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$C \approx 0.85$ scaling and the universal clustering structure of earthquake networks

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Abstract

Earthquake network describes the complexity of seismicity both qualitatively and quantitatively. The procedure of constructing an earthquake network contains as a single parameter the size of the cells, into which a geographical region under consideration is divided. Then, the characteristics of the network depend on the cell size, in general. Here, the dependency of the clustering coefficient, C, of network on the cell size is studied. Remarkably, C of the earthquake networks constructed from the seismic data taken from California, Japan, and Iran well coincide for each value of the scaled dimensionless cell size. It is found that the networks in California and Japan are three-dimensional, whereas the one in Iran is rather two-dimensional. In addition, the values of C of all these three networks monotonically converge to $C \approx 0.85$ as the scaled dimensionless cell size increases, highlighting a universal aspect of the concept of earthquake network.

1 Introduction

The concept of earthquake network has been introduced by the present authors (Abe and Suzuki, 2004a) in order to describe the complexity of seismicity in an efficient way. Since then, a number of its remarkable properties have been discovered. It has been found for example that an earthquake network is scale-free (Abe and Suzuki, 2004a, 2006a; Baek et al., 2012), small-world (Abe and Suzuki, 2004b, 2006a), and hierarchically organized (Abe and Suzuki, 2006b). This network approach has opened a new possibility of extracting information about yet largely unknown dynamics governing seismicity.

The method of constructing an earthquake network (Abe and Suzuki, 2004a) is as follows. Firstly, a geographical region under consideration is divided into cubic cells with the size L, i.e., the length of the side of the cell. A cell is regarded as a vertex if earthquakes with any values of magnitude occurred therein. Then, two vertices associated with two successive events are connected by an edge, which represents the event-event correlation (see the discussion in the paragraph after next). In particular, if two successive events occurred in the same cell, then a tadpole, i.e., a self-loop, is attached to that vertex. In this way, a given seismic data can be mapped to a growing stochastic network.

A convenient method for practically setting up the cells and identifying a cell for each earthquake is as follows. Let θ_0 and θ_{max} be the minimal and maximal values of latitude of the whole region and φ_0 and φ_{max} be the minimal and maximal values of longitude, respectively. These angles are measured in the unit of radian. We define θ_{av} as the sum of the values of latitude of all the events divided by the number of events. The hypocenter of the ith event is denoted by $(\theta_i, \varphi_i, z_i)$, where θ_i, φ_i, and z_i are the values of latitude, longitude and depth, respectively. The north-south distance and the east-west distance between (θ_0, φ_0) and (θ_i, φ_i) are respectively given by $d_i^{NS} = R \cdot (\theta_i - \theta_0)$ and $d_i^{EW} = R \cdot (\varphi_i - \varphi_0) \cdot \cos\theta_{av}$, where R ($\cong$ 6370 km) is the radius of the Earth. The depth is just $d_i^{D} = z_i$, as it is. Starting from the point $(\theta_0, \varphi_0, z_0 \equiv 0)$, divide the region into cubic cells with a given value of the cell size. Then, the cell of the ith event is determined in terms of d_i^{NS}, d_i^{EW}, and d_i^{D}.

The above-mentioned procedure of constructing a network is based on the working hypothesis that successive events are correlated *at least at the statistical level*. This hypothesis comes from the following empirical facts: (i) an earthquake can trigger the next far-off one (Steeples and Steeples, 1996; Peng et al., 2013), and (ii) the distributions of spatial distance and time interval between two successive events significantly deviate from the Poissonian (Corral, 2004, 2006; Abe and Suzuki, 2003, 2005). In addition, analysis of the seismic data shows that the ratio of the number of non-causal pairs to that of non-non-causal ones in terms of the typical values of velocities of seismic waves, in fact, turns out to be statistically negligible. These evidences allow us to frame the working hypothesis. Also, an event may trigger not only the next one but also the one after next etc.. Such a "non-Markovian" formulation is actually possible, but we avoid extra complication like that, here.

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Clearly, a full earthquake network is a directed one with multiple edges as well as tadpoles. When its small-worldness is studied, it should be reduced to a simple network, where the directedness is ignored, tadpoles are removed, and each multiple edge is replaced with a single edge.

The method of earthquake-network construction explained above contains a single parameter, which is the cell size, L. Quantities characterizing a network depend on this parameter, in general. Since there is no a priori principle for determining the cell size, it is of crucial importance to clarify how change of the cell size affects the network characteristics. This problem has been studied in recent works (Abe and Suzuki, 2009[1]; Lotfi and Darooneh, 2012). It has been found (Abe and Suzuki, 2009) that the clustering coefficient, C (its definition to be given in the next section), converges to a specific value

$$C \approx 0.85, \tag{1}$$

as the dimensionless cell size increases. Since all characteristic quantities of a network do not possess physical dimensions, the cell size should be made dimensionless. In the work (Abe and Suzuki, 2009), the following two different definitions of the dimensionless cell size have been considered:

$$l_2 = L/(L_{\mathrm{LAT}} L_{\mathrm{LON}})^{1/2}, \tag{2}$$

$$l_3 = L/(L_{\mathrm{LAT}} L_{\mathrm{LON}} L_{\mathrm{DEP}})^{1/3}, \tag{3}$$

where L_{LAT}, L_{LON}, and L_{DEP} are the dimensions of the whole geographical region under consideration in the directions of latitude, longitude, and depth, respectively. One could

[1] In this paper, there are errors in the Iranian case. They should be corrected as follows: $(L_{\mathrm{LAT}}\, L_{\mathrm{LON}})^{1/2} = 2348.86\,\mathrm{km}$ and $(L_{\mathrm{LAT}}\, L_{\mathrm{LON}}\, L_{\mathrm{DEP}})^{1/3} = 583.45\,\mathrm{km}$.

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consider a more general form such as $L/(L_{\mathrm{LAT}}^{\alpha} L_{\mathrm{LON}}^{\beta} L_{\mathrm{DEP}}^{\gamma})^{1/(\alpha+\beta+\gamma)}$, where α, β, and γ are nonnegative parameters with at least one of them being nonzero, but we do not introduce such additional parameters, here.

The seismic data analyzed in the work (Abe and Suzuki, 2009) are of California, Japan, and Iran. The values of C of the earthquake networks in these regions converge to the universal one in Eq. (1) as l_2 and l_3 increase. However, for small values of l_2 and l_3, C of the network in Iran significantly deviates from those in California and Japan.

In this paper, we readdress ourselves to this problem concerning the cell-size dependence of the clustering coefficient of earthquake network. We will show that the networks in California and Japan are three-dimensional, whereas the one in Iran is quite two-dimensional, and an impressive universal behavior emerges if this issue of dimensionality is appropriately taken into account.

2 Universal clustering structure

To calculate the value of the clustering coefficient, we reduce a full earthquake to a simple network in the small-world picture, as mentioned in the preceding section. Such a network is characterized by a symmetric adjacency matrix, A, having the property: $(A)_{ij} = 1$ (0) if the ith and jth vertices are connected (unconnected). The clustering coefficient, C, is then expressed as follows:

$$C = \frac{1}{N} \sum_{i=1}^{N} c_i \tag{4}$$

with

$$c_i = \frac{(A^3)_{ii}/2}{k_i(k_i-1)/2}, \tag{5}$$

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where N and k_i are the total number of vertices and the value of connectivity, i.e., the number of edges, of the ith vertex, respectively. C ranges from 0 to 1 and measures the tendency of triangle formation in the network, as can be seen in the A^3-structure in Eq. (5). The larger C is, the more clustered the network is. One of the important features of a small-world network is that such a network has a large value of C compared to the Erdös-Rényi classical random network (Watts and Strogatz, 1998).

We have constructed the earthquake networks by taking three different data sets of (i) California; available at http://www.data.scec.org, (ii) Japan; at http://www.hinet.bosai.go.jp, and (iii) Iran; at http://irsc.ut.ac.ir/. The periods and the geographical regions covered are as follows: (i) between 00:06:28.31 on 1 January 2005 and 13:52:11.55 on 11 November 2006, 28.00° N–39.41° N latitude, 112.10° W–123.54° W longitude with the maximal depth 35.00 km, (ii) 00:02:52.93 on 1 June 2007 and 05:43:19.30 on 27 July 2007, 20.26° N–49.31° N latitude, 120.55° E–155.11° E longitude with the maximal depth 552.00 km, and (iii) between 03:08:11.10 on 1 January 2006 and 18:26:21.90 on 31 December 2008, 23.89° N–43.51° N latitude, 41.32° E–68.93° E longitude with the maximal depth 36.00 km, respectively. The total numbers of events in these periods are adjusted to be 22 845. This adjustment enables us to avoid the effect of data-size dependence (Abe, Pastén, and Suzuki, 2011). Also, we emphasize that the subsequent results are not sensitive to the present choice of the periods.

What we wish to stress here is as follows. Seismicity in Iran occurs in a relatively shallow region compared to those in Californian and Japan. In fact, we have ascerteined that 80 % of the events occurred in the region shallower than d = 13.70 km in California, d = 40.70 km in Japan, and d = 20.50 km in Iran. On the other hand, the values of the corresponding "epicenter region", λ (the square root of the product of the dimensions in the directions of latitude and longitude covering 80 % of the events), are 1153.51 km in California, 3171.84 km in Japan, and 2348.86 km in Iran. Therefore, the values of d/λ are 0.012 in California, 0.013 in Japan, and 0.009 in Iran, showing that seismicity in Iran is, in fact, shallower compared to those in California and Japan.

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Taking this fact into account, we come back to the values of $(L_{\mathrm{LAT}}L_{\mathrm{LON}})^{1/2}$ and $(L_{\mathrm{LAT}}L_{\mathrm{LON}}L_{\mathrm{DEP}})^{1/3}$ in Eqs. (2) and (3). They are respectively as follows: (i) 1153.51 km and 359.78 km, (ii) 3171.84 km and 1770.87 km, and (iii) 2245.73 km and 566.25 km.

Now, we examine the dependency of the clustering coefficient, C, on the dimensionless cell size.

The result obtained is as follows. In Fig. 1, we present the plots of C with respect to l_3 in Eq. (3). All three curves of C approach the common value in Eq. (1). It is seen, however, that the curves of California and Japan coincide quite well, whereas that of Iran deviates from them for smaller values of l_3. On the other hand, taking into account the fact that seismicity in Iran is rather two-dimensional, we present in Fig. 2 the plots of C with respect to $l = l_3$ for California and Japan but $l = l_2$ for Iran. There, one observes a remarkable behavior that all three curves nicely collapse to a single one. This is the main result of the present work and highlights a universal aspect of the concept of earthquake network.

3 Conclusions

We have reexamined the dependency of the clustering coefficient, C, of earthquake network on the dimensionless cell size, which is the one and only parameter contained in the network construction. Analyzing the networks constructed from the data sets taken from California, Japan, and Iran, we have found that C exhibits a universal behavior and approaches $C \approx 0.85$ as the cell size increases. This discovery has been based on the fact that seismicity in Iran is rather two-dimensional, in contrast to those in California and Japan.

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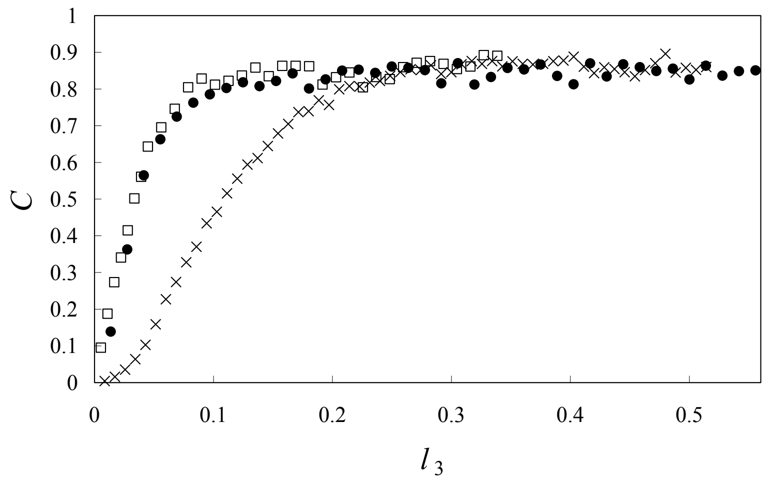

Fig. 1. Dependence of the clustering coefficient, C, on the dimensionless cell size, l_3, (•: California, □: Japan, ×: Iran). All quantities are dimensionless.

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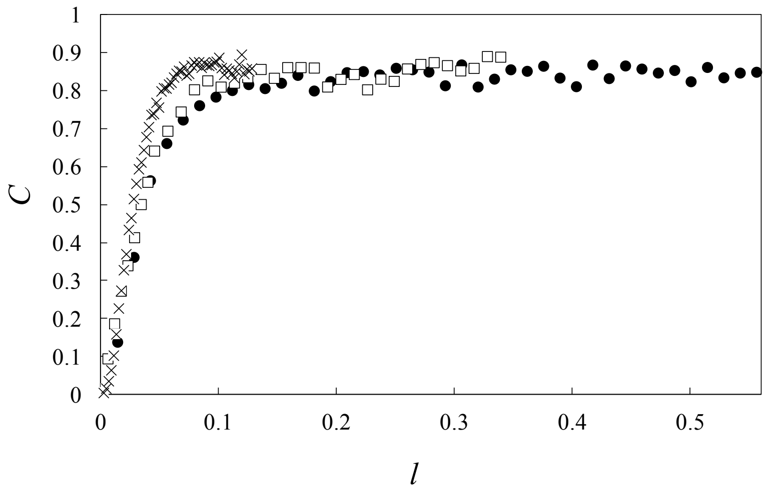

Fig. 2. Dependence of the clustering coefficient, C, on the dimensionless cell size, $l = l_3$ for California and Japan, but $l = l_2$ for Iran (•: California, □: Japan, ×: Iran). All quantities are dimensionless.

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