We are thankful to the reviewer for useful comments. A response to these comments, including the list of performed changes, is given below.

S1. As suggested by the reviewer, we have commented the choice of sunspot numbers as an indicator of solar activity:

Solar activity is estimated in the paper with the Wolf (ISSN, sunspot) numbers, involving the number of groups and the number of spots in each particular group. The number of groups reflects the emerging magnetic field and is an indicator of activity. The number of spots within a group depends on the magnetic field as such and also on the interaction between the magnetic and velocity field. In this paper, we had limited ourselves to sunspot numbers. This reviewer suggests that we could study in the same way Group Sunspot Numbers (GSN, Hoyt & Schatten, 1998) in a further paper. Reviewer 2 (Svalgaard) is more pressing, and therefore we have already undertaken some analyses that we include in the revised paper and that provide comforting responses to both reviewers 1 and 2 (page 9, lines 22-28). We agree that the study of GSN is worth discussing in this paper (hence a new paragraph, page 18 - lines 4-14 and a new Appendix B page 21). On the other hand, the paper mainly focuses on changes of the irregularity index with respect to smoothing, which leads to evidence of different "QBO" epochs. Therefore, from a pedagogical point of view, we place the graphs with the computation of λ of *ISSN* and *GSN* with different embedding dimensions in the new appendix B and comment them in the discussion. Despite differences in inhomogeneities and potential problems with the two series, the main results are quite similar, excluding the possibility of an artefact due to the choice of an imperfect time series. The irregularity index of GSN exhibits two different regimes with a clear transition in the period 1915-1940. This strengthens the result obtained for ISSN and published in Shapoval et al (2013) and further supports our approach.

The abbreviation WN is changed to ISSN.

S2. We agree with the reviewer that smoothing of the data over 27*k days, where k is large, leaves some traces of the periodicity related to solar rotation, since the duration of the rotation slightly deviates from 27 days. Following the suggestion of the reviewer, we have looked for possible remaining traces of the 27-day signal in the Fourier spectra of the preprocessed data.

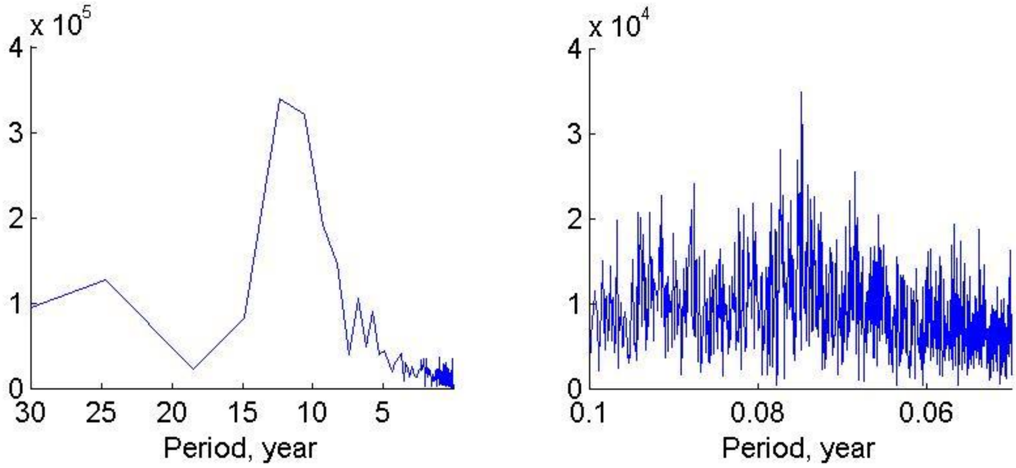

Figure 1. Spectrum (left) of the daily ISSN (1855-1930) and a zoom on the window (right) around solar rotation (27 day) periodicities. Periods rather than frequencies are given on the horizontal axis.

Figure 1 (left) gives a general view of the power spectrum with a prominent peak at approximately 11-year Schwabe cycles. The zoom around 27 days (0.074 yr) shows energy at the solar rotation period (right). Averaging over 162 days largely eliminates solar rotation periodicities, as shown by Figure 2 (left). There is more remaining energy when the averaging is over 648 days. This observation is natural, since averaging (convolution with a boxcar) pins the spectrum down and removes a frequency interval, whose length decreases with increased averaging (properties of the sinc function in Fourier space).

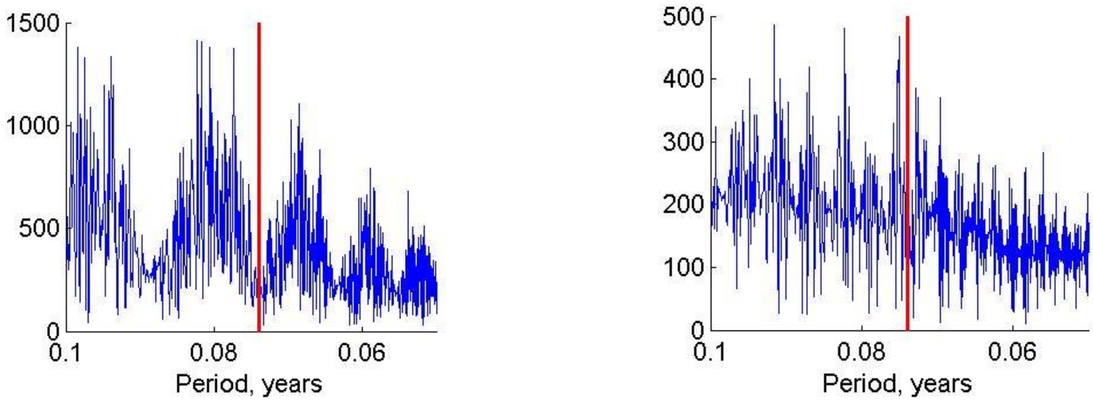

Figure 2. Part of the spectrum of the daily ISSN (1855-1930) averaged first over 162 (left) and 648 (right) days. The red lines indicate 27 day (0.074 yr) period.

In the paper we do not aim at complete elimination of signatures of solar rotation from the data but at their reduction. Their existence does not invalidate our technique. This point is clarified now in the text (page 10, lines 4-8). We have added the reference to Kitchatinov and Olemskoy (2005) and to other papers at the beginning of section 3 (in red).

S3. There are several questions in S3. We answer them separately.

(i) HSV as such are not of a great importance. Our functional λ can achieve its extrema on ascending and descending phases. If it is the case, HSV appear because of a certain similarity between ascending and descending phase. That is why we do not discuss the physics underlying the essence of HSV. On the other hand, we look for a simple time series with properties observed for *ISSN* (section 4). This new paragraph is in the paper on Page 16, lines 18-22.

(ii) When the signal is close to zero (this is the case of Figure 5, but the graph of the model signal is not presented) and the embedding dimension is 1, many points lying at distance 1 are transformed by the translation mapping to points lying at distance 11. As a result, the logarithm of the ratio, log10(11) appears as the value of the irregularity index. This result comes from the very simplicity of the model. Nevertheless, to get reasonable conclusions on sunspot numbers, which are more sophisticated than the simple model signal, we turn from embedding dimension *m*=1 to *m*=2.

(iii) As we understand the main questions of the reviewer (in S3) are "why is our λ large at cycle minima, and can we repeat our observation with a simpler tool? Some suggestions of the answers are in the review. Really, an explicit relation between lambda and level of solar activity is not evident. At least, it is *not* a decreasing function of activity. We agree with the reviewer that *ISSN* and its variation are connected to each other. However the coefficient of variation and its simple modifications do not demonstrate the properties of the irregularity index (see also answer to reviewer 3). Introducing the irregularity index, we try to define such a variation, which seems to be what the reviewer asked for. We believe that, because the irregularity index varies (i) in time and (ii) with smoothing, we see new properties of solar activity. Understanding the physics that underlie the changes of patterns of the irregularity index with time will require further research. At the moment, we can neither accept nor reject that "the maxima of lambda at activity maxima and minima may also arise because of the broadest latitudinal extension of the activity at these phases which might cause higher irregularity".

As to QBO, in this paper we emphasize primary changes of the irregularity index lambda as a function of smoothing. 5.5-year oscillations as such are not surprising because they are generated by 11-year modulation (a simple simulation with a sine-curve supports this statement). It is the dependence on smoothing that needs to be explained. Of course, we recognize the possible non-uniqueness of the solution (page 17, lines 14-17); however, we construct a simple model that incorporates the basic features of ISSN. In the framework of this model, there are 5 parameters to be tuned (page 13, lines 14-20). They correspond, in particular, to the activity level, the lifetime of sunspots and the period of the intermediate oscillations. Tuning them one by one (sections 4.2.2 and 4.2.3), we are able to generate a transition from increasing to decreasing HSV as a function of smoothing only with the parameter that reflects the strength of the 600-700 day variations. That is why, as done by other authors when this period range emerges, we link different regimes of HSV to quasi-biennial variations.

As the reviewer mentioned, a jump of the irregularity index to a new level prior to a general change of solar activity can be considered as a precursor. This consideration underlies another paper "Shapoval, A., Le Mouël, J.-L., Courtillot, V., Shnirman, M.: Is a sudden increase of irregularity of sunspot numbers a precursor of a return to low solar activity?" submitted to the Journal of Geophysical Research.

S4. We agree with the reviewer. But Figures 3 and 10 indicate that in the 1930s, lambda for ISSN and *aa* went in opposite directions (a more or less step-like change) to new levels of values (of oscillations), even in spite of the absence of preliminary smoothing. We find this observation of simultaneous opposite changes of ISSN and *aa* important and therefore worth noting in the paper (page 17, line 30 - page 18, line 7; Figure 10; page 19, lines 23-29). However (also as an answer to the other reviewers), we have eliminated most references to *aa* from the abstract, in order to remove emphasis on that topic at present. Since there is a regime change of *aa* in the 1930-s (exactly at the time when the irregularity index of ISSN coming into a new regime), we find it reasonable to suggest that our irregularity index does measure some physical phenomena (and not artifacts reflecting inhomogeneity of ISSN; see also response to reviewer 2). But at this stage we do not have a more convincing physical argumentation.

S5. The reviewer would like more comments to elucidate how the procedure works. We have attempted to improve this, though we do not have all the answers yet to the remarkable numerical results we have obtained. Let us specify a few things further. The irregularity index is likely to coincide with the Lyapunov exponent for chaotic systems. We checked this conclusion for logistic maps that possess a positive Lyapunov exponent. The irregularity index is also meaningful for the description of stochastic processes. The key point is that white noise is characterized by a positive irregularity index, which decreases to zero when the embedding dimension m increases. But the theoretical zero-value of the irregularity index as well as that of the Lyapunov exponent cannot be approached in a real computation, since we cannot have infinitely (space-)close points. Therefore, the dependence of the irregularity index on m allows one to estimate whether a stochastic component is present in the signal.

Initial smoothing of the signal simplifies the irregularity index and amplifies the importance of the main extrema: by this we mean that originally noisy lambda curves evolve to a simplified quasi-periodical structure with much less noise. They look like quasi-sinusoidal curves, with smooth maxima at solar cycle extrema. This is what we call HSV, since the period (or rather pseudo-period) is about half of the length of a Schwabe cycle. Responses of the *main* extrema to changes of N are worth studying by themselves. This is done with parameter R that measures the respective mean amplitudes of lambda maxima at solar cycle minima (that we did not expect to occur originally) with respect to those at solar maxima (the ones that might have been expected a priori). In other words, R is a measure of HSV (page 11, lines 9-11). A decrease in R corresponds to an increase in HSV. The meaning of the $R(N)$ function is studied with our simple model in section 4. We find that the secondary maxima of the irregularity index nearly disappear with larger smoothing of the signal for the model without intermediate oscillations (Figures 8 and 9). However, if the intermediate oscillations are strong, then secondary maxima (observed at the signal minima) are amplified with a growth of N (Figures 6b and 6c).

We are also thankful to the reviewer for the technical comments.

T1. We changed 1867 to 1870 in the description of table 1. It was a misprint. The minimum in late 1866 is not used in the computation of R. It is for the correct time values that our R values have been calculated and they still stand.

The computation of *R* is straightforward when the signal behaves as a sine-like curve and the minima and maxima are easily detectable (see the curve in Figure 3c as an example). We think that during cycles 11-14 HSV is observed clearly only from the maximum of cycle 11 to the maximum of cycle 12. The computation of *R*, which is equal to 0.67, supports this statement. On the other hand, HSV is easily seen within cycles 11-14 with smoothing *N* equal to 324 and 648. This results in rather similar and relatively large values of *R*: 0.83 and 0.79.

We do not discuss in the paper the left boundary of the regime characterized by amplification of HSV with smoothing, which is difficult to detect. We concentrate on the right boundary, at which lambda changes to an epoch with messy HSV for any value of smoothing.

T2. We unified the notation. The changes are in red.

T3. We wrote an introductory sentence, split the formal description into small paragraphs, each of which determines one quantity, and named these paragraphs. We also added : "With respect to the Lyapunov exponent, in order to determine the irregularity index, we relax the requirement that close points in the phase space must be remote along the time axis (page 7, lines 6-7)."

T4. We re-typed the symbol.

Anonymous Referee #1

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Specific comments

S1) I have doubts about the suitability of the sunspot number in its present form. It is an interesting contradiction that its importance is tremendous in long-term studies although its physical meaning is really ambiguous. The problem is that the numbers of sunspot groups and the individual sunspots are indicators of two different physical processes which are mixed in the present form of the time series. The number of groups is an indicator of the activity level i.e. the amount of the emerging field, whereas the number of spots within the groups depends on the fragmentation of the flux ropes which is a matter of interaction between the magnetic and velocity fields. This is one of the motivations to revise this time series. One possible solution is the Group Sunspot Number (GSN) compiled by Hoyt and Schatten. I do not suggest to repeat the procedure with GSN in the present paper but it may be worth considering in a further analysis and the ambiguous background of the recent sunspot number might be mentioned here. By the way, the official name of the sunspot number is recently International Sunspot Number (ISSN), I would recommend using it. The following address also recommends the correct reference to it:
http://sidc.oma.be/sunspot-data/SIDCpub.php
in the following way:
SIDC-team, World Data Center for the Sunspot Index, Royal Observatory of Belgium,
Monthly Report on the International Sunspot Number, online catalogue of the sunspot
index: http://www.sidc.be/sunspot-data/, 'year(s)-of-data'

S2) The pre-procession (Third section, Data analysis) has not been executed in P1. One should obviously get rid of the signal of the rotation which is an observational effect but the oldest active regions live as long as about three rotations. If after their decay another AR emerges at the same location its contribution is not an observational artifact but a component of the non-irregular behavior of the sunspot activity. See among others the paper of Kitchatinov and Olemskoy (2005, Ast.Lett. 31, 280) about the active longitudes, (by the way, their rotation rate is different from that of the Carrington frame). The question: is it possible that a N=162 smoothing leaves some signatures of the rotation in the time series? This could be checked with a power spectrum. I have the impression that smoothing with high N may be an exaggeration. A brief explanation is welcome.

S3) P1 reports a half-Schwabe variation (HSV) emerging in the irregularity index by computing with m=5. The high values at activity minima are conspicuous and also the difference between the years before and after the thirties, this is also reported in this paper. However, there may be an interesting relationship between the values of lambda and the activity levels at minima. In P1 Fig.3 the lambda is high at the very weak activity minima prior to the 1930s but it drops at minima after the 30s where the minima are not as weak as earlier. The best example is the last minimum prior to cycle 24, this was an extremely inactive minimum, sometimes several weeks passed without observable spots and the minimum-lambda was high again (is this a signature of a new regime?) . This is not so obvious in the present paper because of the averaging but the trend is similar. My question: is it possible that the behavior of lambda is just a consequence of a variation which is recognizable without any irregularity analyses? Furthermore, the maxima of lambda at activity maxima and minima may also arise because of the broadest latitudinal extension of the activity at these phases which might cause higher irregularity (?) . It is conspicuous that Fig.5 of the recent paper does not exhibit a variation of the lambda peaks at activity minima presumably because the model does not contain a modulation of the minimum levels. A comment on the mentioned (or any further) alternative explanations would be welcome. I dont claim that the explanation with QBO cannot be correct, I just conjecture that there may be a more simple interpretation which could be checked more easily and directly. Can the authors exclude it?

S4) I do not know whether the comparison with the aa-index can convince the readers. It has two distinct components, the coronal holes are the sources of recurrent disturbances for several rotations (Bartels) and if the 27-day signal is not filtered out from the time series then it may more strongly predominate the regularity than in the sunspot dataset because it is definitely an observational effect whereas sunspots may repeatedly emerge at the active longitudes which is a physical effect. The other component of the variations is the series of eruptive events, this is expectedly even more irregular than the simple sunspot emergence. Moreover, the solar impacts are also modulated by a further observational effect, the semiannual variation (Russell-McPherron, Rosenberg-Coleman) so the aa is more contaminated with known non-solar regularities than the sunspot index. The lambda of aa-index has definitely minima at activity minima (Fig.10), presumably because of the regular signal of the coronal holes in the poloidal phase. Is this possible? The pre-procession and the irregularity curves of the sunspot index and aa index are different but the authors guess: "...but the same singularity in solar behavior could be at the origin of both." This does not seem to be a corroboration, just a conjecture, it deserves a more convincing argumentation.

S5) By reading the text I was wondering what is the answer to the question of the main title, in other terms, what is the heuristic potential of the irregularity analysis in this case. We use a time series, the sunspot index, which is a Sun-as-a-star parameter disregarding many relevant details, we carry out two sophisticated procedures (the analyses of irregularity and autoregression) and draw a conclusion about the role of the mid-term variations. The procedures are similar to a black box. For instance the reader do not see the role of the averaging although it is an emphasized conclusion that the R parameter increases with growing N in the time interval 1870-1910. What is the meaning of an N-dependent R? What is behind the m-dependence of the lambda fluctuation? Some comments would elucidate how the procedure works.

Technical comments:
T1) Table 1 and Fig.3 do not seem to support the claim that R increases with increasing N between 1870-1910. For N: 162-324-648 parameter set R is: 0.67-0.83-0.79. I scrutinized Fig.3 and I have the impression that lambda_min at N=162 is 0.19 rather than 0.23. If so, R=0.75 at N=162, thus the series R is 0.75-0.83-0.79 for increasing N. This seems to be a mere fluctuation. It is also disturbing that the start of this earlier interval is indicated earlier than 1870 in Fig.3 and the plot is not in accordance with the data of Table 1. Please, check the data and the plot.

T2) There are apparent contradictions between the definitions of R, please check the use of greek letters: capital-Delta and small-delta and the possible reciprocal versions of R by comparing: i) page 167 last but one paragraph, ii) the caption and marks of Fig.3 (including lambda_mid) and iii) the caption of Table 1 (where the definition of delta_S_max is also suspicious).

T3) Please, consider a more straightforward description of the irregularity index. Section 2.2 contains too many indexes with their combinations, and variables. It can remain as it is but at the beginning a brief summary could enlighten the train of thought to facilitate the reading.

T4) A simple typological remark: I was embarrassed to see "theta" instead of the Euler’s number (e) in the first equation in 2.1.1, and it turned out only in larger zoom that it is really "e", I would prefer a different font here and in section 4.1, third row for clarity.