Nonlin. Processes Geophys. Discuss., 1, C195–C202, 2014
www.nonlin-processes-geophys-discuss.net/1/C195/2014/

Nonlinear Processes in Geophysics Discussions
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Interactive comment on "Effective coastal boundary conditions for tsunami wave run-up over sloping bathymetry" *by* W. Kristina et al.

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Received and published: 31 May 2014

Thank you for the review given by Anonymous Referee #1. The suggestions and detailed comments by the Anonymous Referee #1 are answered as follows.

1. We already have reference to Brocchini and Peregrine (1996) on page 328.

2. On page 320, line 4-15, the paragraph will be revised as follows:
The shoreline position and wave reflection in the model area (sloping region) are determined using an analytical solution of the nonlinear shallow water equations (NSWE) following the approach of Antuono and Brocchini (2010) for unbroken waves. The decomposition of the incoming wave signal and the reflected one

is also described in Antuono and Brocchini (2007,2010) for the calculation of the shoreline and wave reflection. Nevertheless, the method in their paper is applied by determining the incoming wave signal with the solution of the Korteweg-de Vries (KdV) equation. The novelty of our approach is the utilization of an observation operator at the boundary $x = B$ to calculate the incoming wave elevation towards the shore from the numerical solution of the LSWE in the simulation area. For any given wave profile and bathymetry in the simulation area, the numerical solution can be calculated and the signal arriving at $x = B$ can be observed. Afterwards, the data are used to calculate the analytical solution of the NSWE in the onshore region and the reflected waves.

We also have fixed the typo in the citations.

3. Equation (1) is Miles' variational principle (Miles, 1977) that can be rewritten in terms of velocity potential Φ and wave elevation η as follows

$0= \delta \int_0^T \mathcal{L}[\Phi,\eta]\mathrm{dt} = \delta \int_0^T \int_{x_s}^L \int_{-h_b}^{\eta} \left(\partial_t \Phi + gz + \frac{1}{2}|\nabla\Phi|^2\right)\mathrm{dzdxdt}$

Arbitrary variations of the functional with respect to η gives result

$\partial_t\Phi + g\eta + \frac{1}{2}|\nabla\Phi|^2 = 0$ at $z = \eta$.

This is Bernoulli equation which states that the pressure at the surface of the water should vanish (it is the assumed pressure condition for the variational formulation of full surface wave problem).

We do not fully understand the remark from Anonymous Referee #1. In the full water wave problem, there is no depth-averaged flow. From Eq. (1), we derive a Boussinesq version with a simplified vertical structure (Eq. (2)). There is no arbitrary constrant, as the surface potential ϕ is specified by the initial condition.

Without such an initial condition, of course, ϕ only appears under space or time gradients. Hence, also the numerical implementation is fine, because it requires an initial condition for ϕ (and not its gradient). Of course, if one specifies the initial velocity u, this initial condition for ϕ is indeed specified up to a constant, but once ϕ is specified that initial constant is automatically fixed as well.

4. In the linear wave theory, it is assumed that the bottom variations and surface elevations are small compared to other dimensions. Equation (3) arises from Miles' variational principles (Miles, 1977) in Eq. (1) as follows

   $0= \delta \int_0^T \mathcal{L}[\phi, \Phi, \eta, x_s]\mathrm{dt}$
   $= \delta \int_0^T \int_{x_s}^L \Big(\phi \partial_t \eta - \frac{1}{2} g((h+b)^2 - b^2) - \int_{-h_b}^{\eta} \frac{1}{2} |\nabla \Phi|^2 \mathrm{dz}\Big) \mathrm{dxdt}$

   with velocity potential $\Phi = \Phi(x,z,t)$, surface potential $\phi(x,t) = \Phi(x, z=\eta, t)$, where $\eta = h - h_b$ is the wave elevation and $h = h(x,t)$ the total water depth above the bathymetry $b = -h_b(x)$ with $h_b(x)$ the rest depth. Time runs from $t \in [0,T]$; partial derivatives are denoted by ∂_t et cetera, the gradient in the vertical plane as $\nabla = (\partial_x, \partial_z)^T$ and the acceleration of gravity as g.
   The second term in the equation above is the potential energy and the third term is the kinetic one. We follow Klopman, et al. (2010) for approximating the velocity potential Φ and thus we get Eq. (3) with the functions $\breve{\beta}(x), \breve{\alpha}(x)$, and $\breve{\gamma}(x)$ are given by Eq. (4).

   The linear model is obtained by assuming that the the bottom variations and surface elevations are small, thus the kinetic energy in the equation above are obtained by integration in the vertical z-axis from $z = -h_b$ to $z = 0$ in $x \in [B, L]$ as follows

   $0= \delta \int_0^T \mathcal{L}[\phi, \Phi, \eta, x_s]\mathrm{dt}$
   $= \delta \int_0^T \int_B^L \Big(\phi \partial_t \eta - \frac{1}{2} g((h+b)^2 - b^2) - \int_{-h_b}^{0} \frac{1}{2} |\nabla \Phi|^2 \mathrm{dz}\Big) \mathrm{dxdt}$

   By replacing the approximation for the velocity potential and rewriting ϕ and η as $\breve{\phi}$ and $\breve{\eta}$, we get Eq. (5a) with the functions $\breve{\beta}(x), \breve{\alpha}(x)$, and $\breve{\gamma}(x)$ are given by Eq. (6). The linear variables mentioned in the article refer to the variables in the linear model.

   We will revise the last paragraph on page 323 and state as follows:
   We a priori divide the domain into two intervals, $x \in [B, L]$, where we model the wave propagation linearly, and $x \in [x_s(t), B]$, where we keep the nonlinearity. To be precise, in the simulation area from $x \in [B, L]$, we linearize the equations and thus the wave propagation in this domain is modeled by linear shallow water shallow water equations and a linear yet dispersive Boussinesq model. In the model area $x \in [x_s(t), B]$, we only consider depth-averaged shallow water flow. Thus, a non-dispersive and nonlinear shallow water equations are used to model the wave propagation in this region. Hereafter, we write $\breve{\phi}$ and $\breve{\eta}$ for the linear variables and also the definitions of $\breve{\beta}$, $\breve{\alpha}$ and $\breve{\gamma}$ simplify accordingly. Consequently, by applying the corresponding approximations to variational principle (3), the (approximated) variational principle becomes . . . .

   On page 324 line 9, we will add explanation as follows:
   Hence, the coefficients in Eq. (4) simplify to their linearized counterparts in the simulation area where the linear Boussinesq equation holds (while these coefficients disappear in the model area where the nonlinear depth-averaged shallow water equations hold).

5. Equations (13a) and (14) are only identical for the coupling between linear (Boussinesq and shallow water) model with the nonlinear shallow water model. In the linear domain (where we have $\breve{\phi}$ and $\breve{\psi}$), Eqs. (13a) and (13b) together must be applied to transfer the information from the nonlinear domain (where we

only have ϕ) to the linear domain. While in the nonlinear domain, the coupling condition is only given by Eq. (14).
In general, the coupling conditions in both domains will not be identical. For example, the coupling conditions between linear potential flow and shallow water model are derived by Klaver (2009).

We will revise the paragraph on page 327 line 11-15 as follows:
Note that the coupling conditions (13)-(14) are used to transfer the information between the two domains. The coupling conditions (13) gives the information of $\breve{\phi}$ and $\breve{\psi}$ in simulation area, provided the information of ϕ from model area. Meanwhile, the coupling condition (14) gives the information of ϕ in model area, provided the information of $\breve{\phi}$ and $\breve{\psi}$ from simulation area.

6. We will revise page 330 line 25 and state as follows:
This article follows the approach of Antuono and Brocchini (2010) which uses this incoming Riemann variable as boundary data and solve the dimensionless NSWE by direct use of physical variables instead of using the hodograph transformation introduced by Carrier and Greenspan (1958). We do, however, clarify the mathematics of the boundary condition at the shoreline.

7. We will revise page 339 lines 21 and state as follows:
This superposition is also described in Antuono and Brocchini (2007, 2010) and actually in line with our EBC concept since the linearity holds in the simulation area.

8. Page 340, lines 2-5 will be deleted. The citation to Antuono and Brocchini (2007, 2010) has been done on the previous page (comment 7).

9. Yes, the linearity ratio δ is in agreement with the definition of ϵ on page 332, line 20. For consistency, the parameter δ will be replaced with ϵ in the whole article.

Page 332, line 20, we will revise as follows:
We expand it in perturbation series around the rest solution (23) with the assumption of small data at $x = B$. Using the linearity ratio $\epsilon = A/h_0$ (A is the wave amplitude), we say a wave is small if $\epsilon \ll 1$ and expand as follows:

Equation (58) is used to determine the location of the seaward boundary condition. In the subsequent study cases, we choose the value of $\delta \ll 1$ to calculate the value of h_0.

10. Page 346, the first paragraph of the conclusions will be revised as follows:
We have formulated a so-called effective boundary condition (EBC), which is used as an internal boundary condition within a domain divided into simulation and model areas. The simulation area from the deep ocean up to a certain depth at a seaward boundary point at $x = B$ is solved numerically using the linear shallow water equations (LSWE) and the linear variational Boussinesq model (LVBM). The nonlinear shallow water equations (NSWE) are solved analytically in the model area from this boundary point towards the coastline over a simplified sloping bathymetry. The wave elevation at the seaward boundary point is decomposed into the incoming signal and the reflected one, as described in Antuono and Brocchini (2007,2010). The advantages of using this EBC are the ability to measure the incoming wave signal at the boundary point $x = B$ for various shapes of incoming waves, and thereafter to calculate the wave run-up and reflection from these measured data. To solve the tsunami wave run-up

in nearshore area analytically, we employ the asymptotic technique for solving the NSWE over sloping bathymetry derived by Antuono and Brocchini (2010), applied to any given wave signal at $x = B$.

11. Ryrie's model decouples the longshore problem from the onshore one. By doing this, we neglect any effect on the onshore motion of interaction between onshore and longshore motion. It is justifiable for waves incident at a small angle to the beach. Ryrie (1983) numerically solves the 2D shallow water equations for motion on a sloping beach generated by a single bore and by a periodic succession of bores, both incident at small angles. Brocchini and Peregrine (1996) use Ryrie's approach to get 2D analytical solution of shallow water equations for periodic unbroken waves (extending the solution of Carrier and Greenspan (1958)).
    We still think that the extension of the EBC method in 2D can be done by using Ryrie's approach. The onshore problem is solved using the same approach of Antuono and Brocchini (2010), and the solution of the decoupled longshore problem is left for further study.

References

Antuono, M. and Brocchini, M.: The boundary value problem for the nonlinear shallow water equations, Stud. Appl. Math., 119, 73-93, 2007.

Antuono, M. and Brocchini, M.: Solving the nonlinear shallow-water equations in physical space, J. Fluid Mech., 643, 207-232, 2010.

Brocchini, M. and Peregrine, D. H.: Integral flow properties of the swash zone and averaging, J. Fluid Mech., 317, 241-273, 1996.

Carrier, G. F. and Greenspan, H.P.: Water waves of finite amplitude on a sloping beach, J. Fluid Mech, 4, 97-109, 1958.

Klaver, F.: Coupling of numerical models for deep and shallow water, University of Twente, M. Sc. thesis, 2009.

Klopman, G., Van Groesen, E., and Dingemans, M.: A variational approach to Boussinesq modelling of fully non-linear water waves, J. Fluid Mech., 657, 36-63, 2010.

Miles, J. W.: On Hamilton's principle for surface waves, J. Fluid Mech., 83, 153–158, 1977.

Ryrie, S. C.: Longshore motion generated on beaches by obliquely incident bores, J. Fluid Mech., 129,193–212, 1983.

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