<u>Nonlinear Processes in Geophysics</u>

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Title: On improving ensemble transform Kalman filter assimilation with nonlinear observational operator

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We thank the anonymous reviewer for his/her very helpful and insightful comments that lead to significant improvement of the quality of this manuscript. We have tried our best to address all the comments. In the supplement, we use boldface to indicate the comments from the reviewer and italics for our responses.

To reviewer 2

General comments

This paper addresses some issues associated with Ensemble Transform Kalman Filter (ETKF) in applications to nonlinear observation operators. In particular, the paper proposes the use of second-order Taylor expansion in approximation of nonlinear observation operator to improve error covariance inflation in ETKF. The proposed methodology is applied to the Lorenz 40-variable model.

Overall, the paper clearly describes the improvements and demonstrates the benefit of introducing the second order information. Most of the mathematical description is focused on the improvements of the error covariance inflation methodology for the ETKF.

Response: *Thank you for your thorough review of our manuscript and we appreciate your encouraging comments.*

Specific comments

1) Introduction: Although the title of the paper indicates it is focusing on the ETKF applications and improvements, it would be beneficial to describe the treatment of

nonlinearity in general ensemble data assimilation outside of ETKF, including the Maximum Likelihood Ensemble Filter (Zupanski 2005) and the particle filters (van Leeuwen 2009).

Response: *Following this comment, we have added the following sentences in the third paragraph of the introduction: "In general ensemble data assimilation, Maximum Likelihood Ensemble Filter (MLEF) minimizes a cost function that depends on a general nonlinear observation operator to estimate the state vector, which is equivalent to maximize the likelihood of the posterior probability distribution (Zupanski, 2005). Particle filter uses a set of weighted random samples (particles) to approximate the posterior probability distribution that may depend on a nonlinear observation operator (Leeuwen, 2009)."*

Also, the proposed methodology implicitly assumes the use of incremental minimization (e.g. a form of truncated Newton method), with outer and inner loops. This should be clearly stated, since this is only one possible approach to iterative minimization, with many more efficient methods available in mathematical optimization and control theory.

Response: *Following this comment, the sentences "It is worthwhile to point out that the proposed methodology implicitly assumes the use of incremental minimization with outer and inner loops. There may be other efficient methods available in mathematical optimization and control theory." are added in the six paragraph of the introduction.*

2) Impact of higher order nonlinear Taylor approximation: The utility of the nonlinear difference between observation operators (e.g., Eq.(7)) is not adequately presented. For general nonlinear or even non-smooth radiative transfer operators (Steward et al. 2012), the utility of higher-order elements in Taylor expansion may be questionable. Also, the development of the second order term may be time consuming and difficult in case of complex observation operators, and this aspect should also be discussed. I believe that the paper would benefit if these issues are also addressed in discussion.

Response: *Thanks for the valuable comments. In the revised version, we discussed these caveats in the third paragraph of section 4.3.*

3) Realistic applications: Since the ultimate goal of data assimilation is to be applied with realistic high-dimensional systems and observations, the conclusion should include some discussion of the outlooks into the applicability of the proposed improvements of ETKF in realistic situations.

Response: *The following sentences are added in the conclusion section "The proposed method is computationally feasible to assimilate satellite observations with radiative transfer models as the nonlinear observation operators (see Appendix E) which are broadly used in atmospheric, ocean and land data assimilations."*

References:

van Leeuwen, P. J., 2009: Particle Filtering in Geophysical Systems. Mon. Wea. Rev., 137, 4089–4114.

Steward, J. L., I. M. Navon, M. Zupanski, and N. Karmitsa, 2012: Impact of Non-Smooth Observation Operators on Variational and Sequential Data Assimilation for a Limited-Area Shallow-Water Equation Model. Quart. J. Roy. Meteorol. Soc., 138, 323-339.

Zupanski, M., 2005: Maximum Likelihood Ensemble Filter: Theoretical Aspects. Mon. Wea. Rev., 133, 1710–1726.

Response: *These references are added in the revised version.*

Technical corrections

4) Abstract, line 8: This statement is not correct. Iterative minimization with advanced Hessian preconditioning would require very few minimization iterations (1-2).

Response: *In the revised version, the sentence is changed to "One problem in the minimization of a nonlinear objective function similar to 4D-Var is that the nonlinear operator and its tangent-linear operator have to be iteratively calculated if the Hessian is not preconditioned or the Hessian has to be calculated several times. This may be computationally expensive."*

5) Introduction, p.544, L.24: "... satellite radiance data : : :"

Response: *Comment is followed.*

6) Introduction, p.546, L.3-5: Not clear what the sentence wants to say. Given that degrees of freedom of the ensemble forecast error covariance are governed by the number of ensembles, it is only natural to define the minimization space in the ensemble domain. The way to deal with insufficient degrees of freedom is to consider hybrid variational-ensemble error covariance, which is outside of the paper's considerations.

Response: *Following this comment, the sentence is deleted in the revised version.*

7) Introduction, p.547, L.7-10: Linearization typically doubles the number of operations, and thus increases the computational cost (e.g. del(x*y)=x*del(y)+y*del(x)). This should also be taken into account when discussing the cost.

Response: *The computational cost is discussed in Appendix E. In the revised version, the paragraph "On the other hand, computing the first and second derivatives requires additional number of operations, but it is manageable." is added in the end of Appendix E.*

8) Section 2.2.2: Mathematical derivation should be followed by a brief verbal description of the meaning and implications of equations, as this is the main novelty of this paper.

Response: *We have added more descriptions of the mathematical derivation in section 2.2.2.*

9) p. 562, L.13: Although it is true that most observation operators are localized, there are some that are not. How would this impact the computation of the second order term?

Response: *In the revised version, we discussed these problems in the fourth paragraph of section 4.2 as follows "For the observation operators which are not localized, the computation of the second-order term may be complex."*