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Interactive comment on "Self-breeding: a new method to estimate local Lyapunov structures" *by* J. D. Keller and A. Hense

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This manuscript aims to show how to estimate "local Lyapunov vectors" by bred vectors generated with a new technique, dubbed self-breeding.

In my view, the kind of problem considered may be potentially interesting but the manuscript cannot be accepted for several reasons I discuss in more detail below. My main criticism is that the paper is confusing and contradicting at several points. The manuscript basically discusses the relationship between two types of eigenvectors, but the authors are not aware of this fact, and that this problem has been the subject of some of papers before. I miss a careful mathematical formulation of the problem in this paper. Without it, it makes no sense to introduce more specialized techniques, such as the bred vector orthogonalization proposed in the manuscript.

I'm inclined to think that the authors better rethink their research programme, rather than prepare a revised version from the current one. This is my recommendation since the manuscript is not conceived in the correct framework. This topic deserves of a much deeper understanding and an accurate formalization of the mathematics behind to be of certain usefulness.

My criticisms are listed next:

(1) The manuscript is very confusing and misleading concerning the mathematical definitions. Local LVs are not well defined, with contradicting definitions (more details below). Another example, the adjective "forward" is used for referring to objects that in Legras and Vautard paper (the main theoretical citation of the manuscript) are termed backward.

(2) The local Lyapunov structures the authors pursue to estimate are not well explained and motivated.

(3) This research and the results are more understandable in a linear framework. Even if the manuscript deals with finite vectors, the amplitudes adopted (0.005-0.1) are small enough to be well described by the linear theory. In sum, the proposed self-breeding method is equivalent to obtain the leading eigenvector of a certain matrix (see below).

(4) There is a number of points where the manuscript is confusing:

(4a) The explanation of the self-breding is too short and no rationale for the self-breeding is provided.

(4b) In Sec. 5, I assume four-dimensional state means the vector has the largest components at four sites. The authors are basically computing the eigenvector of M with the largest eigenvalue; Note that all solutions in Fig. 3 are very similar (up to a sign flip) indicating approximately linear dynamics.

(4c) The problem of comparing the operators M and MˆT M (or M MˆT) has been discussed, for instance, in (Yoden & Nomura, 1992), see also Pazó (2009). The decreasing localization strength of the self-breeding BV with rescaling interval, observed in Fig.4, is perfectly consistent with the result in Pazó (2009), where the Lorenz96 model was also studied.

(4d) The orthogonalization in Sec. 2.4. is said to be performed only in the N_BV-dimensional subspace of the BVs. Is it similar to a Gram-Schmidt orthogonalization? The details are not provided, but in any case it is not understandable why information from the whole interval t=1,..,N_t is needed in Eq. (6).

(4e) Roughly speaking Sec. 5.2 intends to compare the orthogonal eigenvectors of M MˆT, and the set of self-breeding BVs as described in Sec 2.4). It is difficult to understand what the authors are doing in Fig. 5 since no formula is included. What I can infer is that the 16 dimensions the untransformed vectors may project at most on the Lyapunov vectors, Fig. 7(a), is probably related with the 16 positive LEs of the system. Not surprisingly, BVs can only capture unstable directions.

(4f) The citation to (Pazó, 2013) does not correspond to what is said in the text.

(4g) The ensemble transform in Sec. 5.3 is not sufficiently explained.The final result, Fig. 7(b), is not very surprising (at least as it is explained). At the light of the previous criticisms, these comments are nonetheless superfluous.

DEFINITIONS

The first time the authors use the term "local LV" refer to (Fujisaka, 1983), where the term is not used due to its pointless meaning in the one-dimensional system studied there.

The second time "local LV" is used, the authors refer to (Szunyogh, 1997), where "local LVs" is intended to mean the classical set of orthonormal vectors obtained numerically through the Bennettin's algorithm via periodic orthonormalization of infinitesimal perturbations. These vectors are called 'backward' Lyapunov vectors in (Legras & Vautard,

1996), and in the literature they have been also called Gram-Schmidt vectors to emphasize their origin. The backward LVs coincide with the eigenvectors of M_2=M MˆT (see, Ershov & Potapov, 1992) if the linearized operator M that governs the evolution of infinitesimal perturbations is taken to evolve perturbations from t=-\infty (or a remote past in numerical simulations).

The authors use again the term "local LV" by the end of section 4, with a definition that is not consistent with the previous one. Now they call 'local LVs' to the eigenvectors of M_2=M MˆT, but now with M being the propagator for a *finite* time. This kind of vectors are called 'backward singular vectors' in Legras & Vautard (1996), 'final (or evolved) singular vectors' in Kalnay (2003), or 'left finite-time LV' in (Okushima, 2003). (Note that the forward singular vectors or optimal vectors have been much more studied in the literature, but these are the eigenvectors of MˆT M.)

(The authors use an average of M_2 generated from 51 near trajectories, what is a very questionable step in mathematical terms. In the limit of very close trajectories this average has no effect, and I prefer to skip this in my discussion, to keep it in a coherent framework.)

The eigenvalues of M2, e_i, permit to obtain the exponential growth rates lambda_i, via lambda_i=ln e_i /(2 N_t) (note the factor 2). The authors call these numbers 'local Laypunov exponents' in disagreement with the definitions in the references cited: Szunyogh (1997) and Fujisaka (1983). The quantity denoted by lambda_i is sometimes called finite-time LE (though this term has been used as well for other quantities in the literature). Contrary to what the authors say, for recovering the (global?) LE one must not take the average over target time steps, but to take the limit of the operator M to the remote past. No average is then needed due to the Oseledec theorem. As expected, for the rescaling intervals used, lambda_1 is in Fig. 1, systematically larger that the LE (approx. 1.78 for this system). More details on this convergence for the Lorenz96 model can be found in (Pazó, 2009); there, the truly forward case is considered, but no essential difference should arise with respect to the backward case considered here (though the authors call it mistakenly forward). Moreover, the strong localization of the singular vector observed in Fig. 2 was analyzed also in (Pazó, 2009).

The expression 'global Lyapunov vector' is used throughout the manuscript without being defined. I found it confusing, and I could not find that term in the references provided.

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