F. Uboldi and A. Trevisan, 12 November 2014
Answer to Editorial Comments on the revised version of the paper:
Multiple scale error growth in a convection-resolving model

Dear Editor,
we modified the manuscript following your suggestions. Below you find our modifications and answers to specific points.

Editor's comments are reported here in *green italic*.

Reviewer 1, Comments #1 about lack of considering model error in your study.
Generally I concur with your point that the introduction of model errors would make it more difficult to study the chaotically growing errors that is the subject of your study. Two related, relatively minor points:

a) Please consider adding a quick note also in the abstract, clarifying that this is a "perfect model" type analysis of chaotically growing errors
b) As we understand from a number of studies (including those by Nicolis, Vannitsem, and others) initial value and model related errors evolve in a convoluted manner. For the sake of completeness you may refer to this, to acknowledge another limitation of your approach?

We modified the text following the Editor's suggestions:

a) **Abstract, line 5**: we inserted "By means of perfect model twin experiments...";
b) We now make reference to these works at the beginning of **Section 2, page 3, line 80-83**: "Limited-area models are forced by lateral boundary conditions, that represent an error source in addition to initial state errors and to errors intrinsic to the model dynamics (model error), which are present in all operational systems. Moreover, initial and model errors evolve in a convoluted manner (a relevant subject in recent studies, e. g. Nicolis et al., 2009; Carrassi and Vannitsem, 2011)."

Reviewer 1, Comment #2 – amplitude vs other features of errors?
I find this a more substantive question. I find your point A somewhat vague. And your response B incomplete. What I mean here is that you do use very special, dynamically constrained BV vectors as initial perturbations – clearly because you believe initial perturbation structures (and not only amplitudes) matter. This is the perspective from where I find your point B incomplete. Perhaps you could consider at what lead time forecasts would be used in data assimilation or forecasting applications (whatever your focus is) and relate your point to that? Clearly, error converge to a dynamically constrained, growing subspace – if your focus is beyond this transitional period, your point B may be valid, but you don't reflect on this and the reader may be confused. I find your points C and "final comment" convincing. Wonder if you need to strengthen the manuscript by incorporating these points?

We agree on the fact that, also in the revised version, our text was unclear on this point, so that a reader could actually get confused. We then modified the manuscript, clarifying the explanations on the initial conditions of the experiment trajectories, and on how bred vectors are initially seeded. We include these modifications here:
Page 7, Section 5, lines 224-236, we inserted**:**
"The experiments described in Sect.4 suggest to wait an initial 3-hour period for both control and ``true" states to approach the system attractor and for the bred vectors to acquire structures representative of the unstable directions of the system. Remark that the bred vectors are initiated as random noise (Sect. 5.2) at 21:00 UTC on 25 September 2006 and that, at this very initial time, the error is due to features present in the external model field. Instead, after 00:00 UTC on 26 September, forecast error and bred vector structures are those typical of the convection-resolving model dynamics, making it possible and significant to compare them.
The procedure used in the experiments is very similar to what is often done in the operational practice, when the state used as the initial condition for a non-hydrostatic model is taken from a hydrostatic model that also provides boundary conditions. This procedure implies the absence, in the initial control state, of small and fast scales typical of a convection-resolving model. These dynamic scales, present in reality, are also present in our experiments (after 00:00 UTC) as we chose to give enough integration time for them to develop."
Sect. 5.2, lines 259-265, we substituted the phrase "All perturbations..." with:
"Bred vectors are initiated ("seeded") by adding perturbations to the control state (either that of trajectory 18H or that of trajectory R21) at 21:00 UTC on 25 September. Remark that all initial perturbations are randomly generated point by point on the 3-D domain on variables T, U, and V, so that they do not have a particular spatial structure at this time. Only their amplitude (TUV norm) is imposed to be the same used for rescaling (see below). The spatial structure that bred vectors have at later times (when comparisons are performed) is the result of the breeding technique, combining model evolution and rescaling."
Sect. 6.1, page 9-10, line 290-305:
"We start discussing Fig. 1, that shows the forecast error as a function of time for the two control trajectories

starting from initial errors **with the same spatial structure, but with different amplitude**: one, R21, an order of magnitude smaller than the other, 18H. For both trajectories the growth rate is positive in the initial phase, until about 05:00 UTC for trajectory 18H and 09:00 UTC for trajectory R21, approximately corresponding to episode (1) of Sect. 3, and in the last part of the simulation, from 11:00 UTC to the end, when the intense phenomena progressively extend from North--Eastern Italy to almost the whole domain.
We observe that, whereas the ratio of the initial errors in the two control trajectories was initially as large as 10 (in view of the hypothetical assumption that the analysis error could be reduced by an order of magnitude) after 21 h the ratio of the forecast errors has become smaller than 1.5. The error, during the first 10 h in fact grows much faster in the small initial error trajectory (R21), whereas during the second episode the error growth of the two trajectories is very similar. This is because the growth rate is dominated by (small scale) fast instabilities when the initial error is small **(regardless of its initial spatial structure, which is the same in both cases)**, but, as forecast error grows in time, these components saturate and slower instabilities associated to larger dynamical scales become dominant"

Reviewer 2, Comment #1 about role of model error
I concur that during forecast integrations, model error is not present since the same model is used in the forecast process as that in generating "truth". On the other hand, if the initial condition is chosen to be a state produced by another model (l.205, "free MOLOCH evolution initiated by an external model state"), an initial drift may occur during the forecast integration that is model error related. Can you please consider if that is really the case, and whether that may have any impact on the results?
In Sect 6.2 of the manuscript we discuss the role of non-growing features persisting in the simulation and inherited from the initial external model field. These error features, as you suggest, are the outcome of the dynamical readjustment process that brings the initial condition state from one model to a state compatible with the other model dynamics and in this sense they are "model error" related. As stated in the conclusions, this kind of error features would not be present if a continuous assimilation cycle, without recurrent re-initialization from the external model was implemented, a procedure that we think would be much preferable.
Finally, since the focus of the paper is really not the model error, we think it would not be appropriate to divert too much of a reader's attention onto this topic by discussing it at length in this manuscript, beyond the modifications we already made, agreeing with the suggestions of the Referees and the Editor.

Additional Comments from the Editor:
l.244 – "All perturbations are randomly generated, point by point on the 3-D domain, on variables T, U, and V" – you are referring here to the "seed" that you start up the breeding procedure, is that correct? Please clarify
Yes. We modified the manuscript in **Section 5.2, lines 259-265** (included above), improving the explanation on how bred vectors are initially generated.

l.316 – "orthogonal projection of the forecast error on the subsets of BVs" – note that Wei and Toth (2003) used the same or very similar technique to compute what they call Perturbation versus Error Correlation Analysis (PECA) scores. In your perfect model experiment, you know and use real forecast errors while they used an estimate of forecast errors (forecast minus verifying analysis): Wei, M. and Z. Toth, 2003: A New Measure of Ensemble Performance: Perturbation versus Error Correlation Analysis (PECA). Mon. Wea. Rev., 131, 1549-1565. Your Fig 6a is analogous to their Fig. 4f.
We inserted a reference to this work in **Section 5.2, line 270-272**:
"A similar technique was used by Wei and Toth (2003) to compare forecast error estimates with member subspaces in an ensemble prediction system."

l.426 – "after subtracting the BV-estimated component, the growth of fast instabilities is reactivated and rapidly brings the error to its previous level in the subsequent free evolution" – while I understand what you say here the word "reactivate" may confuse some. Wonder if a better word / expression could be used here, e.g., "fast perturbation growth fueled by strong instabilities recreates related fine scale errors to bring overall errors to their level before the removal of the fine-scale errors" – or something similar?
We modified the manuscript, **Section 7.1, line 441-443**:
"The error reduction obtained by subtracting the error projection on the BV subspace brings some small-scale error features below their saturation level: in this condition fast error growth can be fueled by strong instabilities."

l.446 – "In the real case study at hand, it is impossible to compute the entire set of Lyapunov vectors with positive growth rate that would give us all the information on the instabilities of the system." You point out that it is computationally impossible to compute the full spectrum. Had you been able to determine the full spectrum, you would still need a tool to identify the fastest growing vectors corresponding to particular types of instabilities that dominate once faster growing vectors nonlinearly saturate. You may still need to use breeding or some similar *tool to do that? Point is that it is not only due to computational constraints that make you use the breeding algorithm here?*

We modified the text in the **Conclusions, lines 471-474**. It now reads:

"It is impossible to compute the entire set of Lyapunov vectors with positive growth rate that would give us all the information on the instabilities characterising the linear growth regime of the non-hydrostatic model. To quantify the instabilities we then used the breeding technique, which, moreover, enables selecting those that are relevant for forecast errors of a given typical amplitude.

l.453 – "the number of BVs that are active during the convective episodes is not extremely large" – this is vague, can you be more specific?

We modified the text in the **Conclusions, lines 476-482** as:

"When the initial error is comparable to the present day analysis error, the unstable subspace, as estimated with the breeding technique, has the following properties: the growth rate always decreases with BV index and the number of BVs that are active during the convective episodes is not extremely large: just one or two in this case during the night and the morning; only in the afternoon their number increases, to 8 after 12:00 UTC and to 12 (at least) after 15:00 UTC (Fig. 2), in agreement with the flow-dependent instabilities evident in the forecast error evolution (Fig. 1)."

Fig. 5 – Wonder if an alternative display of the data may be easier to access for the readers? Have you considered switching rescaling amplitude (X axis now) and lead time (symbols, if I understand it right) in the figure (i. e., showing data as a function of lead time, with curves for different amplitudes)?

The same data are plotted in two different ways: in Fig. 4, as a function of lead time, and in Fig. 5, as a function of rescaling amplitude. This is done to point out different features in the results: the different behaviour at different times (Fig. 4) and the "optimal" rescaling amplitude which can only be found when the initial error is the large one (Fig. 5). This is explained in the main text, Section 6.2. Moreover, we modified the **captions of Figure 4 and 5** to point this out:

Caption of Fig. 4, we added a **last line**: "The same data are plotted as a function of rescaling amplitude in Fig. 5".

Caption of Fig. 5. we inserted, in the **second line**: "as in Fig. 4, but plotted here as a function of rescaling amplitude". And we added a **last line**: "The same data are plotted as a function of lead time in Fig. 4"