Manuscript prepared for Nonlin. Processes Geophys.
with version 5.0 of the LaTeX class copernicus.cls.
Date: 12 August 2014

Spatial analysis of oil reservoirs using DFA of geophysical data

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Abstract. We employ Detrended Fluctuation Analysis (**DFA**) technique to investigate spatial properties of an oil reservoir. This reservoir is situated at Bacia de Namorados, RJ, Brazil. The data corresponds to well logs of the following geophysical quantities: sonic, gamma ray, density, porosity and electrical resistivity, measured in 56 wells. We tested the hypothesis of constructing spatial models using data from fluctuation analysis over well logs. To verify this hypothesis, we compare the matrix of distances among well logs with the differences among **DFA**-exponents of geophysical quantities using spatial correlation function and Mantel test. Our data analysis suggests that the sonic profile is a good candidate to represent spatial structures. Then, we apply the clustering analysis technique to the sonic profile to identify these spatial patterns. In addition, we use the Mantel test to search for correlations among **DFA**-exponents of geophysical quantities.

1 Introduction

To a great extend the information about petroleum reservoirs is obtained from well logs that measure geophysical quantities along drilled wells, see Asquith and Krygowski (2004). As a rule data is spatially sparse and presents strong fluctuation, therefore we have to rely on statistical methods for evaluating indices that describe the characteristics of the reservoirs, see for instance Hardy and Beir (1994) and Hewitt (1998). The question about what methods are more appropriate to fulfil this task is still open. In this work we investigate the use of fluctuation analysis to tackle this problem.

The well log data is the most valuable information that can be obtained from geological volumes and from oil reservoirs. However, the cost of drilling imposes severe limitations in the number of wells. In this situation we are faced with the problem of uncovering geophysical properties over long field extensions from data collected along few drilled wells. To perform this task we have to rely on data statistics that guarantees similarities among geological structures. One goal is to draw contour lines expressing the variation of proprieties in the subsurface by evaluating interpolation from well logs data. This will be justified if correlations show consistent spatial patterns. The question of this article is: can we use **DFA**-exponent to discover spatial patterns. In other words, is **DFA**-exponent spatially correlated in such way we can employ it as a spatial parameter.

In the last decade new techniques from the physics of complex systems were introduced in geophysics, see Lovejoy and Schertzer (2007); Dashtian et al. (2011b). The Detrended Fluctuation Analysis **DFA** is a powerful fluctuation analysis technique introduced by Peng et al. (1995) that was developed to deal with non-stationary time series. This tool is similar to the Hurst method, see for instance Mandelbrot (1977), that is used to compare an aleatory time series with a similar Brownian series, as well as, to evaluate correlation and anti-correlation in a series. **DFA** technique has been used in many areas of geophysical literature, in Padhy (2004) it is used to obtain information from seismic signals. In references Andrade et al. (2009); Chun-Feng and Liner (2005); Gholamy et al. (2008); Tavares et al. (2005) **DFA** is employed to interpret and filter images of seismograms. In reference Ribeiro et al. (2011); Lozada-Zumeta et al. (2012); Marinho et al. (2013); Dashtian et al. (2011) this technique is used, as in this manuscript, in the analysis of well logs.

When we treat with complex systems that have a huge amount of data the **DFA** method is attractive because it allows to summarize data into a suitable parameter. The **DFA** parameter summarizes fluctuation information of a time series, this parameter is related to the autocorrelation properties and the spectrum of frequency of the data. The **DFA** exponent in this sense is an overall measure of its complexity. This simple procedure allows a fast comparison between large samples. Furthermore, the first step in oil research is a geographical analysis of the surface. To have characteristics of the geological structure of the subsurface projected into a single measurement on the ground level is an useful information. In addition, the spatial correlation between these quantities allow us to have a better understanding of the lithology which is crucial in oil prospection.

The case study employed in this work is an oil reservoir and we apply the **DFA** technique over data logs of drilled wells. The oil reservoir is situated at Bacia de Namorados, an offshore field in the Rio de Janeiro State, Brazil. The five geophysical measurements available in the well logs are: sonic (DT, sonic transient time), gamma ray (GR, gamma emission), density (RHOB, bulk density), porosity (NPHI, neutron porosity) and electrical resistivity (ILD, deep induction resistivity). The manuscript can be summarized as follows. In section 2 we perform three tasks: show the geologic data in some detail, introduce briefly the mathematics of the **DFA** and present the statistical methods we use in this work: spatial correlation, Mantel test and k-means clustering analysis technique. In section 3 we show the results of the spatial correlation function and the Mantel test; we estimate that the sonic profile is the best candidate to model spatial patterns. In addition, we apply clustering analysis to this geophysical quantity to create a spatial model. Finally, in section 4 we conclude the work and give our final remarks.

2 Model background

2.1 The geologic data

The geologic data used in this work are from well logs located in the oil field of Bacia de Namorados, Rio de Janeiro State, Brazil. The wells are situated in an area of approximately $100km^2$ and distant $150km$ from the coast. The spatial arrangement of the well logs is illustrated in figure 3.3 and the matrix of distance among pairs of well i and j is done by $d_{i,j}$. The number of records for each well is not constant, the sonic register was recorded in ($N = 17$) well logs, gamma ray ($N = 53$), density ($N = 51$), porosity ($N = 48$), and, finally, resistivity ($N = 54$). The time series of the geophysical quantities of each well log has around $N_S \approx 1000$, the exact value depends on the measurement, this data series length guarantees a good statistic for the use of **DFA** method Kantelhardt et al. (2001). An example of a segment of the time series corresponding to each of the five geophysical variables is visualized in figure 2.1.

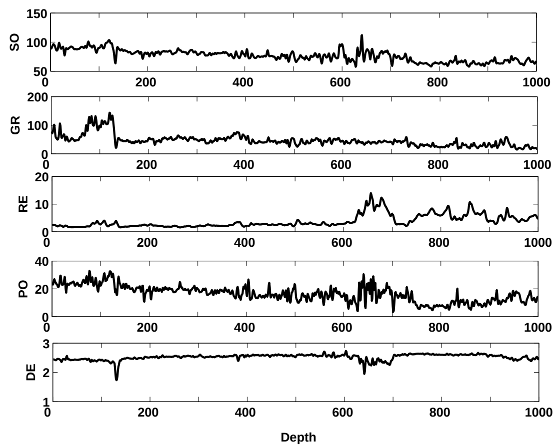

Fig. 1. A segment of a typical measurement, for an arbitrary well, of the geophysical properties versus depth (in meters): sonic (SO), gamma ray (GR), density (DE), porosity (PO), and resistivity (RE).

2.2 The Detrended Fluctuation Analysis DFA

The **DFA** is a fluctuation analysis technique, see for instance Peng et al. (1994) and Kantelhardt et al. (2001). We present a concise description of the **DFA** algorithm, comprehensive introduction of the method is in Peng et al. (1995) and Ihlen (2012). Consider a time series $x_t = (x_1, x_2, ..., x_{N_S})$ with N_S elements. To calculate the **DFA** algorithm we initially integrate the series $x(t)$ producing a new variable $y(t)$:

$$y(t) = \sum_{i=1}^{t} |x_i| \tag{1}$$

In the second step of the algorithm we perform an equally partition of the time series into boxes of length n. A data fitting is performed inside each box a using the least square method, the generated auxiliary curve is called the local trend $y_n(t)$ of the data. In the third step we detrend the integrated series, $y(t)$, to execute this procedure we subtract $y(t)$ from the local trend $y_n(t)$. The root mean square fluctuation is found with help of the relation:

$$F(n) = \sqrt{\frac{1}{N_S} \sum_{i=1}^{N_S} (y(t) - y_n(t))^2}. \tag{2}$$

The fourth step consists in estimating Eq. (2) over all boxes of size n. Usually $F(n)$ increases with n, a linear increasing of $F(n)$ with n in a log-log scale is a typical signature of a fractal behavior. The exponent α of the relation:

$$F(n) = n^{\alpha} \tag{3}$$

is known as the **DFA**-exponent. The most important equation of this theoretical development is Eq. (3) that provides a relationship between the average root mean square fluctuation, $F(n)$, as a function of the box size n. In this work we have computed α with the help of the algorithm available in Matlab, a similar algorithm is also available in C-language, Peng et al. (1995). In figure 3.3 we show, as an illustration, the curve of $F(n)$ versus n for two distinct well for gamma-ray and sonic data.

We performed a similar analysis for the available well logs of all geophysical quantities. For 98% of cases the correlation coefficient of the adjusted line in the log-log plot fulfil the relation $R^2 \leq 0.95$, for R the linear correlation coefficient Sokal and Rohlf (1995). The cases that do not follow this condition were discarded from the statistics.

2.3 Statistical Analysis

In the paragraphs that follow we show the statistical methods explored in the paper. All statistical analysis were performed using R language, see the reference R-project (2008).

2.3.1 Spatial correlation

To test the spatial correlation among variables, the most simple statistics is the correlation function, $Corr(\tau)$, for τ the correlation length. To test spatial correlation between **DFA**-exponent and distance we start ranking all $d_{i,j}$ of the distance matrix. We compute the difference of the matrix of DFA-exponent: $\Delta^t\alpha_{i,j} = |\alpha_i^t - \alpha_j^t|$ for all geophysical variable g^t. The quantity $\Delta\alpha^t$ is ordered according to distances τ. $Corr^t(\tau)$ is estimated as follows:

$$Corr^t(\tau) = \frac{\sum_{l=1}^{Num} \Delta\alpha^t(d)\Delta\alpha^t(d+\tau)}{Num\, sd(\Delta\alpha^t)} \tag{4}$$

where the sum in equation is performed over all possible pairs Num. To compute $Corr(\tau)$ the quantity $\Delta\alpha$ is transformed to $\Delta\alpha \rightarrow \Delta\alpha - \mu$ for μ the average of $\Delta\alpha$, the correlation function is evaluated over zero means series. The standard deviation, $sd(\Delta\alpha)$, in the denominator normalizes adequately the function such that $Corr(0) = 1$.

2.3.2 Mantel test

Mantel test is a statistical tool to test correlation between two symmetrical matrices of the same rank. The rationale of this test is to employ matrix elements in the same way as vectors of objects, in this way the Mantel test is quite similar to the Pearson test that search for a correlation between two vectors. In the Mantel test matrices are transformed into vectors to evaluate the linear correlation, see Sokal and Rohlf (1995).

We compute two distinct sets of tests: in the first we check for correlation between the matrix of distances of the well logs $d_{i,j}$ and the differences matrix of DFA-exponent $\Delta^t\alpha_{i,j}$ of any geophysical variable g^t. In a second moment we compare the DFA among geophysical quantities applying Mantel test between matrices of $\Delta^t\alpha_{i,j}$ and $\Delta^s\alpha_{i,j}$ of geophysical quantities g^t and g^s. Of course we evaluate this test only over pairs i and j of well logs that have available data for both g^t and g^s.

2.3.3 Clustering analysis

For the geophysical quantities that show spatial correlation we search for spatial patterns. In this article we use k-means - a standard tool of clustering analysis to perform this task. The k-means methodology works by creating groups using a metric criterion. The user of the method chooses a fixed number k of subsets, or clusters, and an optimization algorithm selects elements according to the distance to k centroids.

In our study, we find that only one geophysical quantity present significant spatial correlation, the sonic variable. To use k-means methodology it is necessary to have at least three input variables. For obtaining the two additional parameters we employ the following strategy: we use the upper and lower values of the error interval of the fitting of the curve defined by Eq. (3).

We use a Monte Carlo test, or a randomization test, to check if k-clusters method creates groups that are closer, in a metric sense, than groups generated by an aleatory way. We define an index Ω of neighborhood in the following way. Consider the map of the field with all wells. Over each well we attach a geometric ball (or a disk) of radius b. The wells that are spatially closer share overlapping balls in opposition to distant wells. This schema of overlapping balls is used to measure if two wells that are in the same k-group are close or not. For all pairs of well logs we perform the computation: if the balls of two well logs overlap and belong to the same group we count $\Omega \rightarrow \Omega + 1$ otherwise we do nothing. The index Ω is normalized by the number of groups and the maximal number of elements in each k-group. After that we shuffle the well logs over the k-groups and compute $\Omega_{shuffled}$ over the shuffled data. The idea of this method is to compare if the k-groups are more distant from each other than groups chosen at random. We estimate a p-value as the probability of Ω being larger than the $\Omega_{shuffled}$ distribution.

3 Results

To check for spatial correlation we use three independent statistical tests: the spatial correlation, the Mantel test and the clustering analysis. To improve the visualization of our analysis we introduce a couple of spatial pictures of the DFA-exponent computed over the well logs, figure 3.3. We depict five figures, one for each geophysical variables: porosity (PO), resistivity (RE), gamma ray (GR), density (DE), and sonic (SO) as indicated in the picture. The spatial image uses arbitrary distance unities x and y, to help the perception of the system we depict contour plots with colours, regions sharing the same color assume close DFA values.

3.1 Spatial correlation

We initially compute the function $Corr(\tau)$ for $0 \leq \tau \leq 80$ for all geophysical variables; we checked that 80 is a number large enough to $Corr(\tau)$ decay and start oscillating around zero. We expect that in case α variables of any geophysical quantity g^t shows spatial correlation the function $Corr(\tau)$ should decrease with τ. To analyze the decay of $Corr(\tau)$ of the geophysical variables we fit a linear curve and test how significant is its decay. The result of the fitting of the geophysical quantities is shown in table 1, this result indicates that the only quantity that reveals a significant decay is the sonic data, all the other quantities show $p > 0.05$ for the linear fitting test.

3.2 Mantel test

Table 1 it also shows the results of the analysis of the Mantel test for all geophysical variables. Here it computes the correlation between two matrices: $d_{i,j}$, the matrix of distance between two wells, and $\Delta\alpha_{i,j} = \alpha_i - \alpha_j$, the matrix of difference between **DFA**-exponent α for the same wells. The correlation parameter of the test is indicated by r while p is the p-value of the significance test. In agreement with the output of the correlation function analysis the smallest p-value is attributed to the sonic variable. This result justifies the use of sonic data for constructing spatial patterns, the subject of the next section.

We use Mantel test not only to analyze the correlation between distances and **DFA**-exponent, but also to perform a comparison between distinct geophysical quantities. That means we compare matrices $\Delta\alpha^t$ and $\Delta\alpha^s$ of geophysical quantities g^t and g^s. The result of this analysis is shown in table 2. We plot only the p-value of the test in the table, the major agreement observed was between variables: resistivity and porosity, which is followed by density and sonic.

3.3 Clustering analysis

The sonic variable has revealed a good candidate to generate spatial patterns. In figure 3.3 we plot the oil reservoir area with well logs, the axis x and y represent the spatial coordinates, we use metric arbitrary units. The points in the figure represent the coordinates of the well logs. In figure 3.3a we use the fixed number of clusters $k = 3$ while in figure 3.3b we use $k = 4$. Elements in the same cluster are indicated by a common symbol, these two pictures suggests that sonic variable is indeed a good geophysical quantity to model spatial formations.

Table 1. The results of spatial correlation: the decaying of the spatial correlation and Mantel test. The linear fitting of the correlation function is indicated in table as well as the output of the Mantel test. The result indicates that only sonic data is appropriate for constructing spatial analysis. The geophysical quantities are indicated in the table: sonic (SO), density (DE), gamma ray (GR), electrical resistivity (RE), and porosity (PO).

	Spatial correlation			Mantel test	
	F	ρ	p	r	p
PO	0.002	0.00003	0.96	-0.021	0.64
RE	0.11	0.002	0.74	0.016	0.51
GR	1.05	0.015	0.31	-0.028	0.73
SO	9.03	0.12	0.004	0.181	0.06
DE	0. 64	0.01	0.43	0.023	0.34

Table 2. This symmetric table shows the p-value of the Mantel test of hypothesis for correlation among the **DFA**-exponent of geophysical quantities. The test is performed between each pair of five geophysical variables: porosity (PO), resistivity (RE), gamma ray (GR), density (DE), and sonic (SO).

	RE	GR	DE	SO
PO	p = 0.088	p = 0.74	p = 0.95	p = 0.21
RE	-	p = 0.73	p = 0.62	p = 0.44
GR	-	-	p = 0.62	p = 0.61
DE	-	-	-	p = 0.13

To test how good is the spatial formation of the clustering analysis, we employ a Monte Carlo test. We estimated the proper Ω value and found $p = 0.005$ for $k = 3$ and $p = 0.16$ for $k = 4$ using an optimal ball size b. We checked the k-means clustering technique for the other quantities: sonic, resistivity, porosity and gamma-ray. We use $3 \leq k \leq 6$ for all these geophysical data set and we found no $p > 0.05$, that means, no evidence of significant spatial cluster formation. This result is an indirect evidence that only sonic variable is a good choice to formation of spatial patterns.

4 Final Remarks

The issue of this manuscript is to test the hypothesis that we can use **DFA**-exponent α from log wells as integrated indices projected over the earth surface to reveal spatial structures. Each α is an index that summarizes the structure of fluctuation of a geophysical quantity over geologic layers of thousand meters deep. The challenge is to use the information of the fluctuation from a set of distinct well logs distributed over several kilometers to construct spatial patterns.

The results of Mantel test and spatial correlation function indicate that the only geophysical parameter we can rely on this global approach to model spatial patterns is the sonic. We use partitioning by k-means, a standard technique of cluster analysis appropriate to represent spatial models. A visual

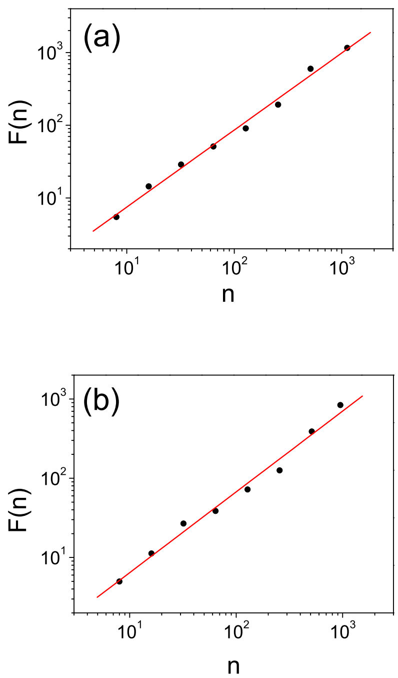

Fig. 2. A typical plot illustrating **DFA** scaling property: $F(n)$ versus n, the curve of Eq. 3. The good fitting of most curves in a log-log scale reveals the fractal characteristic of geophysical data. In (a) the well 2 of gamma ray data and in (b) the well 17 of sonic data.

inspection of the spatial patterns, as well as a Monte Carlo test, verify that sonic data forms good spatial models for $k = 3$ and 4. In opposition, other geophysical quantities do not show significant results in Monte Carlo test.

In addition to spatial analysis, we also used Mantel test to search for correlations among geophysical quantities. In a previous work Ribeiro et al. (2011), using the same data set, but applying a different methodology, it was found that the only pair of geophysical variables that shows significant correlation was density and sonic $(p = 0.01)$. In this work the pairs of quantities that show greater significance were porosity and resistivity $(p = 0.088)$ is closely followed by density and sonic $(p = 0.13)$. The paper Ferreira et al. (2009) has also found a major correlation between sonic and density using a standard correlation matrix. For both methodologies the pair density and sonic seems to be correlated, this property is probably related to the trivial fact that sound speed increases with density, see for instance Feynman and Leighton (1964). a result that is close to our result. As the methodologies of these works are not the identical, we do not expect the same result, indeed, small discrepancies are acceptable in statistical treatments. This last result is in agreement with Dashtian et al. (2011) that have used cross-correlation analysis

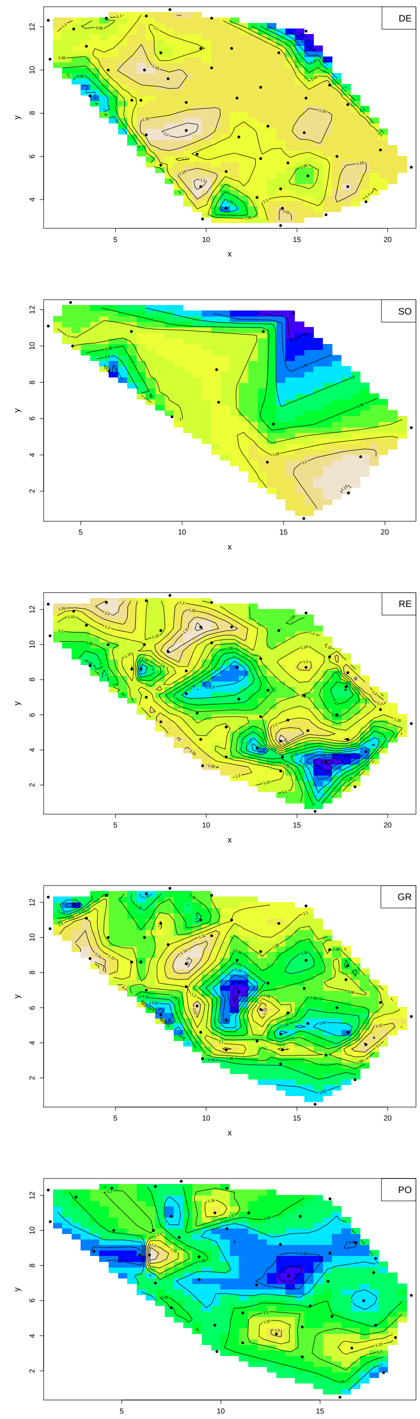

Fig. 3. Contour plots of DFA values over spatial data of oil reservoir of Campo dos Namorados, RJ, Brazil. We depict five figures one for each geophysical variables: porosity (PO), resistivity (RE), gamma ray (GR), density (DE), and sonic (SO). The dots correspond to well logs, we use arbitrary length unities x and y.

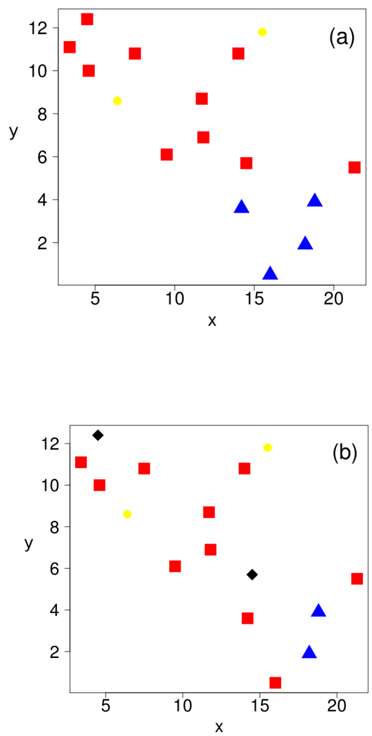

Fig. 4. Clustering analysis patterns for sonic data (a) $k = 3$, and (b) $k = 4$. Both figures show a satisfactory cluster formation in this data as confirmed by Monte Carlo test. We use arbitrary length unities x and y.

between well logs and found that sonic, porosity and density are more correlated among them than with gamma-ray.

To conclude the work we go back to the initial question of the manuscript: is it possible to create spatial models using fluctuation analysis? The sonic variable has shown enough spatial correlation to perform this task, but the density, which is the quantity the most correlated to sonic does not share the same property. However, a visual inspection in the couple of figures 3.3 suggests that the porosity has a consistent spatial distribution. In a future work we intend to test the combination of distinct geophysical quantities in the formation of spatial patterns.

Acknowledgements. The authors thank the ANP (National Oil Agency - Brazil) by the concession of the geologic data and Kleber C. O. Costa and Franscisco W. F. Silva for the use of data of their Master Dissertations. The authors, also, gratefully acknowledge the financial support of CNPq (INCT-Nano(Bio)Simes, Procad-Casadinho), CAPES (Rede NanoBioTec and PNPD) and CTPERTRO/FINEP.

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