

1 On quartet interactions in the California Current system

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7 8 **Abstract**

9 SSH altimetry observations for 1992 to 2009 off California are used to show that observed
10 quasi-zonal jets were likely driven by near-resonance interactions between different scales of
11 the flow. Quartet (modulational) instability dominated and caused non-local transfer of energy
12 from waves and eddies to bi-annual oscillations and quasi-zonal jets. **Two types of quartets**
13 **were identified: those composed from scales corresponding to (a) quasi-zonal jets, annual and**
14 **semi-annual Rossby waves and mesoscale eddies, and (b) bi-annual oscillations, semi-annual**
15 **Rossby waves and mesoscale eddies.** The spectral centroid regularly shifted into the domain
16 of low-order modes. However, the spectrum of SSHs does not demonstrate a power behavior.
17 This says that the classical inverse cascade **is absent**. For a case with bottom friction, quartet
18 instability required the existence of a certain level of dissipativity in the flow.

19 20 **1 Introduction**

21 In recent years there has been growing recognition of the existence of persistent quasi-zonal
22 jets (QZJs) in the mid-latitude atmosphere as well as in the atmospheres of gaseous planets
23 (an intensive review of such QZJs is given by Baldwin et al. (2007)). These jets were clearly
24 detected in the atmospheres of planets and Earth because they were intense and had a clear
25 signature in cloud systems although mechanisms of their generation were probably different.
26 This is not the case for the Earth's oceans because oceanic QZJs are latent (i.e. masked by
27 more intense non-zonal flows), and can only be extracted from observations and model results
28 by statistical procedures. Therefore, the role of QZJs in ocean dynamics and the nature of

1 these jets are not clear due to their weak signal. The principle question here is what [a]
2 physical mechanism[s] can generate the QZJs.

3 A number of the generation of QZJs includes: modulation instability (as example see
4 Connaughton et al. 2010), β -plume (Centurioni et al., 2008), inverse cascade (Galperin et al.,
5 2010) and others. Most results were derived from analysis of 2D models of QZJs which
6 yielded useful analytical results (Loesch, 1977; Newell, 1969; Rhines, 1975; Vallis and
7 Maltrud, 1991; Nadiga, 2006; Galperin et al., 2010; Connaughton et al., 2010; Wang et al.,
8 2012; 2013 among others). However these results were obtained for simplified jet models,
9 and it is difficult to use these models to detect mechanisms of oceanic jet generation which
10 might exist in nature.

11 Srinivasan and Young (2012) demonstrated that zonal jets could be reproduced with a linear
12 theory and did not rely upon the existence of an inverse cascade. The results presented here
13 do not agree with Srinivasan and Young (2012) because in our opinion nonlinearity played a
14 very important role in the jet generation; therefore, results were not compared to those
15 obtained by Srinivasan and Young (2012).

16 Alternative hypotheses based on 3D models, have been proposed to explain mechanisms of
17 QZJ generation. See Hristova et al. (2008), Berloff et al. (2009); (2011); Tanaka and
18 Akitomo (2010); Wang et al. (2012); Qiu et al. (2013) among others. For example, Hristova et
19 al. (2008) hypothesized that QZJs, observed in the eastern region of subtropical gyres may be
20 related to radiating instabilities of eastern boundary currents.

21 Connaughton et al. (2010) have suggested using a modulational instability concept for
22 explanation of quasi-zonal jets observed in the ocean and atmosphere. Weak, long-wave
23 modulations of a periodic nonlinear wave train can grow exponentially under certain
24 conditions. This is known as the modulational instability and was first identified by Benajm
25 and Feir (1967) for surface gravity waves on deep water. Modulational instability is a
26 ubiquitous instability in nature and is not restricted to hydrodynamics. It also occurs in plasma
27 physics, electrodynamics and nonlinear optics (Zajharov and Ostrovsky, 2009). Modulational
28 instability is one of mechanisms for generating zonal flows in magnetically-confined plasmas
29 and in planetary atmospheres and oceans (Onishenko et al. 2008, Connaughton et al., 2010
30 and others).

31 Connaughton et al. (2010) found that in a 2D case for very weak primary waves, the unstable
32 waves were close to being in three-wave resonance with the primary waves. They have

1 revised the linear theory of Gill (1974) using a 4-mode truncation and emphasized the role of
2 the carrier amplitude/nonlinearity, the role of the deformation radius and the role of resonant
3 wave interactions in the case of a stronger nonlinear carrier wave.

4 Ivanov et al. (2010) demonstrated that annual and semi-annual Rossby waves could be
5 interpreted as weak nonlinear waves because their nonlinearity parameters changed by as
6 much as several units. They used a zonal component of propagation speed (which varied
7 from zero to several cm/s), the wave steepness, q , ($0,1-0,2 < q < 2-3$, the steepness was less than
8 1 for a considerable part of the wave evolution time), and the length of spatial phase
9 coherence for detection of different regimes of Rossby wave evolution. It follows that
10 nonlinear interactions between these waves should be studied with a view to understanding
11 whether and how they might influence quasi-zonal jets.

12 Following Connaughton et al (2010) we will study the weakly nonlinear stage of the
13 modulational instability ($q < 1$) and demonstrate that quartet interactions can generate quasi-
14 zonal jets. From the theoretical point of view these zonal jets subsequently become unstable
15 and acquire the interesting two-dimensional structure of a double (Karman-like) vortex street
16 which should be more stable than a plane parallel shear flow with the same zonal profile
17 (McWilliams, 2006).

18 In the present paper modulational instability is identified through existence of quartet
19 interactions when Rossby wave with one year or semi-annual periodicity is considered a
20 carrier wave. This study analyses SSH observations off California and shows that

21 (a) mesoscale flow off California was driven by near-resonance interactions between flow
22 scales. Four wave near-resonance interactions between spatial scales dominated and formed
23 specific structures of the mesoscale flow including a combination of QZJs, biannual
24 oscillations, annual Rossby waves, semi-annual Rossby waves and mesoscale eddies of
25 smaller scales;

26 (b) the SSH signal was transferred from Rossby waves (RWs) and mesoscale eddies (MEs) to
27 lower-frequency currents [biannual oscillations (BAOs) and QZJs]. The existence of two
28 different quartets was found: QZJ-annual RWs- semi-annual RWs-MEs and BAOs-semi-
29 annual RWs-MEs of two different scales. However for both these quartets, the SSH signal
30 was pumped from faster to slower scales, i.e. from annual and semi-annual currents to
31 biannual oscillations and quasi-zonal jets.

1 (c) QZJ (or BAO) generation by modulational instability required the existence of a certain
2 level of dissipativity in mesoscale flows. Dissipativity played an important role because it
3 determined a number of possible near-resonance interactions as well as the latency of
4 observed QZJs.

5 In general, the dimensionality of analyzed QZJs cannot be based on the analysis of SSHs
6 only. However, QZJs are shown to be dissipative structures and their generation was the
7 result of a self-organization process. Following Nicolis and Prigogine (1977), dissipative
8 structures are defined as non-equilibrium thermodynamic systems that generate order
9 spontaneously by exchanging energy with their external environment. Self-organization is
10 generation of regular structures in far-from-equilibrium states of a dissipative system due to
11 interactions of the elements that make up the system (Nicolis and Prigogine, 1977).

12 The remainder of the paper is organized as follows. Section 2 describes the data and methods
13 used. A system of jets (waves and eddies reconstructed from AVISO products) is discussed
14 in Section 3. Robustness and repetition of QZJ generation are demonstrated in Section 3.
15 Sections 4 and 5 include calculation of near resonance and resonance interactions off Central
16 California. The re-distribution of surface elevation between different scales and scales
17 dominating the surface elevation are discussed in Section 6. Although Section 6 is simply a
18 kinematic analysis, it is shown that energy was pumped from Rossby waves and mesoscale
19 eddies to QZJs and BAOs. The role of dissipativity in the process of energy re-distribution is
20 discussed in Section 7. Section 8 shows how different scales forming quasi-zonal jets and bi-
21 annual oscillations were grouped in separate quartets. Conclusions are given in Section 9.

22

23 **2 Data and methods**

24 The SSH anomaly field was produced by the AVISO Project (Archiving Validation and
25 Interpretation on Satellite Data in Oceanography) for the period from Oct. 10, 1992 to May
26 23, 2009 (Collecte Localisation Satellites, 2006). To understand the spatio-temporal
27 complexity of altimetry signals, a double spectral approach (DSA) (Ivanov and Collins, 2009)
28 was applied to the analysis of the SSH anomaly field. Following this approach, the SSH
29 anomaly field $\eta(\mathbf{x}, t)$ is represented by

$$30 \quad \eta(\mathbf{x}, t) = \sum_{m=1}^M a_m(t) \Psi_m(\mathbf{x}), \quad (1)$$

1 where $\mathbf{x} = (x, y)$ is the position vector in a real-space, Ψ_m are basis functions (or M-modes
 2 which extend the classical Fourier polynomials to nonrectangular basins) calculated as in
 3 Ivanov and Collins (2009). Note that $\Psi_m(\mathbf{x}) = \sum_{l=1}^L c_{lm} \sin(kx/L_x) \sin(ky/L_y)$ only for a
 4 rectangular basin. Here L_x and L_y are horizontal basin scales, $k = 2\pi/l$. Fig. 1 shows the
 5 structure of several basis functions for the observational region. About 1000 basis functions
 6 were used to represent SSH with less than 1% rmse. Different combinations of basis
 7 functions formed currents which had different spatial scales.

8 The spectral coefficients a_m were then analysed using a discrete wavelet transform as

$$9 \quad a_{mj}(t) = \sum_{p=1}^{\infty} b_{m,j,p} T_{j,p}(t), \quad (2)$$

10 where a_{mj} is an approximation of a_m with resolution j , $T_{j,p} = 1/2^{j/2} \phi(2^j t - p)$, and ϕ is the
 11 father wavelet. Finally, results were summed around selected time scales j using Eq. (1).

12 A Debaucher 6th order wavelet transform was used in this study. This allowed for the
 13 introduction of six frequency bands (FBs): (1) 1 month ($\omega \approx 2 \cdot 10^{-5}$ rad) to 2 months ($\omega \approx 4 \cdot 10^{-5}$
 14 rad) (referred to as the mesoscale eddy II (MEs-II) band), (2) 2 months ($\omega \approx 4 \cdot 10^{-5}$ rad) to 4
 15 months ($\omega \approx 8 \cdot 10^{-5}$ rad) (ME-I band), (3) 4 months ($\omega \approx 8 \cdot 10^{-5}$ rad) to 8 months ($\omega \approx 16 \cdot 10^{-5}$
 16 rad) (semiannual Rossby waves or SARW band), (4) 8 months ($\omega \approx 1.6 \cdot 10^{-6}$ rad) to 18 months
 17 ($\omega \approx 3,6 \cdot 10^{-6}$ rad) (annual Rossby waves or ARW band), (5) 18 months ($\omega \approx 3,6 \cdot 10^{-6}$ rad) to
 18 36 months ($\omega \approx 7.2 \cdot 10^{-6}$ rad) (bi-annual oscillations or BAO band) and (6) longer than 36
 19 months ($\omega \approx 7.2 \cdot 10^{-6}$ rad) (quasi-zonal jets or QZJ band). Coherent mesoscale structures (jets,
 20 propagating waves and eddies) were selected within these six frequency bands and for a
 21 spatial spectral band limited by the 30th (lower boundary) and 250th (upper boundary) M-
 22 modes.

23 Calculations should be robust for the choice of upper and lower boundaries: the results did
 24 not change when the upper boundary of the band was moved toward higher-order modes, but
 25 changed weakly when the lower boundary was moved to the 15th mode. As discussed in
 26 Ivanov et al. (2010), modes from 1 to 29 were responsible for large-scale seasonal variability
 27 only. Rossby waves with periodicity from 4 months to 18 months cannot contain these modes

1 because this part of spectrum demonstrates no movements westward and therefore were not
2 included in the analyses.

3 Similarly, variations in observational sampling should not affect coherent (long-lived)
4 structures. The length of observational sampling was reduced 4-6 times without changing the
5 structure of quasi-jets. This was a consequence of using the Debaucher wavelet transform for
6 the spectral representation of spectral coefficients a_m .

7 Note that we are going to analyse wave modulation instability in an {M-mode, ω } space
8 although the classical approach (see, for example Zakharov and Ostrovsky, 2009) uses a {k,
9 ω } space. The use of M-mode makes the physical interpretation simpler because M spectrums
10 are one-dimensional. This is an explicit advantage of our approach. More details of the
11 approach can be found in Ivanov and Collins (2009).

12 **Note that (M-mode; ω) space describes the evolution of a signal in a non-rectangular domain**
13 **while (Fourier mode; ω) can only be applied to a rectangular domain. In other aspects, they**
14 **are equivalent. It was shown for example, in Eremeev et al. (1992). Theoretically, we can**
15 **provide the wave amplitude equation for a quartet in (M-mode, ω) space. However**
16 **Marchesiello et al., (2003) who studied numerically mesoscale variability in the observed area,**
17 **demonstrated that approximately 30% of kinetic energy corresponds to non-geostrophic**
18 **motions. Therefore, we worked only with SSHs and did not re-calculate them to velocity.**

19

20 **3 A system of jets, waves and eddies off California**

21 The mesoscale coherent structures observed in SSH observations off California shared
22 overlapping spatial scales but distinct time scales. This allowed for detection and selection of
23 QZJs, BAOs, RWs and MEs. Time-averaged spectra for each of the six frequency bands are
24 shown in Fig. 2a,b as a function of mode number. Figure 2a shows that the SSH signal was
25 very small for $m > 250$ and that QZJs were considerably weaker than annual RW. Figure 2b
26 indicates that BAO were weaker than semi-annual RW. Explicit peaks for spectrum a_m^2 were
27 observed for all frequency bands (Fig. 2a,b) within the domain bounded by 30th and 250th M-
28 modes. This indicated the existence of energy-dominant scales in the mesoscale flow. The
29 instantaneous spectra were very rough with multiple narrow peaks (Fig. 2c). The
30 instantaneous spectrum structure was similar to those observed in weakly nonlinear flows
31 (Rabinovich and Trubetskov, 1989).

1 Characteristic spatial structures for all six frequency bands of the current system are shown in
2 Fig. 3 a,b,c,d,e,f. The coherent structures shown in Fig. 3 for the different frequency bands
3 differed from one another and were not stationary. Temporal behavior is discussed below.

4 QZJs represented eddy chains stretched from east to west and each chain appeared as a
5 number of eddies of the same sign embedded in an envelope of less intensive zonal shear flow
6 (Fig 3a). SSH in the jets reached 4-5 cm and was not less than 2.5-3 cm within the 17 year
7 observational period. The ratio of zonal and meridional scales for the QZJs
8 ($R = L_{zonal} / L_{merid}$) varied from 2-3 (short jets) to 7-8 (long jets). Odd M-modes dominated
9 Eq. (1) which resulted in anisotropy of the QZJ envelope. Direction of QZJ axes varied from
10 west to southwest. The QZJs also moved equatorward with speed of as great as 0.2 cm/s.
11 However, there was no zonal motion associated with the southward movement. Results
12 clearly indicated that QZJs were regular structures with robust recurrence and could not be
13 interpreted as statistical fluctuations.

14 BAOs did not demonstrate any steady motion to either west or southwest and their shape
15 differed somewhat from QZJs (Fig. 3b). The structure of BAOs was often close to the
16 structure of standing waves and therefore they were interpreted as quasi-standing waves. The
17 oscillations were stronger than for QZJs. SSHs for BAOs reached ~ 5-7 cm.

18 Propagating RWs with annual and semi-annual periodicities were detected and are shown in
19 Fig. 3c,d. Jacobs et al. (1993), (1996) had identified semiannual linear Rossby waves and
20 semiannual oscillations in the Pacific Ocean from satellite data for latitudes as high as to
21 35°N and 45°N respectively, when barotropic and first baroclinic modes were fitted to Geosat
22 data. Ivanov et al. (2012b) have analyzed satellite data in the Pacific Ocean within a region
23 from 7°N to 60°N and found semiannual Rossby waves south of 45°N. These waves were
24 nonlinear for most of their existence, i.e. they had relatively large steepness (see, for example,
25 Rhines (2002), (2004)).

26 The annual and semiannual RWs dominated the mesoscale flow off California because SSHs
27 for the waves reached ~ 10-12 cm. The Rossby waves demonstrated robust recurrence in a
28 sequence of dynamical regimes: linear, amplification and saturation. Fig. 3c and Fig. 3d show
29 the spatial structure for annual and semi-annual Rossby waves on Oct. 4, 2006, and Nov. 28,
30 2007, respectively. The waves propagated westward and southwestward. The propagation
31 speed varied with time and had maximum values in the linear regime of up to 7-10 cm/s and

1 minimum value in the saturation regime when a halt of westward wave propagation was
2 observed. Semi-annual waves were more anisotropic than annual waves (compare Fig. 3c to
3 Fig 3d). Comparing alongshore (L_h) and across shore (L_n) spatial scales, annual Rossby
4 waves were characterized by $L_n \sim L_h$ or $L_n > L_h$. For semi-annual Rossby waves $L_h \gg L_n$.
5 The structure of semi-annual waves was close to meridional. Further discussion of the
6 behavior of the Rossby waves can be found in Ivanov et al. (2010).

7 MEs-I and MEs-II with scales smaller than RWs were also observed off California (Fig. 3e,f).
8 They formed an eddy field which was statistically inhomogeneous across-shore and
9 demonstrated mean propagation from the coast to the west or southwest. SSH for these
10 eddies reached several cm. To distinguish Rossby waves and mesoscale eddies, the spatial
11 phase coherence criterion (spatial correlation of the phase of the SSH field) was used [Ivanov
12 et al., 2010].

13 Several features of the coherent structures observed off California were important for the
14 present analysis. First, all structures were generally weakly nonlinear excluding the cases
15 when $\langle \eta^2(t) \rangle = \iint \eta^2(x, y, t) dx dy$ reached maximal and minimal values (Fig. 4). Here,
16 integration was over the computational domain starting on Oct. 10, 1992. Maximum values
17 corresponded to periods when wave steepness $q \sim 2$ (transition to turbulence regime), and
18 minimal values to $q \ll 1$ (linear structures). Interplay between different M-modes was easily
19 detected via the frequency-phase analysis of M-mode structures in phase space (not shown).
20 The interplay between M-modes will be shown through the analysis of triplets and quartets in
21 the next sections.

22 Second, the behaviour of $\langle \eta^2(t) \rangle$ with time showed robustness for the studied process (Fig.
23 4). For all six spectral bands, the behavior of this function was robust with time. Here
24 robustness is a repetition of certain dynamical regimes. Maxima for QZJs corresponded to
25 minima of $\langle \eta^2 \rangle$ for BAOs.

26 Third, a low-frequency modulation (not modulation instability) was visible by eye for all
27 spectral bands shown in Fig. 4 (Rabinovich and Trybetskov, 1993). This modulation became
28 stronger for higher-order spectral bands.

29 Fourth, all structures observed off California with horizontal resolution larger than 50-100 km
30 can be interpreted as weakly nonlinear if $q < 1$ and as strongly nonlinear if $q > 1$. A transition

1 from weak to strongly nonlinear structures was very quick. But q was larger than 1 for most
 2 time. See, for example Ivanov et al. 2010. We plan to study such transitions and dynamics of
 3 strongly nonlinear structures in a separate paper.

4

5 **4 Wavelet analysis**

6 RWs and QZJs demonstrated nonlinear behavior. The observed RW steepness was finite
 7 (most of the time the steepness was ≤ 1) and the QZJs consisted of chains of eddies which
 8 stretched from east to west. These eddy chains may indicate the importance of nonlinear
 9 dynamics, see for example, Dritschell and Scott (2011). Thus it is logical to study nonlinear
 10 mechanisms of QZJ generation and evolution. Numerical and theoretical studies (Gill, 1974;
 11 Lee and Smith 2007 and others) demonstrated that quasi-zonal jets can be generated by
 12 resonance as well as near-resonance interactions between flow scales in simplified models.

13 These interactions should be detected in a phase space which embeds altimetry SSH signals.
 14 M-modes were used as a basis to form an appropriate phase space. Next phase-frequency
 15 analysis of M-mode (wave) dynamics was carried out using amplitude (W_m) and phase (φ_m).
 16 Both variables, W_m and φ_m , were determined for each M-mode using the Morlet wavelet
 17 transform (ψ_τ),

$$18 \quad W_m(\tau, t) = |W_m(\tau, t)| \exp\{i\varphi_m(\tau, t)\}, \quad (2)$$

19 where $W_m(\tau, t) = \int_{-\infty}^{+\infty} a_m(t') \psi_\tau^*[(t-t')/\tau] dt'$, τ is a time scale, and $\omega(m) = \frac{d\varphi_m}{dt}$ is
 20 instantaneous frequency (for details of wavelet techniques see Kumar and Foufoula-Georgiou
 21 (1997)).

22 The absolute value of the amplitude $|W_m|$ describes the evolution of energy-dominant peaks at
 23 certain time scales (or frequencies) ($|W_m|^2$ is also called the wavelet power spectrum, which
 24 in contrast to the Fourier power spectrum, is a function of time). Given the length of the
 25 observational series (nearly 17 years), statistically significant estimates of wavelet amplitude
 26 can be obtained for a domain bounded by $1 \text{ month} \leq \tau \leq 6 \text{ years}$ and $8 \text{ years} \leq t \leq 12 \text{ years}$
 27 (Kumar and Foufoula-Gergiou, 1997).

1 Energy-dominant peaks were detected by examining the structure $|W_m|$. The peaks were
2 selected as to satisfy one of the following conditions.

3 Three m modes participated in near-resonance interactions and formed an appropriate triplet
4 mode if

$$5 \quad m+k=l, \quad |\omega(m, \tau) + \omega(k, \tau) - \omega(l, \tau)| \leq \Omega_1. \quad (3)$$

6 Four modes were members of a mode quartet if

$$7 \quad m+k=l, \quad |\omega(m, \tau) + \omega(k, \tau) - \omega(l, \tau)| \leq \Omega_1, \quad (4)$$

$$8 \quad n-m=l, \quad |\omega(n, \tau) - \omega(m, \tau) - \omega(l, \tau)| \leq \Omega_2. \quad (5)$$

9 Here Ω_1 and Ω_2 represent resonance broadenings which are caused by finite nonlinearity.

10 Nonlinear interactions are effective if $\Omega_1, \Omega_2 \leq \Gamma_m$, where Γ_m is the inverse of the
11 characteristic time of nonlinear evolution of the mth mode (Janseen 2003, Kartashova et al.
12 2010, and others). For resonance interactions - $\Omega_1 = \Omega_2 = 0$.

13 Conditions (4) and (5) are able to describe modulation instability when the lth mode of RW,
14 two mth and nth modes (similar to sideband waves) and the k th mode of lower frequency
15 flow interact with one another in a nonlinear sense (Manfredi et al., 2001). Note that
16 conditions (4)-(5) have to be met during a specific time interval δt (this interval was larger
17 than 2 weeks) because either all modes in a quartet fail to exist after nonlinear interactions or
18 all modes continue to exist in quartets but quartets become unstable and decay due to
19 inhomogeneity of the background flow, vertical temperature gradients etc.

20 Since many of near resonance quartets and triplets lived as long as 14 days or longer, as
21 estimated from analysis of the wavelet power spectrum, it was possible to estimate how
22 surface elevation was re-distributed between different scales and in particular between m-
23 modes for frequency bands corresponding to QZJs, RWs and eddies. Superposition of RW
24 and eddy m-modes (see Eq. 4) results in growth of QZJ mode, identified from the analysis of
25 time evolution of all three m-modes (Eq.4) which are a part of a quartet. Quartets with
26 growth of m-modes (within a mean lifetime) corresponding to QZJs showed that a SSH was
27 re-distributed from high to low frequency m-modes. This was a rough estimate because a part
28 of short-lived m-modes with lifetimes less than 14 days were excluded from our analysis,

1 Conditions (3)–(5) corresponded to near-resonance nonlinear interactions described by the
2 Poisson bracket $\{\eta, \Delta\eta\}$ for quasi-geostrophic flows or $\{\Psi, \eta\}$ without the geostrophic
3 approximation (Ψ is “a stream” function) (Pedlosky, 1987). The “stream” function can be
4 defined through a two scalar potential representation of a three-dimensional incompressible
5 flow. See, for example, Moffatt (1978) or Chu et al. (2003) for applications. Obviously, in
6 this case the two scalar potentials Ψ and Φ defined here are not the same as the stream
7 function and velocity potential of a two-dimensional flow (this is discussed in Chu et al.
8 (2003)). The statistical stability needed for detection of triads and quartets was achieved
9 through special smoothing with *a priori* constraints applied (Ivanov et al., 2012a).

10 Note that horizontal velocity was not estimated from SSH using geostrophic relationships
11 because non-geostrophic motions in the studied coastal oceanic area may contain up to 30%
12 of the total kinetic energy of horizontal circulation (Marchesiello et al., 2003) and frontal
13 dynamics plays an important role in forming circulation in the California Current system
14 (Castelao et al., 2006).

15 For this study, triads were detected which satisfied (4) as the first step, and then checked to
16 see if two m-modes from this triad formed a second triad (5). If the second triad was [not]
17 found, the existence of quartet (4)-(5) [triad (3)] was confirmed. Fig. 5a,b shows a resonance
18 triad (a) and near-resonance quartet (b). In both these cases, energy was transferred to low-
19 frequency time scales. The number of possible near-resonance triads and quartets were
20 defined by the choice of resonance broadening.

21 Our calculations demonstrated existence of quartets of both resonance and non-resonance
22 nature and resonance and no-resonance triplets at the same time.

23 Note that to understand energy re-distribution within the energy spectrum, the energy transfer
24 integrals need to be calculated, for example, see Sagau and Cambon (2008). Unfortunately
25 that cannot be done because equations describing SSH anomaly motions contain terms which
26 cannot be determined from satellite altimetry data only. Therefore simpler characteristics
27 were calculated which described the re-distribution of the altimetry signal between scales (see
28 Section 6). This approach allowed calculation of general characteristics of the re-distribution
29 of energy. Generally, “inverse” transfer from fast motions to slower motions was observed
30 but not from small to large spatial scales.

1 Triplet and quartet interactions showed different mechanisms of energy re-distribution
2 between scales and as well as different time scales for generation of QZJs. For example,
3 Loesch (1977) demonstrated that the time scale required for jet generation by quartet
4 interactions was an order of magnitude longer than that associated with triad interactions.

5

6 **5 Resonance and near-resonance interactions**

7 On average, between 1999 and 2003, at each moment in time, ~40-80 resonance triads
8 ($\Omega_1 = 0$), ~3-5 resonance quartets ($\Omega_1 = 0, \Omega_2 = 0$), about 2×10^4 near-resonance triads
9 ($0 < \Omega_1 < \Omega^*$) and 1.75×10^5 near-resonance quartets ($0 < \Omega_1 < \Omega^*, 0 < \Omega_2 < \Omega^{**}$) were
10 detected. See Fig. 6. For simplicity, hereafter, $\Omega^* < \Omega^{**} = 0.01$ radians. The number of
11 triads and quartets changed with time but not very quickly (Fig. 5a). The number of quartets
12 decreased as $\Omega^* \sim \Omega^{**} \rightarrow 0$, but was always larger than the number of triads if $\Omega^* > \Omega_{cr}$ for
13 reasonable values of resonance broadening (Fig. 5b).

14 The stability of resonance quartets and triads depends on the inhomogeneity of the
15 background conditions of the ocean (density gradient, shear flow, etc.). This inhomogeneity
16 should destroy exact resonances, and most likely exact resonances dominate only in
17 simplified theoretical models. Therefore, in mesoscale oceanic flows, near-resonance
18 interactions are more likely. The intensity of these interactions is determined by the
19 resonance broadening and kinematic features of near resonances.

20 Squared amplitudes for most quartets were also larger (some amplitudes were much larger)
21 than the squared amplitudes of triads (not shown). The dominance of near-resonant quartets
22 compared to resonant and near-resonant triads indicated that there was modulation instability
23 of Rossby waves off California: modulations of the Rossby wave envelope excited zonal
24 flows through a nonlinear mechanism (probably Reynolds stress).

25 The number of near resonance quartets (N^q) and the number of near resonance triads (N^{tr})
26 did not change much with time (Fig. 5a). Quartets dominated off Central California. N^q was
27 much larger than N^{tr} except for values of $\Omega \ll 0.01$ radians (Fig. 5b). Squared amplitudes,
28 a_m^2 , for most quartets were also larger (some amplitudes were much larger) than the squared
29 amplitudes of triads (not shown).

30

1 **6 Re-distribution of surface elevation between scales**

2 Kinematic analysis was applied to understand how mean squared SSH was re-distributed
3 between different scales. The number of quartets for which a signal was transferred from
4 waves and eddies to QZJs and BAOs is shown in Fig. 6a. (Note that this number was
5 considerably less than the total number of existing quartets shown in Fig. 5a.) This indicated
6 a dominant role for four-wave interactions (modulational instability) in the process of forming
7 SSH spectra as compared to three-wave interactions.

8 M-modes for QZJs and BAOs had mean time scales of 4.3 ± 0.7 years and 2.4 ± 0.9 years,
9 respectively (Fig. 6b). Variations of both mean time scales were not large, which reflected a
10 high degree of regularity of QZJs and BAOs off California.

11 Since the wave steepness, q , can be larger than 1 for quite short time periods associated with a
12 transition to turbulence, under some conditions an inverse cascade can occur or coexist with
13 modulational instability. Naturally when $q \ll 1$, pure modulational instability dominates
14 (Connaughton et al., 2010). Therefore it is important to look for the existence of an inverse
15 cascade in our analyses.

16 Two criteria were used to determine and identify whether or not an inverse cascade formed
17 QZJs. First, spectral entropy was used to estimate the re-distribution of SSH signals between
18 different m-modes (Aubry et al., 1991). The spectral entropy, S , was calculated as

$$19 \quad S(M, t) = -(\log M)^{-1} \sum_{m=m_0}^M p_m \log p_m, \quad (7)$$

20 where $p_m(t) = a_m^2 / a(t)$; $a(t) = \sum_{m=m_0}^M a_m^2(t)$, equals one for a uniform distribution of a signal
21 among all $(M - m_0)$ modes and equals zero when the signal was contained in a single mode.

22 Fig. 7a shows the behavior of $S(M, t)$ with time for modes numbered from $m_0=30$ to
23 $M=250$. This figure shows that there was no explicit signal flux from small scales to large
24 scales of SSH during the entire observational period. $S(M, t)$ fluctuated from 0.68 to 0.72.
25 Increasing M to 1000 increased the mean value of S only slightly to 0.72. These results
26 indicated that either there was no inverse cascade of energy or that the cascade was weak. .

27 The centroid for spectrum of η^2 from $m_0=30$ to $M=1000$ was also calculated to identify
28 inverse cascade. Fig. 7b demonstrates the behavior of the spectral centroid with time. A

1 simple analysis of this behavior is that there were time intervals within which the centroid
2 shifted to larger scales. To illustrate the character of this shift, the spectrum of a_m^2 for day
3 2940 (point A on Fig. 8a) when the centroid was 82 (Fig. 8b) was contrasted with the
4 spectrum on day 3150 (point B on Fig. 8a) when the centroid had decreased to 62 (Fig. 8c).
5 The spectra after the shift (Fig.8c) showed a decay of energy for M-modes with $m > 100$.
6 From another perspective, η_m^2 did not demonstrate a power behavior with m (not shown).
7 This indicated that the classical inverse cascade was not observed in our analysis of the
8 satellite data. A specific mechanism for the reduction of the spectral centroid position will be
9 studied in another paper. It may be caused by three dimensional or other effects.

10 The mechanism of QZJ generation due to near-resonance inter-scale interactions is similar to
11 that suggested by Lorenz (1972) for barotropic flow, and was originally called "Rossby wave
12 instability" by Lorenz. A finite-amplitude Rossby wave interacts with a zonal flow
13 perturbation on a β -plane. The surfaces of constant phase move to the west. The zonal flow
14 tends to distort them. At this point, it is easy to see that the waves being transverse, fluid
15 particles, whose velocity is parallel to the wave front, need to turn to the west. This imparts
16 eastward momentum when the fluid particles cross the zonal flow perturbation. This
17 continuous deposition of eastward momentum by the wave produces a positive feedback
18 mechanism leading to exponential growth of the zonal perturbation.

19 However, in contrast to Lorenz (1972), modulational instability dominated the mesoscale
20 flow off California and there was channeling of energy not only to QZJs but also to BAOs.
21 This is a typical self-organized process. Energy was pumped by wind into the ocean and then
22 it was re-distributed between scales so that robust regular structures (QZJs and BAOs) were
23 formed due to nonlinear interactions between scales (Nicolis and Prigogine, 1977).

24 Note that the oceanic eastern boundary current off California is a baroclinic system. SSHs
25 observed from satellites are also a baroclinic system. The baroclinicity can influence the
26 modulational instability in two ways. First, if waves participating in modulation instability
27 seem to be baroclinically stable then their wave characteristic scales and others should be
28 different than for barotropic waves only. Second for Rossby waves generated by baroclinic
29 instability a small modulation with a period of the same order as the internal dynamical
30 timescale can completely destabilize the non-linear system, leading to chaotic solutions even
31 for small supercriticality. See, for example Hart (1979), Hart(1989) and others.

1 **7 Role of flow dissipativity**

2 Lee and Smith (2007) suggested that the resonance broadening $\Omega = \gamma \cdot Rn$, where γ is a small
3 parameter and Rn is a non-dimensional Rhines number, the latter defined for the β -plane
4 system as the ratio of linear and nonlinear time scales. With characteristic length scale L and
5 nonlinear time scale L/U , the Rhines number is equal to $U/(L^2 \beta)$. The resonance
6 broadening seen above is determined by the dissipativity of mesoscale flow caused by bottom
7 friction, vertical viscosity, etc. The Rhines scale depends on the level of dissipativity and
8 inhomogeneity of the background medium (density gradient, shear flow etc.) (Danilov and
9 Gurarie, 2002). This inhomogeneity should destroy exact resonances, and exact resonances
10 probably dominate only for simplified theoretical models. Near-resonance interactions seem
11 more likely for mesoscale oceanic flows. Intensity of near-resonance interactions is
12 determined by resonance broadening and kinematic features of near resonances.

13 Therefore, dissipativity of mesoscale flow should play an important role in re-distribution of
14 energy between scales because its level determines the number of near resonances involved in
15 energy exchange between scales. This can be illustrated by assuming that linear bottom
16 friction with coefficient λ dominated the mesoscale flow. The Rhines number is defined as

$$17 \quad Rn \sim \beta^{-1} \lambda^{-1/2} \varepsilon^{1/2} L^{-1} \quad (8)$$

18 where β and ε are gradients of the local Coriolis parameter and injection rate, respectively.
19 Eq. (8) is similar to that given in Danilov and Gurarie (2002) for a 2D flow although it can
20 also be applied in the case of 3D flows. This is true if the injection rate ε in (8) is modified to
21 include the contribution of open boundary conditions (if any). Since ε was not determined for
22 the CCS, the injection rate in Eq. (8) was not changed.

23 Nonlinear interactions are effective if $\Omega < \Gamma_m$ (Rabinovich and Trubetskov, 1989).

24 For simplicity, we replace $\Gamma \sim \Gamma_m$ and with $\Omega = \gamma Rn$ then λ in the flow should be limited as

$$25 \quad \lambda > \gamma^2 \Gamma^{-2} (\beta/2)^{-2} \varepsilon L^{-2}. \quad (9)$$

26 Since the value of the injection rate is not known (at least for the California Current System),
27 this demonstrates only that there exists a threshold for Ω . Below this threshold, when the
28 number of near-resonance quartets is strongly reduced as λ tends to zero, the energy transfer
29 into QZJs should cease. The same conclusion can be made for BAOs as well.

1 So, QZJs require existence of some finite level of dissipativity. If this level is not reached, no
2 zonal jets can be found in a flow. This conclusion agrees with results obtained by Berloff et
3 al. (2011) who also found that jet generation requires existence of some finite level of
4 dissipation in a system. Another issue is how the choice of dissipativity controls the latency of
5 QZJs.

6 Chelton et al. (2007), (2011) have hypothesized that propagating SSH anomalies off
7 California were a response of the ocean surface to mesoscale eddies because there is
8 "practically" no wave dispersion for such scales. Estimates obtained in this study and Ivanov
9 et al. (2010) have shown that wave dispersion is finite. Rossby waves of annual and semi-
10 annual periodicities which participate in quartets are clearly dispersion waves at least in linear
11 and quasi-linear regimes of propagation (Ivanov et al., 2010). They also participate in
12 modulational instability. Therefore, these SSH anomalies should be interpreted as a result of
13 forcing by nonlinear Rossby waves.

14 Note that bottom friction is very important for generation of broadening of the spectral peaks
15 that correspond to waves. The point is that any turbulent process generates broadening of the
16 peaks, and turbulence is always present in oceanic flows. The broadening of the peaks by
17 turbulence is described in the Appendix of Galperin et al. (2010).

18

19 **8 Phase synchronization/locking events**

20 A number of mechanisms form QZJs in the ocean. QZJs can be generated by near-resonance
21 triple interactions between modes (e.g., see Lee and Smith 2007), anomalous local wind
22 forcing, as assumed in Nakano and Hasumi (2005); or by the instability of background
23 currents (e.g., Berloff et al., 2009). But in each of these cases, if nonlinear interactions
24 including quartets play an important role, phase synchronization/locking events should be
25 expected. These can be easily extracted from observations in phase space (Pikovsky et al.,
26 2001).

27 The M-modes introduced by Eq. (1) generate an appropriate phase space. Hence, the
28 underlying structure of quasi-zonal jets and inter-scale interactions forming these jets can be
29 analyzed through the phase–frequency analysis of M-mode dynamics using amplitude–phase
30 variables. Phase synchronization corresponds to the case when the phase difference between

1 two oscillations is zero. Phase locking assumes this difference is constant (Pikovsky et al.,
2 2001).

3 Using (2) does not solve the problem of phase synchronization/locking events because
4 estimates have shown that in this case only the 1999-2001 phases can be observed.
5 Therefore, another definition of phase synchronization/locking events based on Pereira et al.
6 (2007) is used. They suggested that phase synchronization/locking events in a nonlinear
7 system can be detected through correlations between local maxima and minima of system
8 variables in phase space. In this study, amplitudes of M-modes were used as system
9 variables. Two modes were in phase (anti-phase) if the difference between two times t_j and
10 t_{j^*} corresponding to two neighboring maxima or two neighboring minima was less than some
11 threshold δ , i.e

$$12 \quad \left| t_j - t_{j^*} \right| \leq \delta. \quad (10)$$

13 $\delta = 0$ corresponds to phase synchronization, and $\delta \neq 0$ is phase locking. Note that in
14 comparison with traditional techniques for calculation of phase of a multi-frequency signal as
15 a wavelet or Hilbert transform, the above approach is applicable to any signal that contains
16 distinct marker events.

17 Appropriate histograms calculated as discussed above are shown in Fig. 9. It is clear from
18 these histograms that quartets (four wave interacting structures) were detected in two different
19 spectral bands. First, modes corresponding to QZJs, annual Rossby waves, semi-annual
20 Rossby waves and MEs from the 5th spectral band formed quartets which were responsible
21 for energy transfer from Rossby waves to the QZJs (Fig. 9a,c,e,g). Phases of all these modes
22 were locked that indicates strong interactions between these modes. These quartets
23 corresponded to maximum values of η^2 . The quasi-zonal jets had maximum lengths (long
24 jets) when the number of synchronization events was maximum.

25 Second, modes which corresponded to BAOs (with scales from 2 to 3 years) correlated with
26 semi-annual Rossby waves and mesoscale eddies from the 5th and 6th spectral bands (Fig.
27 9b,d,f,h). Phases of all these modes were locked that indicates strong interactions between
28 these modes. In this case the energy was transferred from Rossby waves to BAOs. Forming
29 BAOs from semi-annual Rossby waves demonstrated the possibility of inverse cascades of
30 energy to accelerate this process. Again these quartets corresponded to maximum values of

1 η^2 . However, since BAOs are not QZJs, they cannot have maximum length when the number
2 of synchronization/locking events was maximum. But modes forming these oscillations have
3 maximum amplitudes too.

4 Note that QZJs and BAOs did not interact with one another over the length of the data record.
5 However, this does not mean that these processes were linear.

6

7 **9 Conclusions**

8 Some evidence of a self-organized process in the CCS is given here. It occurred when QZJs
9 and BAOs were formed due to near-resonance inter-scale interactions through modulational
10 instability. The number of observed quartets was 10 times larger than the number of induced
11 triads. As a consequence, SSH signal was transferred from annual and semiannual Rossby
12 waves and mesoscale eddies to lower frequency current structures such as QZJs and BAOs.
13 This example demonstrates modulational (Benjamin-Feir) instability in which the carrier
14 wave is phase locked with the side bands.

15 Two types of quartets were observed off California. Quartets of the first type contained QZJs,
16 annual Rossby waves, semi-annual Rossby waves and mesoscale eddies from the fifth
17 spectral band. Note that the mesoscale eddy field was strongly smoothed by the AVISO
18 team. Therefore, although the harmonics which participated in nonlinear interactions were
19 identified more or less accurately, fluxes between harmonics have been shifted into the
20 Rossby wave band.

21 The second type of quartets consisted of BAOs, semi-annual Rossby waves and mesoscale
22 eddies from fifth and sixth bands. Energy transfer to BAOs from faster currents seems to be a
23 novel phenomenon and has not been noted in modern oceanographic literature (Baldwin et al.,
24 2001).

25 Note that it was found that BAOs did not interact with QZJs nonlinearly within the given
26 observational series. However this does not mean that BAOs or QZJs should be
27 approximately linear, although they are the lowest frequency currents observed in satellite
28 data.

29 Dissipativity of an oceanic flow controls generation of QZJs and BAOs. This is because the
30 level of dissipativity determined the number of near-resonances causing non-local energy
31 transfer from fast to slower time scales (inverse time cascade). A minimum dissipativity level

1 in the CCS was shown to exist (it was demonstrated for example, for bottom friction) when
2 QZJs and BAOs were generated.

3 These results help in understanding a role of QZJs in the CCS. Although the QZJs have a
4 considerably weaker signature in SSH anomalies than RWs, they acted as a catalyst, helping
5 RWs, MEs and QZJs exchange energy between themselves. In our opinion, QZJs could not
6 serve as barriers for meridional transport in the CCS because they are too weak.

7 Conclusions made in this paper apply only to the California Current System. Other physical
8 mechanisms for generation of QZJs may exist in other regions. For example, Tanaka and
9 Akitono (2010) found the existence of two different mechanisms in a two-layer model of an
10 idealized ocean. Note that our analysis does not support Centurioni et al. (2008), who
11 discussed direct instability of the CCS meanders and hypothesized a possible mechanism of
12 QZJ generation in this region as plumes.

13

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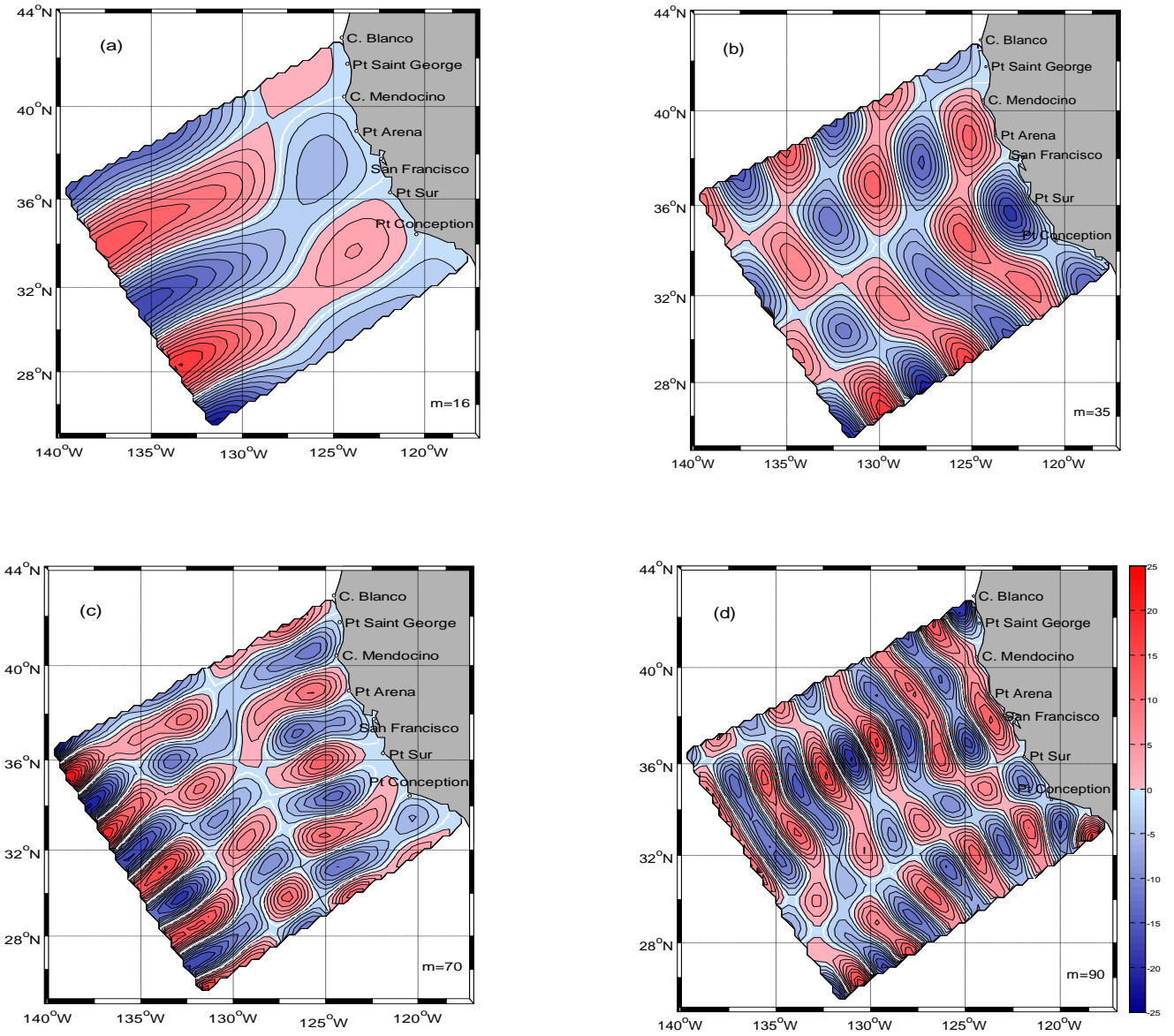
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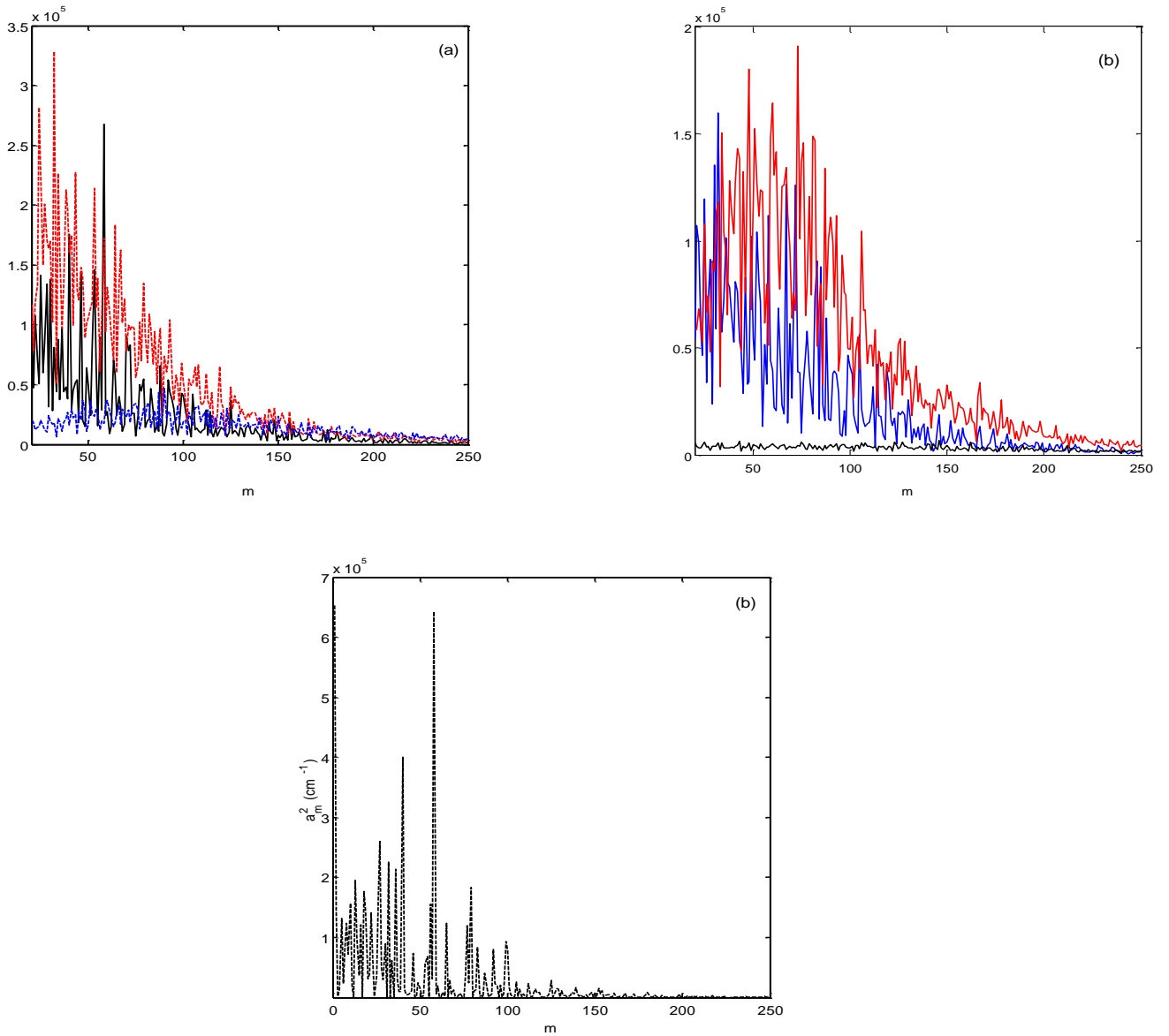


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4 Figure 1. Structure of basis functions (M-modes, $\Psi_m \cdot 10^5$) used for calculations. (a) $m=16$,
5 (b) $m=35$, (c) $m=70$, and (d) $m=90$. White contours are zero isolines.

6



2

3 Figure 2. Spectral characteristics of coherent structures observed off California. (a) and (b)

4 Time averaged energy density ($\overline{a_m^2}$) vs mode number for the six frequency bands used in this

5 study. (a) Black, red and blue correspond to quasi-zonal jets (with scales longer than 36

6 months), annual Rossby waves (with scales from 8 to 18 months), and mesoscale eddies (with

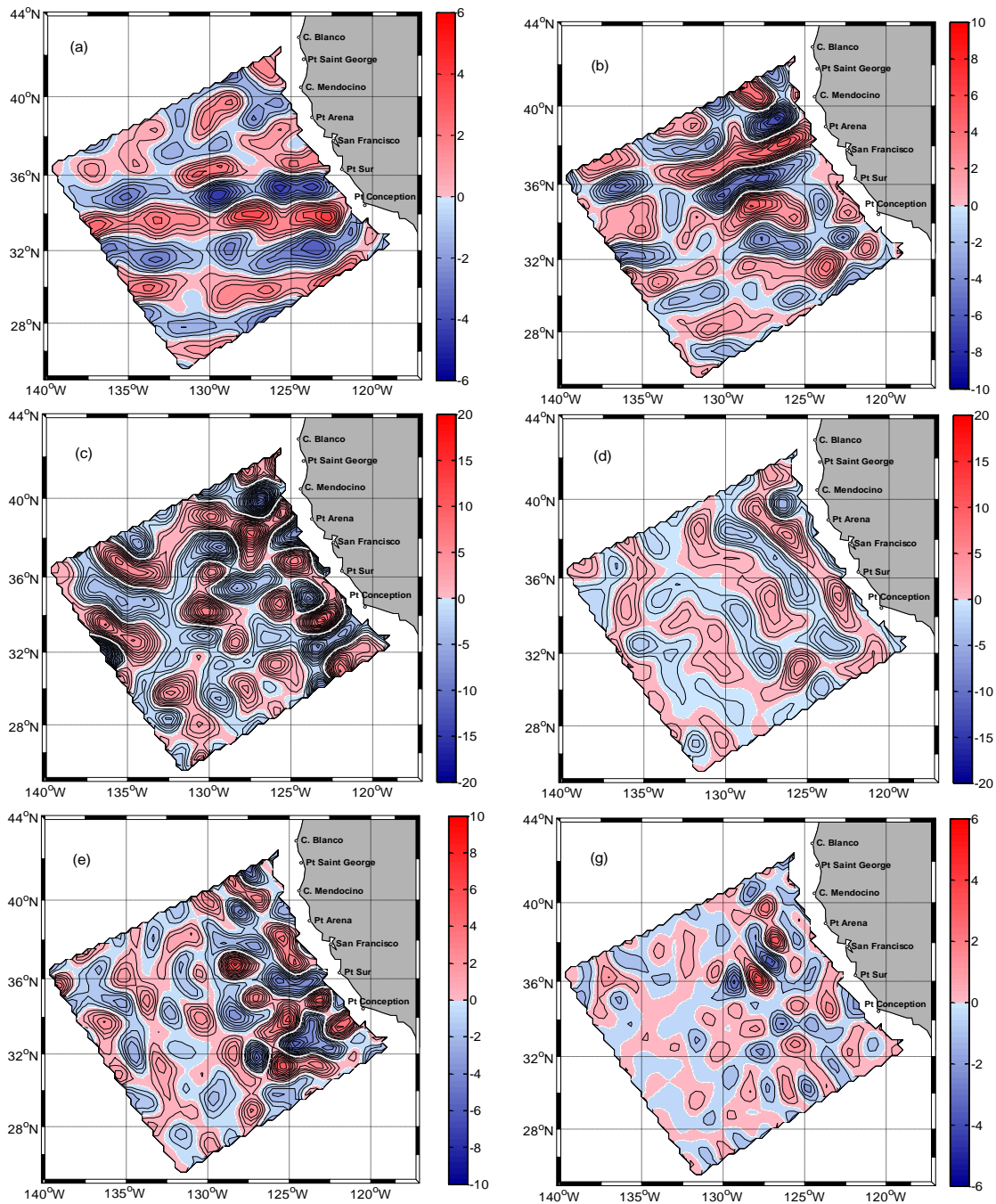
7 scales from 2 to 4 months). (b) Blue, red and black correspond to bi-annual oscillations

8 (with scales from 18 to 36 months), semi-annual Rossby waves (with scales from 4 to 8

9 months), and mesoscale eddies (with scales from 1 to 2 months). (c) An instantaneous spectrum

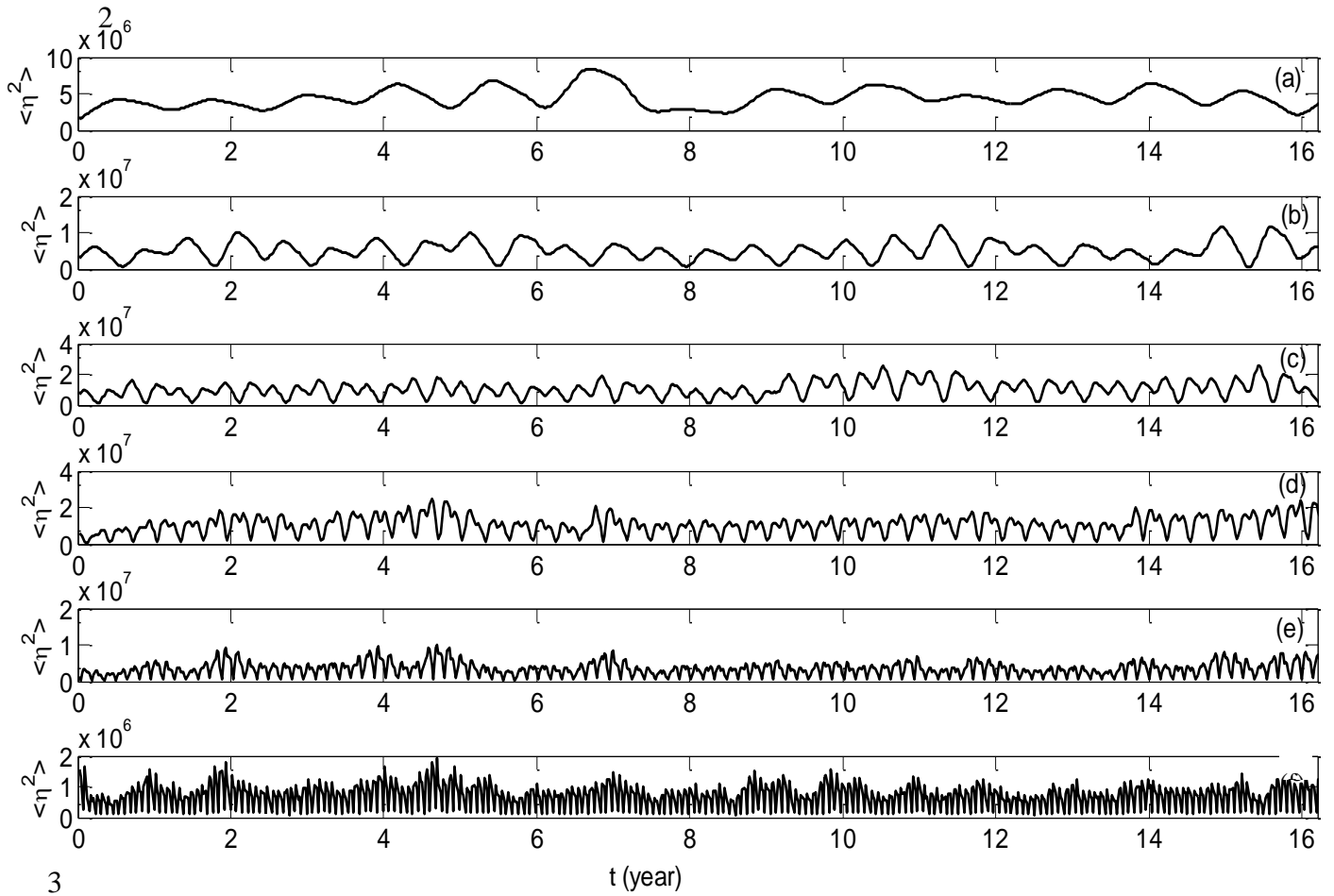
10 a_m^2 .

11



2 Figure 3. Characteristic spatial structure of (a) quasi-zonal jets (May 26, 2006); (b) bi-annual
 3 oscillations (Jan. 11, 2006), (c) annual Rossby waves (Oct. 4, 2006), (d) semi-annual Rossby
 4 waves (Nov. 28, 2007); (e), (g) mesoscale eddies (Nov. 22, 2006 and May 9, 2006). Contour
 5 intervals are 1 cm and 0.5 cm in (a-d) and (e,f), respectively. Note that the range of the
 6 contours varies from ± 6 in panels a, g to ± 10 in b, e and ± 20 in c, d.

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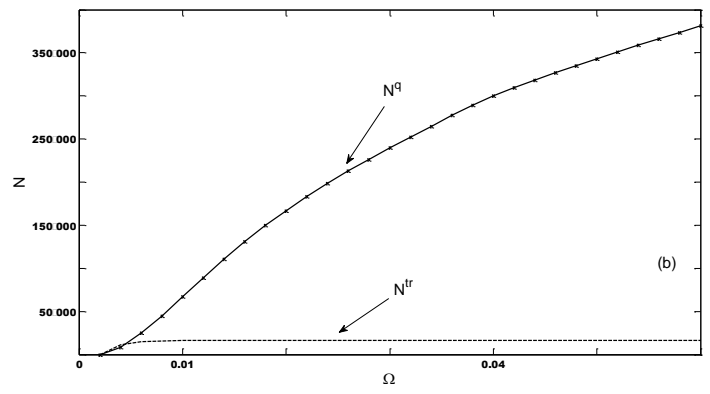
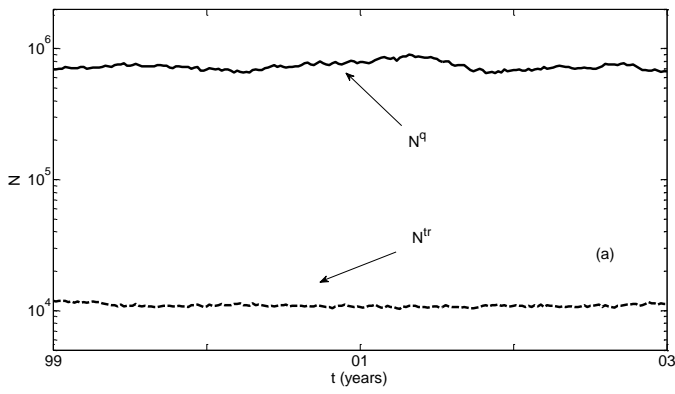


3

4 Figure 4. Temporal behavior of $\langle \eta^2 \rangle$ (cm²) for six spectral bands: (a) longer than 36 months,
 5 quasi-zonal jets; (b) 18 to 36 months, biannual oscillations; (c) 8 to 18 months, annual Rossby
 6 waves; (d) 4 to 8 months, semiannual Rossby waves; (e) 2 to 4 months, mesoscale eddies; and
 7 (f) 1 to 2 months, mesoscale eddies.

8

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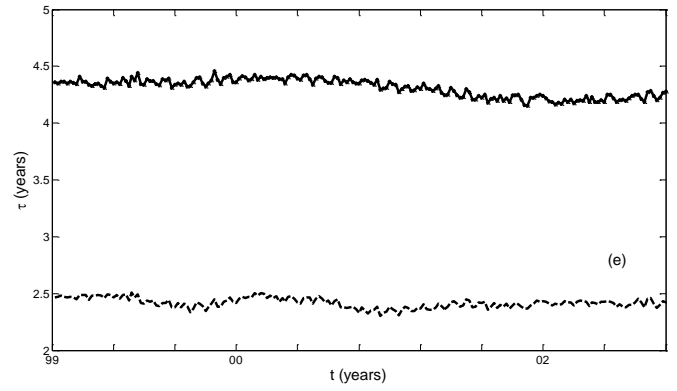
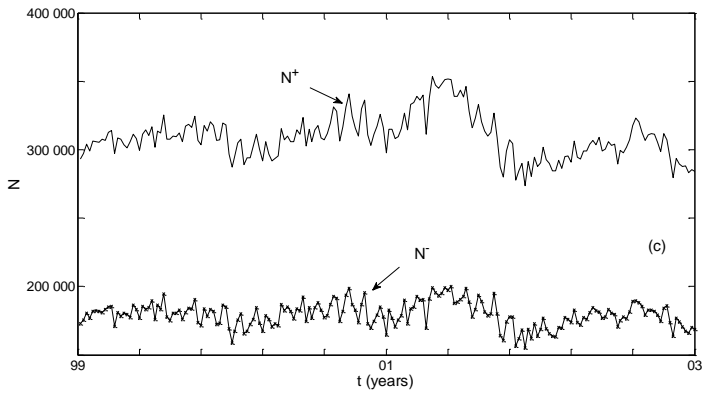


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3 Figure 5. Variability of the number of near resonance triplets, N^{tr} , and near resonance
4 quartets, N^q , as a function of (a) time and (b) resonance broadening, Ω .

5

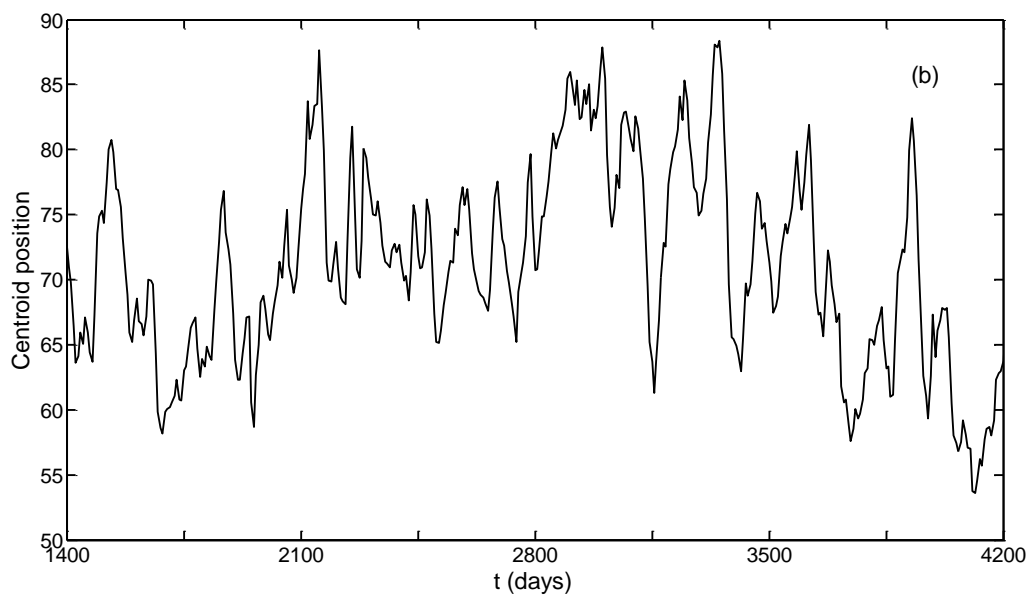
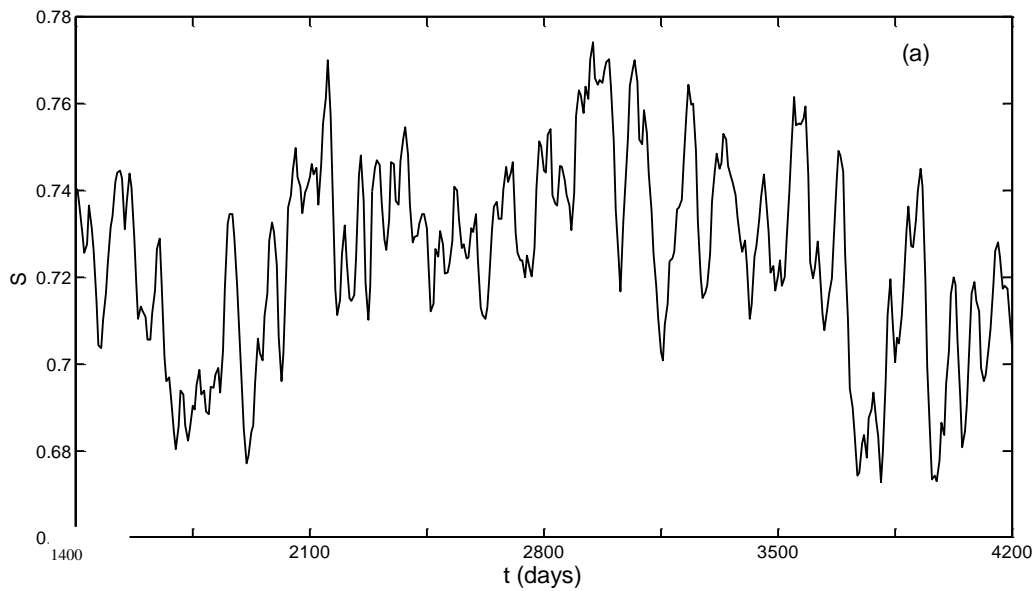
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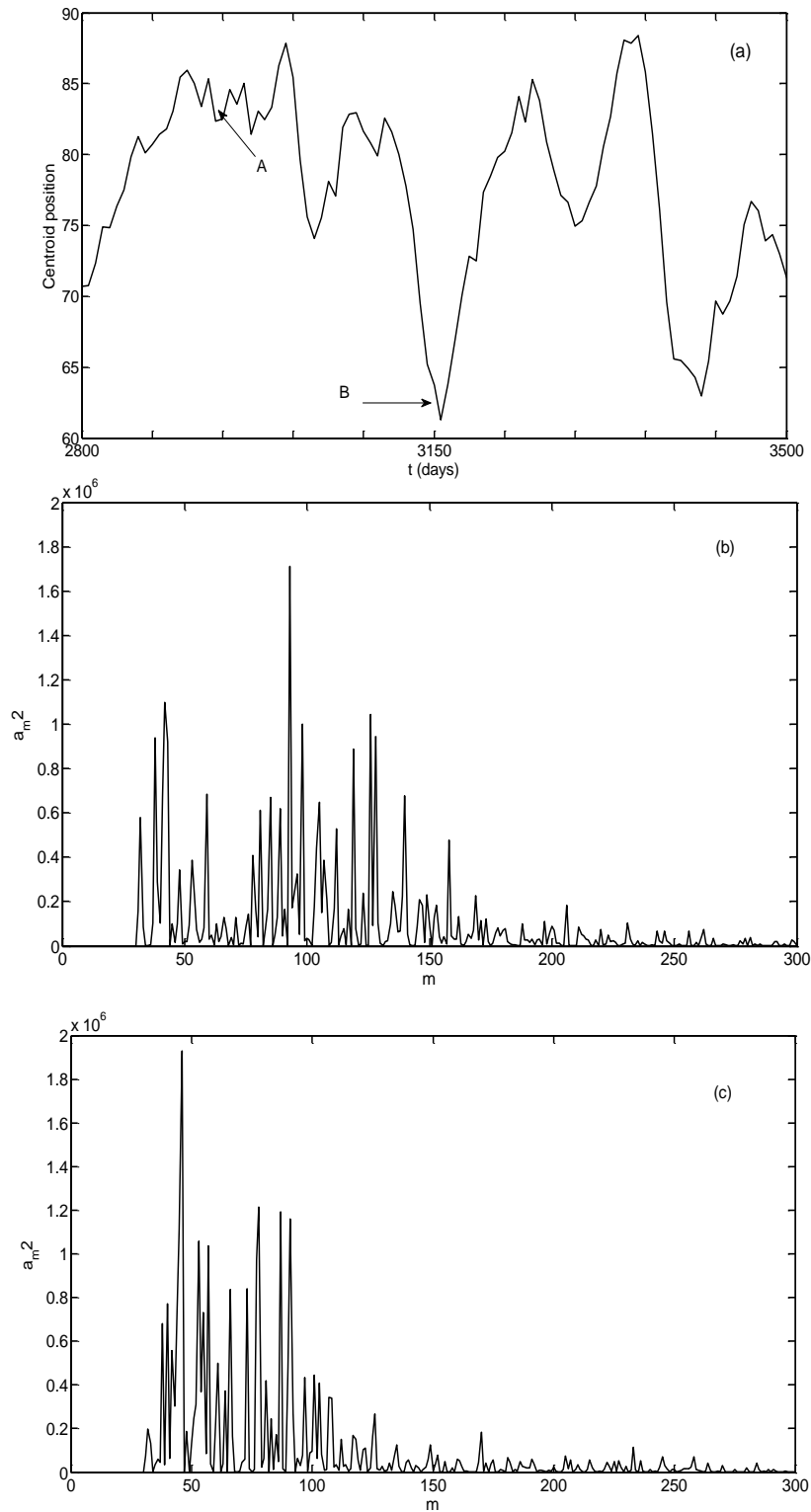
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3 Figure 6. Temporal behavior of the number and time scale of near resonance quartets
4 corresponding to for quasi-zonal jets and biannual oscillations in 1999-2003. (a) Number of
5 near resonance quartets which participated in inter-scale interactions to generate the energy
6 for quasi-zonal jets (QZJs) (N^+) and biannual oscillations (BAOs) (N^-). (b) Mean time scales
7 for QZJs (-) and BAOs (--).

8



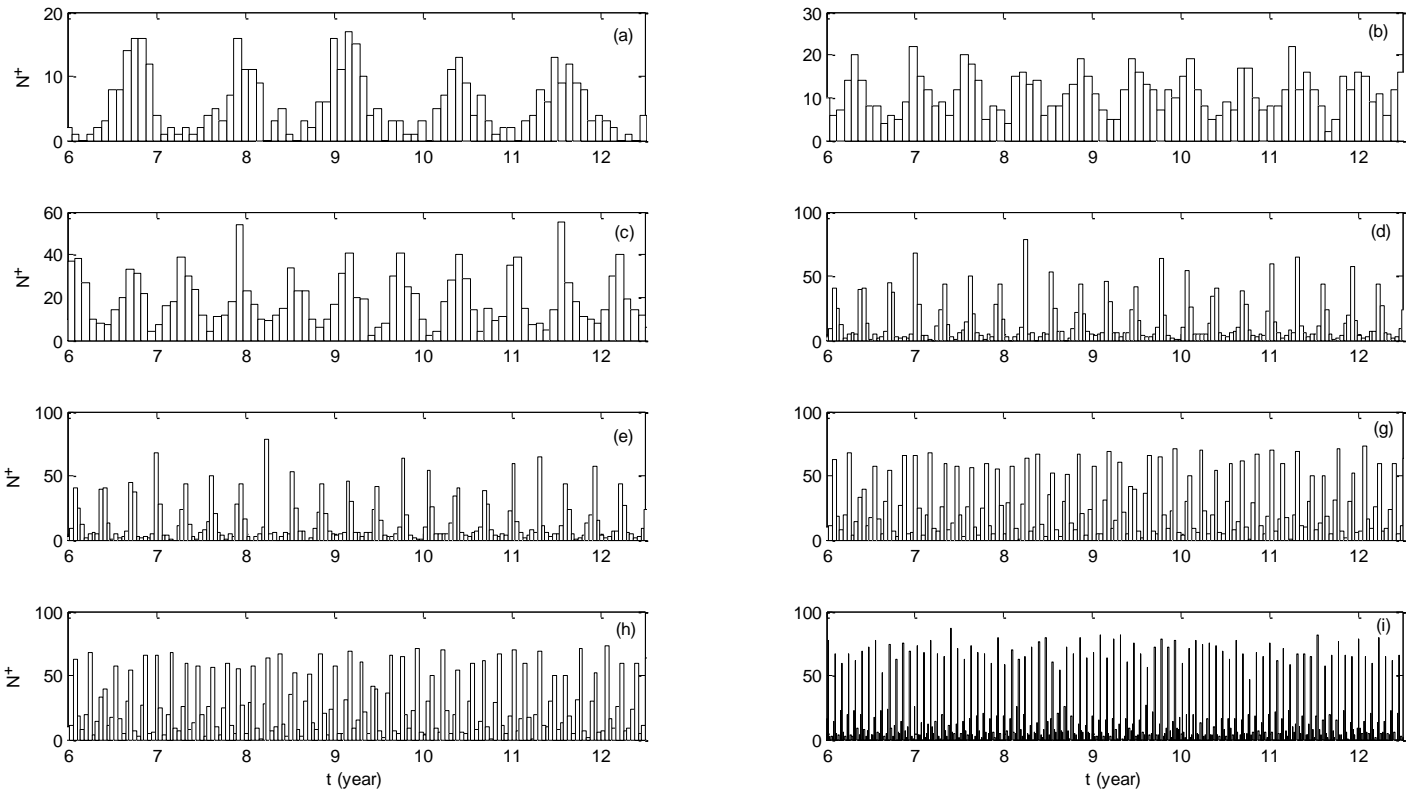
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 2 Figure 7. Evolution of spectral entropy and centroid, October 10, 1992, to May 23, 2009. (a)
 3 Spectral entropy in non-dimensional units. (b) Spectral centroid in m.
 4



1

2 Figure 8. An evolution of the spectral centroid. (a) Spectral centroid, from Aug. 5, 1999 to
 3 July 5, 2000. A and B indicate a time interval within which the centroid moved to lowest
 4 value. (b) Spectrum of a_m^2 for point A on 2940 day. (c) Spectrum of a_m^2 for point B on 3150
 5 day.

6



2 Figure 9. Synchronization/locking events off California. (a),(c),(e),(h) The first quartets
 3 among the quasi-zonal jets, annual Rossby waves , semiannual Rossby (SARWs) waves and
 4 mesoscale eddies from the fifth band ; (b),(d),(g),(f) The second quartets formed by biannual
 5 oscillations, SARWs and mesoscale eddies from the fifth and six spectral bands. N^+ is a
 6 number of synchronized modes.