I pop up into the discussion on the question of operational use of Kalman Filter (KF) and Variational Assimilation (4D-Var). Yes, Kalman Filter is used in operational Numerical Weather Prediction. It is used at the Canadian Meteorological Center (CMC) in the form of Ensemble Kalman Filter (EnKF). And, unless it has changed recently, the US National Centers for Environmental Prediction uses a sequential assimilation algorithm which is essentially a Kalman Filter where the updating of the background with new observations is achieved through minimization of an appropriate objective function (3D-Var). However, I do not remember how the prediction error covariance matrix is evolved in time (if at all). That should be easy to check.

As to the compared numerical cost of KF and 4D-Var, it can depend very much on the numerical implementation of the algorithms (for instance, number of elements in EnKF, or degree of parallelization). Buehner *et al*. (2010a, b) have made a rather clean comparison at CMC, which possesses both algorithms in operational order (but uses at present EnKF). They found that, for the same overall computational cost, the two algorithms produce results that can be different, but are of globally similar quality.

Buehner, M., P. L. Houtekamer, C. Charette, H. L. Mitchell, and B. He. 2010. Intercomparison of Variational Data Assimilation and the Ensemble Kalman Filter for Global Deterministic NWP. Part I: Description and Single-Observation Experiments. Monthly Weather Review. 138:1550-1566.

Buehner, M., P. L. Houtekamer, C. Charette, H. L. Mitchell, and B. He. 2010. Intercomparison of Variational Data Assimilation and the Ensemble Kalman Filter for Global Deterministic NWP. Part II: One-Month Experiments with Real Observations. Monthly Weather Review. 138:1567-1586.