One is a scalar, and in this case we can interpret $K = kH^T R^{-1}$ with $H^T = Id$ and R the observation error covariance, which in our case is diagonal with equal entries. The other one relies in the covariance (correlations) calculated thanks to the PLS regression. In this case, the updating scheme can be seen as a rough approximation of the two steps update for EnKF. As we have already said, we made several changes in the article, thus in the new version only the Kalman-like gain is used. Accordingly, we have added the subsection 5.1 "Prescription of the DBFN gain“ to clarify our choice of the gain matrix K.

d) We agree with the other referee that the relative error is not a good measure of the difference between two states. We suggest the use of the RMS or of the RMS normalized by the standard deviation.

We have changed the figures to consider the RMS error.

e) This point regards the DFBN technique: the authors state the in absence of observations the iterations converge to an homogeneous state. This means that after several iterations the analysis is completely independent of the dynamics equation (F(x) in Eq.(2)).

We think that with our explaination given to the remark a) the reviewer will certainly better understand this point. Indeed, we do not say that without observation the solution is totally independent of the model, since it is stated that the trajectory at convergence is a solution of the model F. Indeed, in the complete absence of observations it is not worth considering the iterations.
We included in the article a better explaination about this point, accordingly

with our answer to the topic a).

We think that with no diffusive term, after several iterations, the model is in some sense "forced" to become equal to the observations in the observed points. Reading the manuscript we have understood that the authors think that with a balance of the diffusive and nudging terms, the trajectory should converge to an actual trajectory-of the model without diffusion. If this is correct the authors should better clarify and also prove that this behavior holds, at least in the model under examination.

What we mean by the paragraphs of lines 3-15 on page 1082 is that the trajectory at the convergence satisfies both the model equations without diffusion terms and the Poisson equation. It does not necessarily mean that it is the solution of the Back and Forth Nudging when considering the model without diffusion. Moreover, we agree with the reviewer when he says that "with no diffusive term, after several iterations, the model is in some sense "forced" to become equal to the observations in the observed points". To see this, we just need to write Eqs.(1) and (2) without the diffusion term and take the difference between them:

$$K(x^{obs} - H(x_\infty)) = 0$$

To show that at convergence the solution satisfies the model equation without the nudging and diffusion terms, we have configured an experiment for which the true state comes from a higher resolution model ($3km$). The projection of the higher resolution model onto our mesh is viewed as the model trajectory without diffusion. This trajectory is assimilated using the DBFN algorithm and then we compare the kinetic energy spectrum for the high resolution model, a typical spectrum for our configuration and the spectrum after the assimilation of the high resolution observation. Figure 1 presents these spectrums. We readily see that the reconstructed spectrum is much closer to the high resolution model than to the typical spectrum for our configuration.

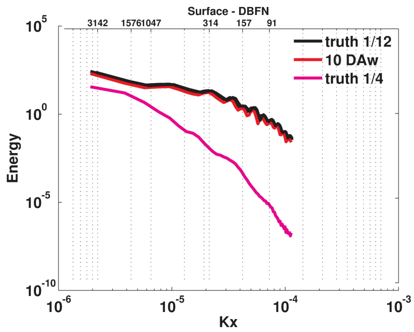

Figure 1: Kinetic energy mean power spectra calculated using the first layer (top) using the forecast of the assimilation experiments using the DBFN and assimilating high resolution observations. Black curve represents the “true“ power spectra at high resolution; Red curve represents the power spectra calculated for the 10 days DAw and Magenta curve represents a typical spectrum for our configuration. In the bottom abscissa the tick-labels stand for longitudinal wave-number (rad/m) while in the top abscissa the tick-labels stand for the corresponding wavelengths in km units.

Remarks

As stated before, we have made major changes in the article. Here we describe how the structure of the article changes:

- Subsection 3.1 "Ocean model configuration" is abbreviated and included into Section 3;
- Subsection 3.2 "Data Assimilation experiments" is transformed into Section 5;
- Section 5 contains four subsections:
  1. "Prescription of the DBFN gain": explains how the matrix K is calculated;
  2. "The 4Dvar background term configuration": details the background term used in the 4Dvar;
  3. "Assimilation cycle": explains how and why the methods are cycled;
  4. "Observation network": describes the assimilated observations and discusses the undetermination of the assimilation problem;
- The Section 5 is shortened and becomes Section 6. Subsection 5.1 "Experiments with scalar nudging coefficients", 5.2.1 "Daily gridded SSH observations" and 5.2.2 "Temporal data sparsity" are removed from the article;
- Subsection 5.2.3 "Intercomparisons" is transformed into subsection 6.1 "Reference experiment";
- Subsection 6.2 "Sensitivity experiments" is created to describe sensitivity tests with respect to the length of the Data Assimilation window. The DBFN and 4Dvar are compared.