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Estimation of flow velocity for a debris flow via the two-phase fluid model

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Abstract

The two-phase fluid model is applied in this study to calculate the steady velocity of a debris flow along a channel bed. By using the momentum equations of the solid and liquid phases in the debris flow together with an empirical formula to describe the interaction between two phases, the steady velocities of the solid and liquid phases are obtained theoretically. The comparison of those velocities obtained by the proposed method with the observed velocities of two real-world debris flows shows that the proposed method can estimate accurately the velocity for a debris flow.

1 Introduction

A debris flow is the gravity flow of soil, rock and water mixtures, which is frequently initiated by a landslide, and is one of the common potential hazards throughout the world. For examples, the debris flow in Zhouqu (China) on 8 August 2010 killed about 1700 people (Wang, 2013), and the debris flow that occurred in Afghanistan on 2 May 2014 killed more than 2000 people (Ahmed and Kakar, 2014). Debris flows can often occur following bush and forest fires. They pose a significant hazard in lots of mountainous, steep areas, and have received particular attention in China, Japan, the USA, Canada, New Zealand, the Philippines, the European Alps, Himalaya-Karakorum, Kazakhstan and Russia.

The typical characteristics of the multi-phase fluid exhibited by a debris flow have been demonstrated by many field observations (O'Brien et al., 1993; Hutter et al., 1996; Hutter and Schneider, 2010a, b). By Takahashi's discussion (Takahashi, 2007), it has been found that a low-viscous debris flow with a density higher than $1400\,\mathrm{kg\,m^{-3}}$ would contain a non-sediment fluid in which the diameter of granules is smaller than $0.05\,\mathrm{mm}$, whereas the high-viscous debris flow with a density higher than $1900\,\mathrm{kg\,m^{-3}}$ would contain a non-sediment fluid in which the diameter of granules is smaller than $2\,\mathrm{mm}$. We refer to these values as the critical diameters for a debris flow. The non-sediment fluid, composed of water and viscous, fine, non-sediment particles, is in the liquid phase in a debris flow, which behaves as non-Newtonian fluid. The

solid phase is composed of particles whose diameter is larger than the critical diameter. This characteristic of a debris flow is aptly described by the two-phase model (Anderson and Jackson, 1967; Iverson, 1997; Iverson and Delinger, 2001; Pitman and Le, 2005; Pudasaini et al., 2005; Pudasaini, 2012). However, the two-phase models describing debris flows are still in development stages. Although, recently there have been substantial advances in simulating real two-phase debris flows (Pudasaini, 2012, 2014), construction of exact solutions are still very challenging (Khattri, 2014).

To understand the dynamics of the debris flow, including its initiation, runout and deposition, finding out the velocity of the debris flow is important, which would be helpful to analyze and forecast the dynamics of the debris flow and then prevent its hazards. The reason for this is that soils or rocks, and fluid involved in a debris flow cause the dynamics of the debris flow to become more complicated, especially the existence of interactions between the solid particles and the fluid (Pudasaini, 2012). As observed in natural debris flow, the velocities of the solid and liquid phases may deviate substantially from each other, essentially affecting flow mechanics (Prochaska et al., 2008; Pudasaini and Domnik, 2009; Pudasaini, 2011, 2012; Revellino et al., 2004; Rickenmann et al., 2006; Teufelsbauer et al., 2009; Uddin et al., 2001; Yang et al., 2011; Zhu, 1992). Pudasaini (2011) presented exact solutions for debris flow velocity for a fully two-dimensional channel flows in which the velocity field through the flow depth and also along the channel have been derived analytically. Several other models have been introduced to estimate the velocity of the debris flow, such as the Fleishman formula (Fleishman, 1970) and the mean velocity formula (Takahashi, 1991; Hashimoto and Hirano, 1997; Julin and Paris, 2010; Hu et al., 2013). These models provide some rough estimations of the flow velocity and are applied to predict the risk of the debris flow. But the assumption of one-phase flow for these models leads to large modelling errors. Few theoretical results have been obtained to estimate the solid- and liquid-phase velocities for a two-phase debris flow (Chen et al., 2004; Chen et al., 2006). Although some empirical formulas are introduced to calculate the velocity of a debris flow at special location, such as the K631 debris flow locating at the Tianshan highway in Xinjiang Province of China and the Pingchuan debris flow locating at the trunk highway from Xichang

mitigation || strange Eng.!

Nevertheless

City to Muli County in Liangshan Yi Autonomous Prefecture, Sichuan Province, China (Chen et al., 2006). Given that, there is no a general formula to calculate the velocity of a debris flow.

In this study, the two-phase flow model is applied to analyze the velocity of a debris flow. To focus on the velocity of the debris flow along the channel, a simplified, one-dimensional, two-phase model is consider here, and the motion equations governing the solid and liquid phases are deduced. Following the discussions of Bagnold (1954), the interaction between the solid and liquid phases is obtained and the velocities of the solid and liquid phases in a debris flow are obtained theoretically. This result provides a new theoretical method for estimating the velocities of the solid and liquid phases for a debris flow, which would be a useful factor for evaluating the damage of a debris flow, estimating its arrival time, simulating its deposition area, predicting its risk, and so on. By comparing the theoretical results for the velocity and the empirical formulas for two natural debris processes, the numerical results show that the proposed method could more accurately provide velocities of solid and liquid phases for a debris flow.

derived

caused by

and

This study is arranged as follows: in Sect. 2, the formulas to calculate the velocities of a debris flow are deduced, and in Sect. 3, the numerical validation of the theoretical results is made by means of two real-world debris flows. The conclusions are presented in Sect. 4 and a complete summary of mathematical notation is provided in Table A1.

Eg.!

2 Velocity estimation of a debris flow

Two difficulties arise in the calculation of the velocity of a debris flow: one is that the diameters of the solid phase particles are in a wide range, and the other is that the interaction between the solid phase particles and liquid phase slurry is difficult to describe exactly. However, recently, by developing a general two-phase debris flow model, Pudasaini (2012) included several important physical aspects of the real two-phase debris mass flows with strong phase-interactions, including the generalized drag, virtual mass force, Newtonian, and solid particle concentration gradient enhanced non-Newtonian viscous stresses. These model equations have also been put in well structured and conservative form. Numerical simulations and possible applications of

these models can be found in Pudasaini (2014), Pudasaini and Miller (2012a, b). In order to deal with the solid particles with different diameters, the diameter-equivalent method (Brunelli, 1987), which treats all particles with different diameters as the particles with the same diameter, is applied in this study.

In order to build a simple model for a debris flow to estimate the velocities of its solid and liquid phases, the following assumptions are made:

1. In this study, the downstream direction is set as the x direction, while the vertical direction to the channel bed is the y direction, see Figs. (a)-(b) (Chen et al., 2006) and Fig. 1. We assume that the velocity along the y direction is uniform, and thus the one-dimensional model for debris flow is mainly considered.

No mass entrainment is considered

2. There are no external materials involved in the debris flow, and there is no transformation between the solid phase particles and liquid phase slurry. Three inner forces are involved in the model: the interactions among the solid phase particles, the interactions in liquid phase slurry and the interactions between the solid phase particles and liquid phase slurry.

3. A debris flow is assumed to be a homogeneous flow (Major and Iverson, 1999; Kaitna et al., 2007).

with these

Under the above assumptions and following the two-phase flow theory (see, e.g., Pudasaini, 2012 for more detail), the governing equations for a debris flow are obtained, which are written separately for the solid and liquid phases, denoted by subscripts s and f, respectively. The mass conservation equations for the two phases are written as

$$\frac{\partial}{\partial t}(\rho_s \varphi) + \nabla \cdot (\rho_s \varphi v_s) = 0, \tag{1}$$

$$\frac{\partial}{\partial t}[\rho_f (1-\varphi)] + \nabla \cdot [\rho_f (1-\varphi) v_f] = 0. \tag{2}$$

for notational convenience use $\phi \to \phi_s$, $1-\phi \to \phi_f$.

Since you consider only spatially 1D problem: $\nabla \cdot \to \frac{\partial}{\partial x}$

Since only 1D flow is considered, better would be:
$v_{sx} \to v_s$
$v_{fx} \to v_f$
$b_{sx} \to b_s$
$b_{fx} \to b_f$
$f_{sx} \to f_s$
$f_{fx} \to f_f$.
explain all notations here. v – depth-averaged velocity, etc.

The momentum equations for the two phases take the forms (with the buoyancy effect considered)

$$\varphi\rho_s\left[\frac{\partial v_s}{\partial t}+(v_s\cdot\nabla)v_s\right]=b_s+f_s-\varphi\nabla P_s, \tag{3}$$

$$(1-\varphi)\rho_f\left[\frac{\partial v_f}{\partial t}+(v_f\cdot\nabla)v_f\right]=b_f+f_f-(1-\varphi)\nabla P_f. \tag{4}$$

repeated!!

For detailed model derivation, and how different types of forces and interactions can arises and should be introduced in a real two-phase mass flow model, we refer to Pudasaini (2012). However, In this study, we are mainly concerned with the one-dimensional model of a debris flow. Thus the motion equation can be re-written:

$$\varphi\rho_s\left(\frac{\partial v_{sx}}{\partial t}+v_{sx}\frac{\partial v_{sx}}{\partial x}\right)=b_{sx}+f_{sx}-\varphi\frac{\partial P_s}{\partial x}, \tag{5}$$

$$(1-\varphi)\rho_f\left(\frac{\partial v_{sx}}{\partial t}+v_{fx}\frac{\partial v_{fx}}{\partial x}\right)=b_{fx}+f_{fx}-(1-\varphi)\frac{\partial P_f}{\partial x}. \tag{6}$$

In order to estimate the velocities of a debris flow using Eqs. (5) and (6), the volume forces (b_{sx} and b_{fx}) in a unit volume, pressures (P_s and P_f), and surface forces (f_{sx} and f_{fx}) in a unit volume beyond pressure (e.g., liquid resistance every phase, apparent mass force derived from acceleration and difference of velocity, and interaction between particles, see, Chen, et al., 2006) firstly need to be given. The pressure for a debris flow can be calculated by

other than

$$P=k\rho v^2, \tag{7}$$

v2

where the density ρ takes the form

$$\rho=\varphi\rho_s+(1-\varphi)\rho_f, \tag{8}$$

pressure

and the non-uniform coefficient k is about 2.4–3.0 for a viscous debris flow; k is about 3.5–4.0 for a thin debris flow (Chen et al., 2011). According to Eq. (7), the pressures of the solid and

liquid phases in the x direction can be rewritten as

$$P_{\mathrm{s}} = k\rho_{\mathrm{s}} v_{\mathrm{s}x}^2, \tag{9}$$

$$P_{\mathrm{f}} = k\rho_{\mathrm{f}} v_{\mathrm{f}x}^2. \tag{10}$$

The velocity of the debris flow in x direction takes the form

$$\overline{v} = \frac{\rho_{\mathrm{s}}\varphi v_{\mathrm{s}x} + (1-\varphi)\rho_{\mathrm{f}} v_{\mathrm{f}x}}{\rho}. \tag{11}$$

In a debris flow, the solid particles move parallel to the liquid phase slurry. By considering the gravity and the buoyancy of solid particles, the volume force of the solid phase is written as

$$b_{sx} = \varphi(\rho_{\mathrm{s}} - \rho_{\mathrm{f}}) g \sin\theta, \tag{12}$$

g?

which is related to the buoyancy reduced normal load (see, e.g., Pitman and Le, 2005; Pudasaini, 2012). The volume force of the liquid phase is written as

$$b_{fx} = (1-\varphi)\rho_{\mathrm{f}} g \sin\theta. \tag{13}$$

debris flow

divided into

In this study, for two-phase in a unit volume, the surface forces on control volume can been classified four parts by Chen et al. (2006). The surface forces of the solid phase f_{sx} on control volume is divided into two parts: the traction of liquid phase slurry outside control volume, f_{sx1}, and the force from the solid phase particles outside control volume, f_{sx2}. The surface forces of the liquid phase $f_{\mathrm{f}x}$ on control volume is divided into two parts: the resistance from the solid phase particles outside control volume, denoted by $f_{\mathrm{f}x1}$, and the resistance from the liquid phase slurry outside control volume, denoted by $f_{\mathrm{f}x2}$. The particle number N in a unit volume is given by

similarly the

for

$$N = \frac{6\varphi}{\pi d_{\mathrm{e}}^3}. \tag{14}$$

d_e?

The cross-section A_0 of the solid phase taken as

$$A_0 = \frac{\pi d_e^2}{4} N = \frac{3\varphi}{2d_e}, \tag{15}$$

on which the pressure difference between the solid and liquid phases is acting, thus using Eqs. (9) and (10), f_{sx1} is written as

$$f_{sx1} = (P_f - P_s)A_0 = \frac{3k\varphi}{2d_e}(\rho_f v_{fx}^2 - \rho_s v_{sx}^2). \tag{16}$$

Further, the traction from the liquid phase slurry outside control volume f_{sx1} and the resistance from the solid phase particles outside control volume f_{fx1} are equal and opposite, i.e.,

OK!

$$f_{sx1} = -f_{fx1}. \tag{17}$$

The force from the solid particles outside control volume mainly appears in the form of impact among all the solid particles. The mechanical effects of impact appear as the dispersion stress among all the solid particles, P_0, and the shear stress among the particles, T_0 (Chien, 1989). Following Bagnold (1954), P_0 and T_0 can be written as

$$P_0 = 0.042 \cos\alpha_i \rho_s (\lambda d_e)^2 \left(\frac{dv_{sy}}{dy}\right)^2,$$

$$T_0 = P_0 \tan\alpha_i,$$

repeated !!

where α_i is the dispersion angle after impact among the solid particles in a debris flow and $\lambda = 1/[(\alpha^0/\alpha)^{1/3} - 1]$ is the linear fraction for the solid particles in a debris flow, where α^0 is the maximum possible static volume fraction for the solid particles. The dispersion stress P_0 and the shear stress T_0 along the downstream direction in a control volume also take the forms (Chen et al., 2006)

(so the pressure)

$$P_0 = 0.013 \rho_s (\lambda d_e)^2 \left(\frac{dv_{sy}}{dy}\right)^2,$$

$$T_0 = 0.028 \rho_s (\lambda d_e)^2 \left(\frac{dv_{sy}}{dy}\right)^2,$$

Why is $T_0 > P_0$? Is something wrong here??

and thus f_{sx2} takes the form

$$f_{sx2} = \int_0^{d_0} (P_0 + T_0)\mathrm{d}y = \int_0^{d_0} 0.041\rho_s(\lambda d_e)^2 \left(\frac{\mathrm{d}v_{sy}}{\mathrm{d}y}\right)^2 \mathrm{d}y. \quad (18)$$

d_0 ??

As the liquid phase slurry in a debris flow can be regarded as a generalized Bingham viscoplastic viscoplastic material (Takahashi, 2007; Chen et al., 2006), the rheological equation of the Bingham material can reflect the internal viscous resistance of liquid phase slurry (Chen et al., 2006), i.e.,

$$\tau = \tau_B + \mu\frac{\mathrm{d}v_{fy}}{\mathrm{d}y} - \rho_f l^2 \left(\frac{\mathrm{d}v_{fy}}{\mathrm{d}y}\right)^2,$$

where l is the moving distance of eddies in the liquid phase slurry under the fluctuation effect, which can be written as $l = \eta y$, where η is the turbulence constant obtained by experiments, and y is the internal depth of the debris flow body. Then the resistance of liquid phase slurry in a control volume f_{fx2} can be written as

$$f_{fx2} = \int_0^{d_0} \tau \mathrm{d}y = \int_0^{d_0} \left[\tau_B + \mu\frac{\mathrm{d}v_{fy}}{\mathrm{d}y} - \rho_f l^2 \left(\frac{\mathrm{d}v_{fy}}{\mathrm{d}y}\right)^2\right] \mathrm{d}y. \quad (19)$$

Now, we assume that the velocity of liquid phase slurry with respect to y satisfies a quadratic function (Chen et al., 2006), i.e.,

$$v_{fy} = ay^2 + by + c, \quad (20)$$

determined

where the coefficients a, b and c are obtained by experiments. Then, using Eqs. (19) and (20), we can further obtain

$$f_{fx2} = -\frac{4\rho_f a^2\eta^2 d_0^5}{5} - ab\rho_f\eta^2 d_0^4 - \frac{\rho_f b^2\eta^2 d_0^3}{3} + a\mu d_0^2 + (\tau_B + \mu b)d_0. \quad (21)$$

There are several model parameters in the proposed model including a, b, c, d_0, k, etc. Constraining these parameters could be challenging. Such parameters, which could also be used as fit parameters, however, do not appear in a real two-phase debris flow model such as that presented by Pudasaini (2012). If the effect of turbulence in the liquid slurry is not considered, then Eq. (21) can be simplified as

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$$f_{\mathrm{f}x2} = a\mu d_0^2 + (\tau_{\mathrm{B}} + \mu b)d_0. \tag{22}$$

f is assumed to be

Further, if the velocity of liquid phase slurry with respect to y submit to linear function, i.e. $a = 0$, then Eq. (22) can be simplified as

$$f_{\mathrm{f}x2} = (\tau_{\mathrm{B}} + \mu b)d_0. \tag{23}$$

and neglecting the y-variation of v_{sy}

Combining Eqs. (16) and (18) yields

$$f_{\mathrm{s}x} = f_{\mathrm{s}x1} + f_{\mathrm{s}x2} = \frac{3k\varphi}{2d_{\mathrm{e}}}(\rho_{\mathrm{f}}v_{\mathrm{f}x}^2 - \rho_{\mathrm{s}}v_{\mathrm{s}x}^2) + \int_0^{d_0} 0.041\rho_{\mathrm{s}}(\lambda d_{\mathrm{e}})^2 \left(\frac{\mathrm{d}v_{\mathrm{s}y}}{\mathrm{d}y}\right)^2 \mathrm{d}y. \tag{24}$$

not used!

Combining Eqs. (17) and (21) yields

$$f_{\mathrm{f}x} = f_{\mathrm{f}x1} + f_{\mathrm{f}x2} = -\frac{3k\varphi}{2d_{\mathrm{e}}}(\rho_{\mathrm{f}}v_{\mathrm{f}x}^2 - \rho_{\mathrm{s}}v_{\mathrm{s}x}^2) - \frac{4\rho_{\mathrm{f}}a^2\eta^2 d_0^5}{5} - ab\rho_{\mathrm{f}}\eta^2 d_0^4 - \frac{\rho_{\mathrm{f}}b^2\eta^2 d_0^3}{3} + a\mu d_0^2 + (\tau_{\mathrm{B}} + \mu b)d_0. \tag{25}$$

repeated!!

Next, we will take steady flow of debris flow (Chen, 1988; Chen et al., 2004; Jan and Shen, 1997) and linear distribution of velocity of liquid phase slurry with respect to y (Chen et al., 2006) as an example. Then Eq. (25) can be written as

$$f_{\mathrm{f}x} = -\frac{3k\varphi}{2d_{\mathrm{e}}}(\rho_{\mathrm{f}}v_{\mathrm{f}x}^2 - \rho_{\mathrm{s}}v_{\mathrm{s}x}^2) + (\tau_{\mathrm{B}} + \mu b)d_0. \tag{26}$$

To simplify the calculation, the velocity variation of solid phase particles along depth of debris flow body is omitted (Chen et al., 2006), then Eq. (24) can be taken the form

$$f_{sx} = \frac{3k\varphi}{2d_e}(\rho_f v_{fx}^2 - \rho_s v_{sx}^2). \tag{27}$$

repeated !!

Substituting Eqs. (9), (12) and (27) into Eq. (5) yields (for steady-state)

$$\rho_s v_{sx}\frac{dv_{sx}}{dx} = (\rho_s - \rho_f)g\sin\theta + \frac{3k}{2d_e}(\rho_f v_{fx}^2 - \rho_s v_{sx}^2) - k\rho_s\frac{dv_{sx}^2}{dx}. \tag{28}$$

repeated !!

Substituting Eqs. (10), (13) and (26) into Eq. (6) yields

$$\rho_f v_{fx}\frac{dv_{fx}}{dx} = \rho_f g\sin\theta - \frac{3k\varphi}{2(1-\varphi)d_e}(\rho_f v_{fx}^2 - \rho_s v_{sx}^2) + \frac{d_0}{1-\varphi}(\tau_B + \mu b) - k\rho_f\frac{dv_{fx}^2}{dx}. \tag{29}$$

Furthermore, Eqs. (28) and (29) can be rewritten as

$$(2k+1)\frac{1}{2}\rho_s\frac{dv_{sx}^2}{dx} = (\rho_s - \rho_f)g\sin\theta + \frac{3k}{2d_e}(\rho_f v_{fx}^2 - \rho_s v_{sx}^2), \tag{30}$$

$$(2k+1)\frac{1}{2}\rho_f\frac{dv_{fx}^2}{dx} = \rho_f g\sin\theta - \frac{3k\varphi}{2(1-\varphi)d_e}(\rho_f v_{fx}^2 - \rho_s v_{sx}^2) + \frac{(\tau_B + \mu b)d_0}{1-\varphi}. \tag{31}$$

Adding Eqs. (30) and (31) together, we obtain

$$\frac{2k+1}{2}\left[\varphi\rho_s\frac{dv_{sx}^2}{dx} + (1-\varphi)\rho_f\frac{dv_{fx}^2}{dx}\right] = \varphi(\rho_s - \rho_f)g\sin\theta + (1-\varphi)\rho_f g\sin\theta + (\tau_B + \mu b)d_0. \tag{32}$$

Integrating from 0 to x for the two sides of Eq. (32) leads to

$$\frac{1}{2}\left[\varphi\rho_s v_{sx}^2 + (1-\varphi)\rho_f v_{fx}^2\right] = \frac{x}{2k+1}\{[\varphi\rho_s + (1-2\varphi)\rho_f]g\sin\theta + (\tau_B + \mu b)d_0\}. \tag{33}$$

Subtracting Eq. (31) from Eq. (30) leads to

$$\frac{1}{2}\left(\rho_{\mathrm{s}}\frac{\mathrm{d}v_{\mathrm{s}x}^2}{\mathrm{d}x}-\rho_{\mathrm{f}}\frac{\mathrm{d}v_{\mathrm{f}x}^2}{\mathrm{d}x}\right)=-\frac{3k}{(2k+1)(1-\varphi)d_{\mathrm{e}}}\frac{1}{2}(\rho_{\mathrm{s}}v_{\mathrm{s}x}^2-\rho_{\mathrm{f}}v_{\mathrm{f}x}^2)-\frac{1}{2k+1}[(2\rho_{\mathrm{f}}-\rho_{\mathrm{s}})g\sin\theta+(\tau_{\mathrm{B}}+\mu b)d_0]. \tag{34}$$

Solving this above equation yields

$$\frac{1}{2}\left(\rho_{\mathrm{s}}v_{\mathrm{s}x}^2-\rho_{\mathrm{f}}v_{\mathrm{f}x}^2\right)=\frac{d_{\mathrm{e}}(1-\varphi)}{3k}[(2\rho_{\mathrm{f}}-\varphi\rho_{\mathrm{s}})g\sin\theta+(\tau_{\mathrm{B}}+\mu b)d_0]\left[\exp\left(\frac{-3k}{(2k+1)(1-\varphi)d_{\mathrm{e}}}x\right)-1\right]. \tag{35}$$

The velocities of the solid and liquid phases for a debris flow are then obtained via Eqs. (33) and (35).

$$\frac{1}{2}\rho_{\mathrm{s}}v_{\mathrm{s}x}^2=\{[\varphi\rho_{\mathrm{s}}+(1-2\varphi)\rho_{\mathrm{f}}]g\sin\theta+(\tau_{\mathrm{B}}+\mu b)d_0\}\frac{x}{2k+1}-\frac{d_{\mathrm{e}}(1-\varphi)^2}{3k}[(2\rho_{\mathrm{f}}-\rho_{\mathrm{s}})g\sin\theta+(\tau_{\mathrm{B}}+\mu b)d_0]\left[1-\exp\left(\frac{-3k}{(2k+1)(1-\varphi)d_{\mathrm{e}}}x\right)\right], \tag{36}$$

$$\frac{1}{2}\rho_{\mathrm{f}}v_{\mathrm{f}x}^2=\{[\varphi\rho_{\mathrm{s}}+(1-2\varphi)\rho_{\mathrm{f}}]g\sin\theta+(\tau_{\mathrm{B}}+\mu b)d_0\}\frac{x}{2k+1}+\frac{d_{\mathrm{e}}(1-\varphi)\varphi}{3k}[(2\rho_{\mathrm{f}}-\rho_{\mathrm{s}})g\sin\theta+(\tau_{\mathrm{B}}+\mu b)d_0]\left[1-\exp\left(\frac{-3k}{(2k+1)(1-\varphi)d_{\mathrm{e}}}x\right)\right], \tag{37}$$

downstream

where x denotes the distance from the calculation point to the initial point in flow area. Although the model solutions (36) and (37) providing the velocity estimates for the solid and fluid phases

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in a debris flow only utilize and retain the impact pressure difference between the solid and the fluid, and the Bingham viscoplastic parameter, they can only provide very basic qualitative picture of the solid and the fluid velocities. Also these solutions do not include any information about the volume of the debris material. Nevertheless, to develop velocity solutions for the solid and the fluid phases in a more consistent and physically more meaningful way, one must use a real and general two-phase debris mass flow model, such as the one developed by Pudasaini (2012), that includes strong phase interactions through the generalized drag, virtual mass force, non-Newtonian enhanced viscous stress, and the evolving volume fraction of the solid-phase.

3 Results and discussion

In this study, we developed a new formula to estimate the solid- and liquid-phase velocities in a debris flow, which is useful for understanding the dynamics of the debris flow. By the discussion in Sect. 2, Eq. (33) provides the total kinetic energy of a debris flow element, which is

!!

$$\{[\varphi\rho_s + (1-2\varphi)\rho_f]g\sin\theta + (\tau_B + \mu b)d_0\}\frac{x}{2k+1}.$$

A bit strange! Why should the total KE increase with τ_B and μ? Seems to be not consistent! Explain!!

The total kinetic energy is combined from two parts: the kinetic energy derived by gravity M_1 and the kinetic energy derived by the yielding stress M_2, which are given by

$$M_1 = [\varphi\rho_s + (1-2\varphi)\rho_f]g\sin\theta\frac{x}{2k+1}, \tag{38}$$

$$M_2 = (\tau_B + \mu b)d_0\frac{x}{2k+1}. \tag{39}$$

However, Eq. (35) provides the kinetic energy difference between two phases – the solid and liquid phases– and it describes the interaction between two phases. The parameter d_c is referred to as the characteristic scale of a debris flow, which is defined by

$$d_c = \frac{d_e(1-\varphi)}{3k}.$$

What scale? What does it mean?!

Following this fact, the kinetic energy change due to the interaction between two phases is divided into two parts: the kinetic energy derived by gravity G_1 and the kinetic energy derived by the yielding stress G_2, which are given by

$$G_1 = (2\rho_f - \rho_s) g \sin\theta d_c \left[1 - \exp\left(\frac{-x}{d_c(2k+1)}\right)\right], \tag{40}$$

$$G_2 = (\tau_B + \mu b) d_0 d_c \left[1 - \exp\left(\frac{-x}{d_c(2k+1)}\right)\right]. \tag{41}$$

→ check sign!

Then the velocities of the solid and liquid phases in a debris flow are given by

$$v_s^2 = \frac{2}{\rho_s}[M_1 + M_2 - (1-\varphi)G_1 - (1-\varphi)G_2],$$

$$v_f^2 = \frac{2}{\rho_f}(M_1 + M_2 + \varphi G_1 + \varphi G_2).$$

→ Next

In this section, we will give some numerical examples to show the dynamics of a debris flow along the channel. Figure 2 shows some numerical results for the solid- and liquid-phase velocities for an example debris flow. The figure indicates that the liquid phase is faster than the solid phase, and the ratio of the velocities for two phases is about 0.790. Such exact solutions have also been presented previously by Pudasaini (2011) for avalanche and debris flows. For such a large velocity difference, at least the drag and the mass force must have been included in the model as in Pitman and Le (2005) and Pudasaini (2012). However, here the model does not consider such effects. The solid- and liquid-phase velocities at a point 300 m along the channel are shown in Fig. 3 for the different solid volume fractions; it can be seen that the velocity of a debris flow decreases as the solid volume fraction increases. However, 10% increase in the solid volume fraction resulted only in very slight decrease in the solid and fluid velocities. The solid- and liquid-phase velocities at 300 m along the channel are shown in Fig. 4 for the different equivalent diameters of solid particles, and here it can be seen that, as the equivalent diameter of solid particles increases, the solid-phase velocity of a debris flow decreases very slowly whereas the liquid-phase velocity increases very slowly. However, 10% increase in the

virtual

we mention that

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in

equivalent diameters of solid particles resulted in almost no change in the solid and fluid velocities. Such discrepancies may have been emerged do to the very simplified model consideration, or some possible inconsistencies in the use of the rheological models considered here. These problems could have been avoided by using more complete and real two-phase debris flow model (Pudasaini, 2012) which includes strong phase interactions.

In order to validate the estimation of velocities, in this section, two real-world debris flow – the K631 debris flow locating at the Tianshan highway in Xinjiang Province and the Pingchuan debris flow locating at the trunk highway from Xichang City to Muli County in Liangshan Yi Autonomous Prefecture, Sichuan Province – are considered. The velocities obtained by observations for the two debris flows, one a viscous debris flow and the other a thin debris flow, are $11.59\,\mathrm{m\,s^{-1}}$ and $9.70\,\mathrm{m\,s^{-1}}$, respectively. Following Chien (1989), particles which diameter is less than 0.1 m in viscous debris flow often form mass and move at certain direction with the same velocity, while particles that diameter is over 0.1 m move at jumping in debris flow channel. However diameter of particle at suspension state in thin debris flow is less than 0.02 m. Thus, the particles more than $2\,\mathrm{cm}$ in diameter are regarded as the equivalent liquid phase slurry, and the others are classified as the solid phase particles (Chen et al., 2006). The related parameters were obtained through analyzing samples at the location. The comparison of the theoretical results and the experiential results shows that the estimation method for the velocities of a debris flow can be effectively used for a real-world debris flow (see Table 1).

4 Conclusions

A one-dimensional model for a debris flow is introduced to estimate the velocities of the solid and liquid phases. By applying the specific form of the volume force and the surface forces for the solid and liquid phases, theoretical results are used to estimate the velocities of the solid and liquid phases. These results are found to be valid by comparing the theoretical results with the experiential formula for two real-world debris flows. Furthermore, the theoretical methods can estimate the velocities of a debris flow with different solid volume fractions and different

for analyzing the

equivalent diameters, which makes the theoretical results more useful for tracing a debris flow, simulating the deposition area and predicting the risk for a debris flow.

dynamics, including the associated KE, and compact forces.

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Table 1. The results of velocity calculation for the K631 (G217 highway) and Pingchuan debris flows.

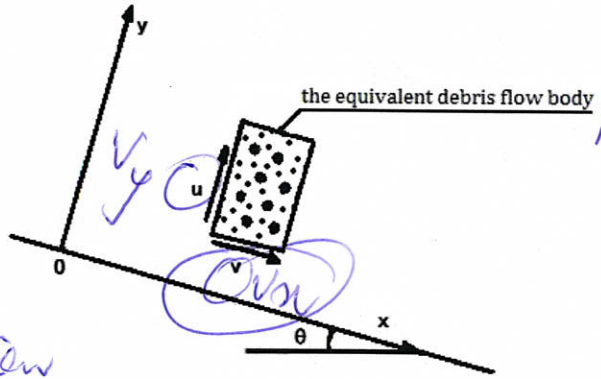
Name	φ	ρ_s $(\mathrm{kg\,m^{-3}})$	ρ_f $(\mathrm{kg\,m^{-3}})$	d_e (m)	v_s $(\mathrm{m\,s^{-1}})$	v_f $(\mathrm{m\,s^{-1}})$	$\overline{v}_1$ $(\mathrm{m\,s^{-1}})$	$\overline{v}_2$ $(\mathrm{m\,s^{-1}})$	$\overline{v}_3$ $(\mathrm{m\,s^{-1}})$
K631	0.0902	2500	1660	0.1033	8.43	11.97	11.59	11.72	11.51
Pingchuan	0.0497	2400	1500	0.0816	8.97	10.41	9.70	11.14	10.30

$\overline{v}_1$ is the velocity of debris flow obtained from field observations, $\overline{v}_2$ is the velocity of debris flow calculated by Chen et al. (2006), and $\overline{v}_3$ is the velocity of debris flow calculated from Eq. (11).

how? Refs.??

Table A1. Notation.

ρ	the density of debris flow
ρ_s	the density of solid phase particles
ρ_f	the density of liquid phase slurry
v_s	the velocity of solid constituent
v_f	the velocity of liquid constituent
d_e	the equivalent diameter of solid phase particles
d_0	the equivalent height of control volume for debris flow
g	the gravity acceleration
P	the pressure of debris flow body
θ	the gradient of debris flow channel
φ	the solid volume fraction
b_s	the volume force of solid phases in a unit volume
b_f	the volume force of liquid phases in a unit volume
P_s	the pressure of solid phases
P_f	the pressure of liquid phases
f_s	the other surface forces of solid phases in a unit volume
f_f	the other surface forces of liquid phases in a unit volume
ν	the velocity of debris flow body
k	the nonuniform coefficient of debris flow body
τ_B	the yielding stress of liquid phase slurry
μ	the viscous coefficient of liquid phase slurry



put (a), (b) from p21 here!

Debris flow configuration

Figure 1. Velocity analysis of the equivalent two-phase debris flow :

x indicates downslope direction, y normal to the slope, θ slope angle, v_x, v_y velocity components along the coordinate axes x and y, resp.!!

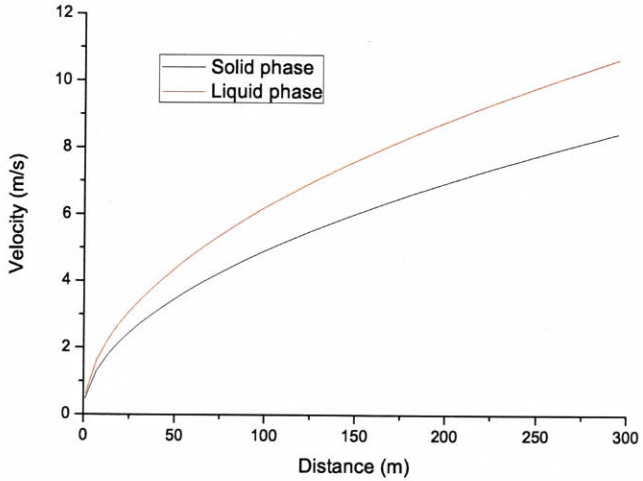


Figure 2. Solid- and liquid-phase velocities of a debris flow along the channel: $\rho_s = 2400\,\mathrm{kg\,m^{-3}}$, $\rho_f = 1500\,\mathrm{kg\,m^{-3}}$, $d_e = 0.10\,\mathrm{m}$, $\varphi = 0.10$, $\theta = 30°$, $(\tau_B + \mu b)d_0 = 100$, $k = 3.72$, $g = 9.8$, $x \in (0, 300)$.

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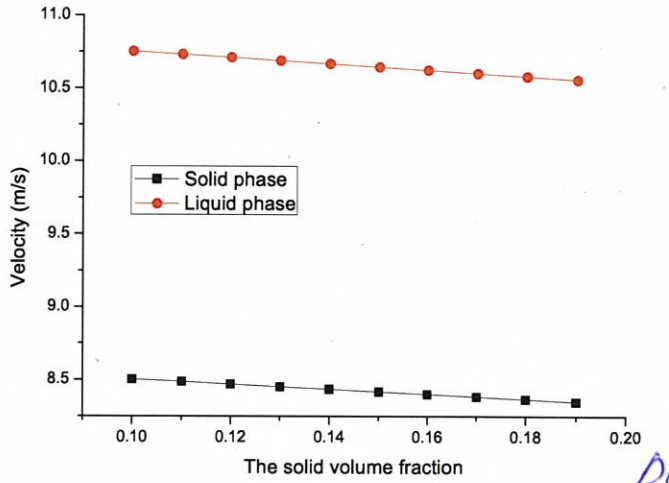
dim?



position $x = 300$ m for different values of the solid volume fraction parameter φ_s.

Figure 3. Solid- and liquid-phase velocities of a debris flow at along the channel: $\rho_s = 2400\,\mathrm{kg\,m^{-3}}$, $\rho_f = 1500\,\mathrm{kg\,m^{-3}}$, $d_e = 0.10\,\mathrm{m}$, $\varphi = 0.10 \sim 0.19$, $\theta = 30°$, $(\tau_B + \mu b)d_0 = 100$, $k = 3.72$, $g = 9.8$, $x = 300$.

Fig. 4: improve similarity