Authors' response to the editorial team

The will like to appreciate the Editorial team and the referees for their participation in shaping this manuscript to a better form. We hereby state that necessary considerations and adjustments that have been made in response to the referee report #1.

1. For points raised in the first paragraph of the referees report. The sessions on Lyapunov exponent and Tsallis entropy remains the same since necessary adjustment has been made in the previous version as pointed out by the reviewer. However proper adjustments have been made to lines 152 to 168 and the paragraph have been reconstructed to its proper shape, with necessary grammatical adjustments. Adjustments have also been made to the conclusion and the main points of the research have been made clearer.
2. Additional text was added to the discussion on semiannual variation, with some physical insight produced. We intend intend to maintain that the fluctuations in TEC are mainly based on the internal dynamics of the ionosphere, since equipment noise and errors mentioned by referee #1 have been majorly taken care of with method used for the extraction of TEC from the RINEX files used in this work.

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The Transient Variation of the Complexes of the Low Latitude Ionosphere within the Equatorial Ionization Anomaly Region of Nigeria.

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Abstract

The quest to find an index for proper characterization and description of the dynamical response of the ionosphere to external influences and its various internal irregularities has led to the study of the day to day variations of the chaoticity and dynamical complexity of the ionosphere. This study was conducted using Global Positioning System (GPS) Total Electron Content (TEC) time series, measured in the year 2011, from 5 GPS receiver stations in Nigeria which lies within the Equatorial Ionization Anomaly region. The nonlinear aspect of the TEC time series were obtained by detrending the data. The detrended TEC time series were subjected to various analyses to obtain the phase space reconstruction and to compute the chaotic quantifiers which are Lyapunov exponents LE, correlation dimension, and Tsallis entropy for the study of dynamical complexity. Considering all the days of the year the daily/transient variations show no definite pattern for each month but day to day values of Lyapunov exponent for the entire year show a wavelike semiannual variation pattern with lower values around March, April, September and October, a change in pattern which demonstrates the self-organized critical phenomenon of the system. This can be seen from the correlation dimension with values between 2.7 and 3.2 with lower values occurring mostly during storm periods demonstrating a phase transition from higher dimension during the quiet periods to lower dimension during storms for most of the stations. The values of Tsallis entropy show similar variation pattern with that of Lyapunov Exponent, with both quantifiers correlating within the range of 0.79 to 0.82. These results show that both quantifiers can be further used together as indices in the study of the variations of the dynamical complexity of the ionosphere. The presence of chaos and high variations in the dynamical complexity, even at quiet periods in the ionosphere may be due to the internal dynamics and inherent irregularities of the ionosphere which exhibit non-linear properties. However, this inherent dynamics may be complicated by external factors like Geomagnetic storms. This may be the main reason for the drop in the values of Lyapunov exponent and Tsallis entropy during storms. The dynamical behavior of the ionosphere throughout the year as described by these quantifiers, were discussed in this work.

1.0 Introduction

The behavior of natural systems like the ionosphere is a function of changes that occur in the underlying dynamics that exists in such system. These underlying dynamics however can be sometimes complex and nonlinear due to superposition of different changes in dynamical variables that constitute it. When the dynamical states of a system changes suddenly due to sudden changes in the external factor affecting the system, then such a system is said to be deterministic.

However, there is no totally deterministic system in nature, because all natural systems exhibit a mixture of both deterministic properties. Although few natural systems have been found to be low dimensional deterministic in the sense of the theory, the concept of low-dimensional chaos has been proven to be fruitful in the understanding of many complex phenomena (Hegger et al., 1999) The degree of determinism or stochasticity in most natural systems is dependent on how much the system can be influenced by external factors, the nature of these external factors among others .The ionosphere like every other natural system possess its intrinsic dynamics and it can also be influenced by other external factors. The typical characteristics of a dynamical system like the ionosphere is expected to naturally show the interplay between determinism and stochasticity simply because of the fact that the ionosphere which has an inherent internal dynamics is also influenced by the influx of stochastic drivers like the solar wind, since it is influenced by external dynamics like every other natural system. This has made pure determinism impossible in the ionosphere, a situation that is common to all natural system and its surrounding.

The intensity of the solar wind coming into the ionosphere varies with the solar activity and an extreme solar activity can lead to geomagnetic storms and substorms drive in high intensity plasma wind at enormous speed and it serves as major stochastic driver leading to storm. The solar wind is driven from the sun into the ionospheric system during the quiet and storm and during relatively quiet periods of each month of the year. However other processes which include various factors like local time variations of the neutral winds, ionization processes, production-recombination rates, photoionization processes, plasma diffusion and various electrodynamics processes. (Unnikrishnan, 2010). The mesosphere and the lower thermospheric dynamics as reported by Kazimirovsky and Vergasova (2009) and also the influence of gravity waves as reported by Sindelarova (2009) can also be of great influence on the internal dynamics of the ionosphere.

Therefore, it is of great importance to study the chaoticity and dynamical complexity of the ionosphere and its variations in all geophysical conditions. However a good number of investigations have been carried out on concept of chaos in the upper atmosphere before now which includes the study on magnetospheric dynamics and the ionosphere. The study of chaos in magnetospheric index time series such as AE and AL were initially carried out by (Vasiliadis et al., 1990, Shan et al, 1999; Pavlos et al, 1992). These previous efforts made by the aforementioned researchers has led to the development of the concept of investigating and revealing the chaoticity and the complex dynamics of the ionosphere, and as a result, studies on the chaoticity of the ionosphere have been conducted, by some investigators like Bhattacharyya (1990) who studied chaotic behavior of ionospheric diversity fluctuation using amplitude and phase scintillation data, and found the existence of low dimension chaos. Also, Wernik and Yeh (1994) further revealed the chaotic behavior of the ionospheric turbulence using scintillation data and numerical modeling of scintillation at high latitude. They showed that the ionospheric turbulence attractor (if it exists) cannot be reconstructed from amplitude scintillation data and their measured phase scintillation data adequately reproduce the assumed chaotic structure in the ionosphere. Also Kumar et al., (2004) reported the evidence of chaos in the ionosphere by showing the chaotic nature of the underlying dynamics of the fluctuations of TEC power spectrum indicating exponential decay and the calculated positive value of Lyapunov exponent. This is also supported by the results of the comparison of the chaotic characteristics of the time series of variations of TEC with the pseudochaotic characteristic of the colored noise time series. Xuann et al., (2006) studied chaos properties of ionospheric total electron content (TEC) using TEC data from 1996 to 2004, and analyze possibility to predict it by using chaos. They found the presence of chaos in the TEC measured in the study area, as indicated by the positive Lyapunov exponent computed from their data. The correlation dimension was 3.6092 from their estimation. They were also able to show that the TEC time series can be predicted using chaos.

Also, Unnikrishnan et al (2006a,b) have analyzed the deterministic chaos in mid latitude and Unnikrishnan (2010), Unnikrishnan and Ravindran (2010), analyzed some TEC data from some Indian low latitude stations for quiet period and major storm period and found in Their results the presence of chaos which was indicated by a positive Lyapunov exponent, and they also inferred that storm periods exhibits lower values compared to quiet periods. The dynamical complexity of magnetospheric processes and the ionosphere have been studied by a number of researchers. Balasis et al., (2008) investigated the dynamical complexity of the magnetosphere by using

Tsallis entropy as a dynamical complexity measure in D_{st} time series also Balasis et al., (2009) investigated the dynamical complexity in D_{st} further by considering different entropy measures. Coco et al (2011) using the information theory approach studied the dynamical changes of the polar cap potential which is characteristic of the polar region ionosphere by considering three cases (i) steady IMF B_z >0, (ii) steady IMF B_z <0 and (iii) a double rotation from negative to positive and then positive to negative Bz. They observed a neat dynamical topological transition when the IMF Bz turns from negative to positive and vice versa, pointing toward the possible occurrence of an order/disorder phase transition, which is the counterpart of the large scale convection rearrangement and of the increase of the global coherence. Further studies in chaotic behavior and nonlinear dynamics is however needed to improve our understanding of the dynamical behavior of the ionosphere of low latitude ionosphere especially over Africa during quiet and storm for different season of the year some as to be able to characterize chaoticity for different season of the year for quiet and storm periods. Recently Ogunsua et al (2014) studied comparatively the chaoticity of the equatorial ionosphere over Nigeria using TEC data, considering five quietest day classification and five most disturbed day classification. They were able to show the presence of chaos as indicated the positive Lyapunov exponents and also were able to show that Tsallis entropy can be used as a viable measure of dynamical complexity in the ionosphere with portions showing lower values of Tsallis entropy indicating lower dynamical complexity, with a good relationship with Lyapunov exponents. They found a phase transition from higher dimension during quiet days to Lower dimension during storm.

The low latitude region where Nigeria is situated is known as the equatorial anomaly region, where the magnetic field B is almost totally parallel to the equator. Off the equator the E region electric field maps map along the magnetic field up to the F-region altitude in the low latitude, this eastward electric field (E) interacts with the magnetic field B at the F region during the day. This results in the electrodynamic lifting of the F-region plasma over the equator, known as EXB drift. The uplifted plasma over the equator moves along the magnetic line in response to gravity, diffusion and pressure gradients and hence, the fountain effect. The fountain effect being controlled by the EXB drift shows the dynamics of the diurnal variation equatorial anomaly (Abdu, 1997; Unnikrishnan 2010). There is a reduction in the F region ionization density at the magnetic equator and also much enhanced ionization density at the two anomaly crests within $\pm 15°$ of the magnetic latitude north and south of the equator (Rama Rao et al., 2006). The equatorial ionization anomaly and other natural processes which includes various ionization processes and recombination; influx of solar wind, photoionization processes and so many other factors that occur due to variations in solar activities, have a great influence on the systems of the ionosphere, due to their effects on internal dynamics of the ionosphere. This portrays the ionosphere as a typical natural system with continuous interaction with its external environment which led to the study of the influence of the sun on the ionosphere (Ogunsua et al., 2014).

The ionosphere possesses a significant level of nonlinear variations that requires more investigation which can be studied and characterized using nonlinear approach like the chaoticity and dynamical complexity for the study of its dynamics. The need to study the daily variation in the dynamical complexity of the ionosphere arises from the established knowledge and understanding which shows that the ionosphere is a complex system with so many variations that can arise from various dynamical changes that can be due to various changes in different processes that contribute to the behavior and nature of the ionosphere. Rabiu et al., (2007) affirmed that characterizing the ionosphere is of utmost importance due to the numerous complexities associated with the region. The scale of these numerous complexities interestingly changes at times from one day to another.

The concept of chaos as previously applied to ionospheric and magnetospheric studies on quiet and stormy conditions are limited.

Most investigations have been based on only quiet and storm conditions for all studies carried out, and none of -the previous works involved the use of quiet and disturbed day classification of geophysical conditions until recently by Ogunsua et al.,(2014), where we considered the comparative use of Lyapunov exponent and Tsallis entropy as proxies for the internal dynamics of the ionosphere. This is the main reason for the consideration of day to day variation of these parameters in this work.

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2.0 Data and Methodology

The data used for this study is the global positioning system (GPS) total electron content (TEC) data obtained from 5 GPS satellite receiver stations. Table 1 shows the coordinates of the stations. These receivers take the measure of slant TEC within 1m^2 columnar unit of the cross section along the ray path of the satellite and the receiver which is given by

$$\text{STEC}=\int_{receiver}^{Satellite} Ndl \qquad (1)$$

The observation of the total number of free electron along the ray path are derived from the frequency L_1(1572.42 MHz) and L_2(1227.60 MHz) of Global Positioning System(GPS), that provide the relative ionosphere delay of electromagnetic waves travelling through the medium (Saito et al.,1998). The Slant TEC is projected to vertical TEC using the thin shell model assuming the height of 350m (Klobuchar,1986).

$$VTEC = STEC \cdot \cos[\arcsin(R_e cos\Theta / R_e + h_{max})] \qquad (2)$$

Where $R_e = 6378km$ (radius of the earth), $h_{max} = 350km$ (the vertical height assumed from the satellite) and $\Theta = elevation\ angle\ at\ ground\ station$

In this study, 5 GPS TEC measuring stations lying within the low latitude region were considered, as shown in table 1. The TEC data obtained for January to December 2011 were considered for this study and the data are given at 1min sampling time. The TEC data were subjected to various analyses which will be discussed in the next section. The day to day variations of the chaotic behavior and dynamical complexity were studied for the entire year. The surrogate data tests for non linearity were also conducted for both the dynamical and geometrical aspects.

3.0 Methods of Data Analysis and Results

3.1 Time series analysis

Time series can be seen as a numerical account that describes the state of a system, from which it was measured. A given time series, S_n can be defined as a sequence of scalar measurement of a particular quantity taken as series at different portion in time for a given time interval(Δt). The time series describe the physical appearance of an entire system, as seen in Fig 1. However it may not always describe the internal dynamics of that system. A system like the ionosphere possesses a dominant dynamics that can be seen as diurnal so the data should be treated so as to be able to see its internal dynamics. The measured TEC time series were plotted to see the dynamics of the system. A typical plot of TEC usually has a dominant dynamics (see fig 1) which may be seen as the diurnal behavior, however, it can also be seen that there is also a presence of fluctuations (which appear to be nonlinear) in the system as a result of the internal dynamics of the ionosphere and space plasma system, due to different activities in the ionosphere. Therefore there is need to minimize the influence of the diurnal variations since we are more interested in the nonlinear internal dynamics of the system in this study, to do so the TEC time series was detrended by carrying out the following analysis below:

Since for the given daily data of 1minute sampling time there are 1440 data points per day. Then there exists a time series t_i, where $i = 1,2,3 \ldots\ldots 1440$ represents the observed time series, and there also exists a set of $u_i$where $i = 1,2,3 \ldots\ldots 1440$, such that the diurnal variation reduced time is given by

$$T_i = t_i - u_j \qquad (3)$$

Where $i = 1,2,3, \ldots\ldots, j = mod(i, 1440)$, if $mod(j, 1440) \neq 0$, and $j = 1440$ if $d(j, 1440) = 0$. This method will give the detrended time series represented by T_i obtained from the original TEC data as shown in fig 2. This method is similar to that used by (Unnikrishnan et al., 2006, Unnikrishnan 2010), the further explanations on the dynamical results can be found in (Kumar et al., 2004). The detrended time series were subjected to further analyses for the Phase space reconstruction and also to obtain the values of Lyapunov exponents, correlation dimension, Tsallis entropy and the implementation of surrogate data test.

3.1.1 Phase Space reconstruction and Non Linear Time Series Analysis

The study of chaoticity and dynamical complexity in a dynamical system requires a nonlinear approach, due to the fact that systems described by these phenomena can be referred to as nonlinear complex systems. The magnetosphere and the ionosphere are good examples of such systems. To be able to study such phenomena some nonlinear time series analysis can be carried out on the time series data describing such a system. The detrended time series of TEC measurement is subjected to some nonlinear time series data analysis to obtain the mutual information and false nearest neigbours, embedding dimension and delay coordinates for the phase space reconstruction, and the evaluation of other chaotic quantifiers namely: Lyapunov Exponents, Correlation dimension, recurrence analysis and Entropy.

The phase space reconstruction helps to reveal the multidirectional aspect of the system. The phase space reconstruction is based on embedding theorem, such that the phase space is reconstructed to show the multidimensional nature as follows:

$$\boldsymbol{Y_n} = (\boldsymbol{s_{n-(m-1)\tau}}, \boldsymbol{s_{n-(m-2)\tau}}, \dots, \boldsymbol{s_{n-\tau}}, \boldsymbol{s_n}) \qquad \textbf{(4)}$$

where Y_n are vector in phase space. The proper choice of embedding dimension (m) and delay Time (τ) are essential for phase space reconstruction (Fraser and Swinney,1986; Kennel et al.,1992) .

If the plot showing the time delayed mutual information shows a marked minimum that value can be considered as a responsible time delay; Fig 3 shows the mutual information plotted against time delay. Likewise, the minimal embedding dimension, which correspond to the minimum number of the false nearest neighbours can be treated as the optimum value of embedding dimension in (Unnikrishnan et al.,2006, Unnikrishnan, 2010). A plot of fraction of false nearest neighbours against embedding dimension can be seen in Fig 4. It was observed that for all the daily detrended TEC time series the choice of $\tau \geq 30$ and $m \geq 4$ values of delay and embedding dimension above these values are suitable for analysis of data for all stations. The choice of $\tau = 30$ and $m = 5$ were mostly used to analyze the dynamical aspects for all the stations. The reconstructed Phase space trajectory is shown in Fig 5

3.1.2 Lyapunov Exponents

The Lyapunov exponent has been a very important quantifier for the determination of chaos in a dynamical system. This quantifier is also used for the determination of chaos in time series, representing natural systems like the ionosphere and magnetosphere (Unnikrishnan 2008, 2010). A positive Lyapunov exponent indicates divergence of trajectory in one dimension, or alternative an expansion of volume, which can also be said to indicate repulsion, or attraction from a fixed point. A positive Lyapunov exponent indicates that there is evidence of chaos in a dissipative deterministic system, where the positive Lyapunov exponent indicates divergence of trajectory in one direction or expansion of value and a negative value shows convergence at trajectory or contraction of volume along another direction.

The largest Lyapunov exponent (λ_1) can be used to determine the rate of divergence as indicated by (Wolf et al.,1985)

Where

$$\lambda_1 = \lim_{r\to\infty} \frac{1}{t}\, ln \frac{\Delta x(t)}{x(0)} = \lim_{r\to\infty} \frac{1}{t} \sum_{i=1}^{t} \ln \left(\frac{\Delta x(t_i)}{\Delta x(t_{i-1})}\right) \quad (5)$$

The Lyapunov exponent was computed for the TEC values measured from Different stations. The evolution in state space was scanned with $\tau = 30, m = 5$, is shown in fig 6. The day to day variations of the Lyapunov exponent was computed for the entire year to so as to study the annual trend of variation. This was implemented using the method introduced by Rosenstein (1993), and Hegger et al., (1994), both algorithms use very similar methods. Lyapunov exponents were also computed for varying time delay at constant embedding dimension and also for varying embedding dimension, to check for the stability with changes in trajectory. These can be seen in fig. 6b and 6c. The day to day values of Lyapunov exponent plotted for the Enugu station and for Toro station are shown in fig 7a to 7b. The plots of the day to day values show the transient variation of the ionosphere and a wavelike yearly pattern.

3.1.3 Correlation Dimension

Another relevant method to study the underlying dynamics or internal dynamics of a system is to evaluate the dimension of the system. The correlation dimension gives a good approximation of this as suggested by Grassberger and Procaccia (1983a; b). The correlation dimension is preferred over the box counting dimension because it takes into account the density of points on the attractor (Strogatz 1994). The correlation dimension D is defined as

$$D = \lim_{r\to 0} \frac{\ln C(r)}{\ln r} \quad (6)$$

The term $C(r)$ is the correlation sum for radius (r) where for a small radius (r) the correlation sum can be seen as $C(r){\sim}r^d$ for $r \to 0$. The correlation sum is dependent of the embedding dimension (m) of the reconstructed phase space and it is also dependent of the length of the time series N as follows

$$C(r) = \frac{2}{N(N-1)} \sum_{i=1}^{N} \sum_{j=i+1}^{N} \Theta\left(r - \left\|y_i - y_j\right\|\right) \quad (7)$$

Where Θ is the Heaviside step function, with $\Theta(\mathrm{H}) = 0$ if $H \leq 0$ and $\Theta(\mathrm{H}) = 1$ for $H > 0$.

The correlation dimension was computed using the Theiler algorithm approach, with Theiler window (w) at 180. The Theiler window was chosen to be approximately equal to the product of m and τ. A similar approach to the computation of correlation dimension was used by Unnikrishnan and Ravindran (2010) to determine the correlation dimension of detrended TEC data for some stations in India which lies within the equatorial region, like Nigeria. Ogunsua et al., (2014) also used similar methods for some detrended TEC from Nigerian stations.

The correlation dimension for data taken for the quietest day of October 2011 and the most disturbed day of October 2011 from Birnin Kebbi GPS TEC measuring station were represented by Fig 8a and Fig 8b respectively. The correlation dimension saturates at $m \geq 4$ for the quietest day of the month and at $m \geq 5$ for the most disturbed day. In this illustration the most disturbed day of this month fall within the storm period of October 2011. The classification of days into quiet and disturbed days in the month of October 2011 enables us to compare the quiet and storm periods together while comparing the quiet days with some relatively disturbed days.

3.1.4 Computation of Tsallis Entropy and Principles of Nonextensive Tsallis Entropy

Entropy measures are very important statistical techniques that can be used to describe the dynamical nature of a system. The Tsallis entropy can be used to describe the dynamical complexity of a system and to also understand the nonlinear dynamics like chaos which may exist in a natural system. The use of entropy measure as a method to describe the state of a physical system has been employed into information theory for decades. The computation of entropy allows us to describe the state of disorderliness in a system, one can generalize this same concept to characterize the amount of information stored in more general probability distributions (Kantz & Shrieber 2003, Balasis et al.,2009). The concept of information theory is basically concerned with these principles. The information theory gives us an important approach to time series analysis. If our time series which is a stream of numbers, is given as a source of information such that this numbers are distributed according to some probability distribution, and transitions between numbers occur with well-defined probabilities. One can deduce same average behaviour of the system at a different point and for the future. The term entropy is used in both physics and information theory to describe the amount of uncertainty or information inherent in an object or system (Kantz and schrieber 2003). The state of an open system is usually associated with a degree of uncertainty that can be quantified by the Boltzmann-Gibbs entropy, a very useful uncertainty measure in statistical mechanics. However Boltzmann-Gibbs entropy cannot, describe non-equilibrium physical systems with large variability and multifractal structure such as the solar wind (Burgala et al., 2007, Balasis et al., 2008). One of the crucial properties of the Boltzmann-Gibbs entropy in the context of classical thermodynamics is extensivity, namely proportionality with the number of elements of the system. The Boltzmann-Gibbs entropy satisfies this prescription if the subsystems are statistically (quasi-) independent, or typically if the correlations within the system are essentially local. In such cases the system is called extensive. In general however, the situation is not of this type and correlations may be far from negligible at all scales. In such cases, the Boltzmann-Gibbs entropy is nonextensive (Balasis et. al., 2008, 2009). These generalizations above were proposed by Tsallis (1988), who was inspired by the probabilistic description of multifractal geometries. Tsallis (1988, 1998) introduced an entropy measure by presenting an entropic expression characterized by an index q which leads to a nonextensive statistics,

$$S_q = k\frac{1}{q-1}\left(1-\sum_{i=1}^{W} p_i^q\right) \tag{8}$$

Where p_i are the probabilities associated with the microscopic configurations, W is their total number, q is a real number, and k is Boltzmann's constant. The value q is a measure of the nonextensitvity of the system: $q \rightarrow 1$ corresponds to the standard extensive Boltzmann-Gibbs statistics. This is the basis of the so called nonextensive statistical mechanics, which generalizes the Boltzmann-Gibbs theory. The entropic index q characterizes the degree of nonadditivity reflected in the following pseudoadditivity rule:

$$\frac{S_q(A+B)}{k} = \left[\frac{S_q(A)}{k}\right] + [\frac{S_q(B)}{k}] + (1-q)[S_q(A)/k][\frac{S_q(B)}{k}]. \quad (9)$$

The cases $q > 1$ and $q < 1$, correspond to subadditivity (or subextensivity) and superadditivity (or superextensivity), respectively and $q = 1$ represents additivity (or extensivity). For subsystems that have special theory probability correlations, extensivity is not a valid for Boltzmann-Gibbs entropy in such cases, but may occur for S_q with a particular value of the index q. Such systems are sometimes referred to as nonextensive (Boon and Tsallis, 2005, Balasis et al 2008, 2009). The parameter q itself is not a measure of the complexity of the system, but measures the degree of nonextensivity of the system. It is the time variations of the Tsallis entropy for a given $q(S_q)$ that quantify the dynamic changes of the complexity of the system. Lower S_q values characterize the portions of the signal with lower complexity. In this presentation we estimate S_q on the basis of the concept of symbolic dynamics and by using the technique of lumping (Balasis et al. 2008, 2009).

A comparison of Tsallis entropy with Lyapunov exponents computed for the same set of data has been carried out in this work, to see the efficacy of the combined usage of both parameters. This is based on the established facts that variations in the values of Tsallis entropy can be linked with that of Lyapunov exponents chaotic behavior in systems as seen in (Baranger et al., 2012; Anastasiadis et al., 2005; Kalogeropoulos et al., 2012;2013). Coraddu et al., (2005) showed the Tsallis entropy generalization for Lyapunov exponents. Further details can be found in Ogunsua et al., (2014),

they were able to investigate the similarities in their response to the complex dynamics of the ionosphere, and this informs the further use of the two quantities as indices to study the day to day variation of ionospheric behaviour in this work.

The values of these entropy measures were also computed in order to study the dynamical complexity of the system under observation (the ionosphere). The day to day values of Tsallis entropy were computed for the entire year for different stations. The day to day values of Tsallis entropy plotted for the Enugu station and for Toro station are shown in fig 9(a and b). The plots of the day to day values show the transient variation of the ionosphere and a wavelike yearly pattern.

3.2 Non linearity Test using surrogate data

The test for non-linearity using the method of surrogate data according to Kantz and Schreber (2003) has proven to be a good test for non-linearity in time series describing a system. It has been accepted that the method of surrogate data test could be a successful tool for the identification of nonlinear deterministic structure in an experimental data (Pavlos et al., 1999). This method involves creating a test of significance of difference between linearly developed surrogate and original nonlinear time series to be tested. The test is done by carrying out the computation of the same quantity on both surrogates and the original time series and then checking for the significance of difference between the results obtained from the surrogates with the original data. Theiler et al (1992) suggested the creation of surrogate data by using Monte Carlo techniques for accurate results. According to this method, typical characteristic of data under study are compared with those of stochastic signals (surrogates), which have the same auto-correlation function and the power spectrum of the original time series. It can be safely concluded from the test of significance carried out on the surrogate and the original data that, a stationary linear Gaussian Stochastic model cannot describe the process under study provided that the behaviour of the original data and the surrogate data are significantly different.

In this work 10 surrogate data were generated from the original data set. The geometrical and dynamical characteristics of the original data were then compared to that of the surrogates using the statistical method of significance of difference which can be defined as

$$S = \frac{\alpha_{Surr} - \alpha_{Original}}{\sigma} \quad (13)$$

Where α_{Surr} is the mean value of the computed quantity for the surrogate data and $\alpha_{Original}$ is the same quantity computed for the original TEC data, σ is the standard deviation of the same quantity computed for the surrogate data. The significance of difference considered for the null hypothesis to be rejected here is greater than 2, which enables us to be able to reject the null hypothesis that the original TEC data describing the ionospheric system can be modeled using a Gaussian linear stochastic model with confidence greater than 95%.

The surrogate data test for all stations used in this study show that the Lyapunov exponent of the surrogate data for the selected days in October are shown in the Table below The results show that the surrogate data test for Lyapunov exponent show a significance of difference greater than 2 for all the selected days for all the stations. Similar results were obtained for Mutual Information, Fraction of False Nearest Neighbours and Correlation Dimension. This result gives us the confidence to reject the null hypothesis that the data used cannot be modeled using a linear Gaussian stochastic model, which shows that the system is a nonlinear system with some level of determinism. Fig. 10 shows the plots comparing the mutual information plotted against time delay for the original detrended data blue with the mutual information for the surrogate data for TEC data measured at Lagos for the quietest day of March 2011, while Fig. 11 is comparing fraction of false nearest neighbours for the same set of data. Tables 2a shows the values of Lyapunov exponents for both original detrended and its surrogate data for TEC measured in Lagos during the quietest days and Table 2b shows the values of Lyapunov exponents for both original detrended and its surrogate data for TEC measured in Lagos during the most disturbed days of October 2011.

3.3 Trend filtering using the moving average approach for the daily Values

The trend of a fluctuating time series can be made clearer to reveal the general pattern of that time series, and to make the fluctuating pattern of the daily variation of the chaoticity and dynamical complexity measures clearer in the work, the moving average method has been employed. The method of moving average filtering has found its applications geophysics (e.g. Bloomfeld 1992; Bloomfield and Nychka 1992; Baillie and Chung 2002), and in other areas like financial time series analysis, microeconomics, biological sciences and medical sciences. The various fields mentioned require different trend filtering method depending on the structure of the time series to be analyzed. Different filtering processes that can be used to reveal the trend includes the moving average filters, exponential filters, band-pass filtering, median filtering etc.

Suppose we have a time series $z[t]$ such that $t = 1,2,3 \ldots \ldots \ldots n,$ where $'n'$ could assume any value. If $z[t]$ consists of a consistent varying trend component that appears over a longer period of time t given as $u[t]$ and a more rapidly varying component $v[t]$. The goal of trend filtering in any research is to estimate either of the two components (Kim et al., 2009). The purpose of trend filtering in this work is to further reveal the general slow varying trend that appears to be obvious in the daily variation of the values of the chaoticity and dynamical complexity of the ionosphere, which might appear to be obviously varying with the yearly solar activity (a quantity with slow varying trend). To make $u[t]$ which represents the general slow varying trend smoother and in the process reduce $v[t]$ we apply the moving average filter.

If we assume $z[t]$ to be our time series representing the daily variation of the values of the chaoticity and dynamical complexity of the ionosphere, then our smoothing with weighting vector/filter w_j will create the new sequence u_j as

$$u[t] = z[t] * w[n] = \frac{1}{2k+1}\sum_{i=-k}^{k} x[n-1]. \quad (14)$$

In this work the Savitzy-Golay method of smoothing proposed by Savitzky and Goley (1967), which is a generalized form of moving average was applied to the trend smoothing of the daily variation of the chaoticity and dynamical complexity of the ionosphere. In this case it performs a least square fit to a small set of $L(= 2k + 1)$ consecutive data to a polynomial and then takes midpoint of the polynomial curve as output. The smoothed time series in this work will now be given as

$$u[t] = z[t] * \omega[n] = \frac{\sum_{i=-k}^{k} A_i * x[n-1]}{\sum_{i=-k}^{k} A_i} \quad (15)$$

where, $\omega[n] = \frac{A_n}{\sum_{i=-k}^{k} A_i}, -k \leq n \leq k$ such that A_i controls the order of polynomial. A similar method was described in Reddy et al., (2010).

The smoothed daily variation and the original data and the plot of the smoothed variation only, for the Lyapunov exponents of the detrended TEC measured at the Enugu and Toro are shown in fig 12(a and b). The smoothed day to day variation for Tsallis entropy for the detrended TEC measured at Enugu and Toro stations respectively are shown in fig 13(a and b).

4.0 DISCUSSION

The results presented in the work reveals the dynamical characteristics of the ionosphere. These characteristics are being discussed in this section, considering the time series treatment and phase space reconstruction; the study of chaos using chaotic quantifiers and the use and comparison of dynamical complexity measures in terms of their response to the variations on ionospheric dynamics. Also being discussed, is the implication of the nonlinearity test using the surrogate data and the comparison of the two quantifiers and their viability as indices for the continuous study and characterization of the ionosphere

The time series analysis shows the appearance of some degree of nonlinearity in the internal dynamics of the ionosphere. The time series plot in Fig. 1 shows the rise in TEC to peak at the sunlit hours of the day, however it can be seen that the rising to the peak exhibited by the ionosphere, which is the dominant dynamics during the day, make it impossible to clearly see the internal dynamics of the system from the TEC time series plot. It can be seen that the TEC time series curve is not a smooth curve with tiny variations, which probably describes a part of the internal dynamics. These visible tiny variations around the edges of the time series plot can be regarded as rate of change of TEC which is a phenomenon that can describe the influence of scintillations in the ionosphere these variations are however more obvious during the night time between 1100th and 1440th minutes of the day (that is, between about 1800 and 2400 hours of the day). It should be noted here that scintillations has been described as a night time phenomena associated with spread-F, and it occurs around pre-midnight and post-midnight periods (Vyas and Chandra 1994; Vyas and Dayanandan 2011; Mukherjee et al.,2012; Bhattacharyya and Pandit 2014). The detrended data shows the internal dynamics of the system more clearly, with a pattern similar to the values around night period mentioned earlier. The post-sunset values (especially at night time) in Fig.1 show a pattern similar pattern with the detrended TEC plot in Fig 2. It has been established that TEC does not decrease totally throughout the night as expected normally through simple theory that TEC builds up during the day, but it shows some anomalous enhancements and variations and this can occur under a wide range of geophysical conditions (Balan and Rao, 1987; Balan et al., 1991;Unnikrishnan and Ravindran, 2010). The delay representation of the phase space reconstruction shows a trajectory that is clustered around its origin, for all the stations, which can be seen as an indication of the possible presence of chaos. The degree of closeness of these trajectories however varies for different days from one station to another, resulting from varying degrees of variations in stochasticity and determinism. The varying degrees of variations in stochasticity and determinism can be attributed to the daily variations and local time variations of photoionization, recombination, influx of solar wind and other factors that may influence the daily variations of TEC (Unnikrishnan 2010).

The positive values of Lyapunov exponent indicate the presence of chaos (Wolf et al., 1985; Rosenstein et al., 1993; Hegger et al., 1999; Kantz and Schreiber, 2003). The presence of chaos was revealed by the positive Lyapunov exponent computed from all stations and this as a result of the fact that the ionosphere is a system controlled by many parameters influencing its internal dynamics. Because of its extreme sensitivity to solar activity, the ionosphere is a very sensitive monitor of solar events. The ionospheric structure and peak densities in the ionosphere vary greatly with time (sunspot cycle, seasonally and diurnally), with geographical location (polar, auroral zones, mild-latitudes, and equatorial regions), and with certain solar-related ionospheric disturbances. During and following a geomagnetic storm, the ionospheric changes around the globe, as observed from ground site can appear chaotic (Fuller-Rowell et al., 1994; Cosolini and Chang, 2001; Unnikrishnan and Ravindran, 2010). The recorded presence of chaos as indicated by the positive values of Lyapunov exponent was found in all the computations, for all the TEC values obtained for the selected days from all the measuring stations used in this work. This can be expected as it agrees with results from previous works that show that there is a reasonable presence of chaos in the ionosphere, even in the midst of the influence of stochastic drivers like solar wind (Bhattacharyya, 1990; Wernik and Yeh, 1994; Kumar et al., 2004; Unnikrishnan et al., 2006a,b; Unnikrishnan, 2010). However the values of Lyapunov exponents vary from day to day due to variations in ionospheric processes for different days on the same latitude as seen in Fig. 7(a and b) with Fig. 12(a and b) showing the day to day variation (upper panel) and the smoothed curve of the day to day variation (lower panel) for the entire year. There are also latitudinal variations due to spatial variations in the various ionospheric processes taking place simultaneously. The ionosphere is said to have a complex structure due to these varying ionospheric processes.

The higher values of Lyapunov exponent during months of low solar activity (the solstices) is an evidence that that the rate of exponential growth in infinitesimal perturbations in the ionosphere leading to chaotic dynamics might be of higher degree during most of the days of those months compared to days of the months with high solar activities showing lower values of Lyapunov exponents (Unnikrishnan 2010, Unnikrishnan and Ravindran, 2010).

The results of the correlation dimension values computed are within the range of 2.7 to 3.2 with the lower values occurring mostly during the storm periods. The lower dimension during the storm periods compared to the quiet days may be due to the effect of a stochastic drivers like strong solar wind and solar flares, that occurs during geomagnetic storms on the internal dynamics of the ionosphere, this could have been as a result of the fact that the internal dynamics must have been suppressed by the external influence. The restructuring of the internal dynamics of the ionosphere might be responsible for low dimension chaos during storm and also the lower values of other measures like the Lyapunov exponents. The relatively disturbed day however might have a higher dimension so long as it is not a storm period, and sometimes a relatively disturbed day of the month might be a day with storm and in this case there is usually a lower value of chaoticity and sometimes lower values of correlation dimension as well. The lower value of chaoticity and dimension in ionosphere during storms indicates a phase transition from higher values during the quiet periods to lower values during storm periods which may be due to the modification of the ionosphere by the influx of high intensity solar wind during the storm period (Unnikrishnan et al., 2006a, b; Unnikrishnan 2010; Unnikrishnan and Ravindran, 2010).

The surrogate data test shows significance of difference greater than 2 for all the computed measures which enables rejection of the null hypothesis that the ionospheric system can be represented with a linear model for all the data used from the stations. However it was discovered that the lower significance of difference corresponds to the lower values of Lyapunov exponents during storm and extremely disturbed periods (see tables 2 and 3). This may be due the rise in stochasticity during the storm period as a result of drop in values of computed quantities like Lyapunov exponents. Our ability to reject the Null hypothesis for all stations however shows the presence of determinism and confirms that the underlying dynamics of the ionosphere is mostly non-linear. This further validates the presence of chaos since the surrogate data test for non-linearity show that out detrended TEC is not a Gaussian (linear) stochastic signal (Unnikrishnan 2010).

The Tsallis entropy was able to show the deterministic behavior of the ionosphere considering its response during storm periods compared to other relatively quiet periods as the rapid drop in values of Tsallis entropy during storm show that there is a transition from higher complexity during quiet period to lower complexity during storms, this response in the values of Tsallis entropy is similar to the response of Lyapunov exponent values during storm. This reaction to storm shown by the values of Tsallis entropy computed for TEC was also described by the reaction of Tsallis entropy computed for Dst during storm periods (Balasis et al., 2008, 2009). A closer observation of the day-to-day variability within a month shows that the values were much lower for storm periods compared to the nearest relative quiet period. For example, the storm that occurred on the 25th of October resulted in lower values of Lyapunov exponent and Tsallis entropy compared to relatively quiet days close to it. The reaction to storm may be due to the influence of stochastic driver like strong solar wind flowing into the system as a result of solar flare or CMEs that produces the geomagnetic storms. Although there is always an influence of corpuscular radiation in form of solar wind flowing from the sun into the ionosphere, the influence is usually low for days without storm compared to days with geomagnetic storms as a result of solar flares, CMEs etc (Unnikrishnan et al., 2006a,b; Unnikrishnan, 2010, Ogunsua et al 2014).

The presence of chaos and high variations in the dynamical complexity, even at quiet periods in the ionosphere may be due to the internal dynamics and inherent irregularities of the ionosphere which exhibit non-linear properties. However, this inherent dynamics may be complicated by external factors like Geomagnetic storms. This may be the main reason for the drop in the values of Lyapunov exponent and Tsallis entropy during storms. According to Unnikrishnan et al., (2006a,b), geomagnetic storms are extreme forms of space weather, during which external driving forces , mainly due to solar wind, subsequent plasmasphere -ionosphere coupling, and related disturbed electric field and wind patterns will develop. This in turn creates many active degrees of freedom with various levels of coupling among them, which alters and modifies the quiet time states of ionosphere, during a storm period. This new situation developed by a storm, may modify the stability/instability conditions of ionosphere, due to the superposition of various active degrees of freedom.

The observation from the day-to-day variability of Lyapunov exponent and Tsallis entropy also show irregular pattern for all stations. These irregular variations might be due to the same factors

Formatted: Space After: 10 pt, Line spacing: Double, Adjust space between Latin and Asian text, Adjust space between Asian text and numbers mentioned before (i.e internal irregularities due to so many factors described and also due to variation in the influx of the external stochastic drivers). The day-to-day variability for the entire year shows a "wavelike" pattern with the values dropping to lower values during the equinox months especially during March-April equinox, this can be seen as a form of semiannual variation, possibly resulting from the higher energy inputs during equinoxes. This is because solar wind is maximized at the equinoxes which might result in higher energy input that will eventually suppress the internal dynamics to give lower values of chaoticity. The modification of the ionosphere as a result of the higher energy input resulting from the maximized influx of solar wind has been reported to be responsible for the lower values of chaoticity when averagely compared to the days of the year with lower solar wind inputs as reported by Unnikrishnan et al., 2006; 2010; Ogunsua et al., 2014 . The ~~wavelike~~ semiannual pattern has been found to be similar for different stations as seen in Figs. 7 & 12 and Figs. 9 &13 for Lyapunov exponents and Tsallis entropy respectively. Figs.9 and 13 show the smoothed curves for Lyapunov exponent and Tsallis entropy respectively, with the drop in values at equinoxes showing more clearly. The phase transition in chaoticity and dynamical complexity is also responsible for the wavelike variations, with values of Lyapunov exponent and Tsallis entropy dropping during the equinoxial months, and this may be due to the influence of the daily influx of the solar wind having higher values during equinoxes due to the proximity of the Earth to the sun during this period compared to the solstice months.

The wavelike pattern observed has been described to be as a result of self organized critical (SOC) phenomenon, a phenomenon which has been found to exist in both the magnetosphere and the ionosphere or the space plasma system in general, due to coupling between the two systems, since the magnetosphere couples the ionosphere tightly to the solar wind (Lui, 2002).

Many literatures has shown the existence of chaos in the SOC in the magnetosphere (chang et al.,1992,1998,1999; Consolini et al., 1996 Chapman et al., 1998; Freeman and Watkins 2002 and; Koselov and Koselova, 2001. Uritsky et al., (2003) and; Chang et al., (1992). The existence of SOC in space plasma system involving both the ionosphere the the magnetosphere was described by (Lui, 2002; Chang et al., 2002; Chang et al., 2004).

The variation along the latitude also shows the inconsistence and complexity of the ionospheric processes. This is the reason why for the same day of the month the values of Lyapunov exponent vary from one station to another. Lyapunov exponent however, appears to respond better to changes in solar activities compared to Tsallis entropy with more distinct results. This may be due to the fact that Tsallis entropy being not only a measure of complexity, but also a measure of disorderliness in a system might not be as perfect in describing chaos as Lyapunov exponent. Kalogeropoulos (2009) and Baranger et.al (2002) observed that Tsallis entropy has a relationship that is not totally linear in all cases at different level of chaos with Lyapunov exponent as a measure of chaos.

There are also many variations in the internal dynamics of the ionosphere that could lead to changes in chaotic behavior. The variations of Lyapunov exponents during quiet days might be as a result of different variations in the intrinsic dynamics of the ionosphere. Difference in variation pattern at different stations for the same quiet day might also be due to the same reason. It can be affirmed that the ionosphere is a complex system that varies with a short latitudinal or longitudinal interval such that even stations with one or two degrees of latitudinal differences might record different values on the same day for both quiet and disturbed periods and that the same might also occur for storm periods. This is illustrated by the different pattern of variation of TEC recorded from different stations within such a close range as used in this study.

These Latitudinal variation in the values of Lyapunov exponents and Tsallis entropy can be further described by the behavior of the TEC because there can be a more sporadic rate of change in TEC as seen in the time series plots as a result of irregularities in the internal dynamics of the ionosphere, which might be as a result of plasma bubbles. Irregularities develop in the evening hours at F region altitudes of magnetic equator, in the form of depletions, frequently referred to as bubbles. The edges of these depletions are very sharp resulting in large time rate of TEC in the equatorial ionosphere, even during magnetically quiet conditions. The large gradient of the equatorial ionization persists in the local post-sunset hours till about 2100 h LT. (DasGupta et al., 2007; Unnikrishnan and Ravindran, 2010). The TEC data for one station might experience an extremely sharp rate of change in TEC that may be due to some plasma bubbles in that region while the TEC from the other station stays normal. These variations in the various internal dynamics like plasma bubbles leading to scintillation can cause variations in the dynamical response of the TEC. Hence, the irregular variation in the values of the Lyapunov exponent and Tsallis entropy even in quiet periods for two relatively close stations may be due to these irregularities. This might also be responsible for the quiet days in the same station having lower values of Lyapunov exponent compared to higher values recorded for disturbed days without the external influence of storms.

The variations of these chaos and dynamical complexity parameters might also be as a result of the anomalous TEC enhancements that might occur at nights (Balan and Rao (1987); Balan et al., 1991). These effects can also be seen more clearly in the Tsallis entropy values for the five period window for quiet day of January, 2011, because the night time value is higher and it also show a much higher series of fluctuations during this period compared to other periods. As mentioned in Unnikrishnan and Ravindran (2010), the irregular changes in the dynamical characteristics of TEC from the results of Lyapunov exponent and Tsallis entropy also may be due to the collisional Raleigh-Taylor instability which may give rise to a few large irregularities in L band measurements (Rama Rao et al., 2006; Sripathi et al., 2008) all these can be seen as internal factors responsible for variations in the dynamical response of TEC as recorded from the values of the Lyapunov exponents and Tsallis entropy completed for days without storm which might be quiet or disturbed according to classification and also could account for higher values of these qualifiers during disturbed days compared quiet days. During storms however, the values were much lower

Earlier we, (Ogunsua et al., 2014) showed the appearance and variation of chaoticity quiet and disturbed day classification by international most quiet day (IQD) and internal most disturbed day (IDD) classification, as compared to quiet and storm period used by Unnikrishnan (2006;

2010). We were able establish that a relatively quiet day may be less chaotic compared to a relatively disturbed day unlike the result presented by Unnikrishnan (2006; 2010) for quiet and storm period. Also the combined use of both Lyapunov exponent and Tsallis entropy for the first time was found to have a high correlation mostly above 80%, which has stimulated the interest for further research using the two diagnosis for the study of ionospheric dynamics.

This work on the other hand presents the results for day to day variation and has revealed a seasonal trend for both Lyapunov exponents and Tsallis entropy, which appear in wavelike in form, with troughs during the two equinoxes. This was established for different stations used in this research work. The results show the appearance of seasonal trend in spite of the sporadic daily variation resulting from various changes in the internal dynamics. The seasonal trend has provided another possible evidence of higher energy input during equinoxes, since it reveals the effect of the annual energy input to the ionosphere. The day to day response these parameters has also revealed the variations in the underlying dynamics of the system.

As a similarity between the present work and Ogunsua et al. (2014) the relationship between Lyapunov exponent and Tsallis entropy can also be seen from this work, as the two quantifiers exhibit similarities in their response to the dynamical behavior of the ionosphere with phase transition at the same periods of time for all stations. A further investigation of this relationship shows that all the daily values of Tsallis entropy correlates positively with the values of Lyapunov exponent at values between 0.78 and 0.83.

The ability of these quantifiers to clearly reveal the ionospheric dynamical response to solar activities and changes in its internal dynamics due to other factors is a valid proof of the authenticity of the use of these chaotic and dynamical measures, as indices for ionospheric studies.

5.0 Conclusion

The chaotic behaviour and dynamical complexity of low latitude ionosphere over some parts of Nigeria was investigated using TEC time series measured Simultaneously at five different stations namely Birnin Kebbi (geographic coordinates $12°32'N,\ 4°12'E$; dip latitude $0.62°N$), Torro (geographic coordinates $10°\ 03'N, 9°04'E$; dip latitude $-0.82°N$), Enugu (geographic coordinates $6°26'N$, $7°30'E$; dip latitude $-3.21°N$), Lagos (geographic coordinates $6°27'N$, $3°23'E$; dip latitude $-3.07°N$) and Yola (geographic coordinates $9°12'N, 12°30°E$; dip latitude $-1.39°N$) within the low latitude region. The detrended TEC time series data obtained from the GPS data measurement were analysed ~~studied for~~ using different chaoticity and dynamical complexity parameters. ~~using phase space reconstruction techniques, computation of Lyapunov exponents and correlation dimension. Tsallis entropy was used for the study of dynamical complexity of the ionospheric system described by the TEC data.~~

~~The detrended TEC time series were subjected to further analysis for phase space reconstruction from which the choice of time delay of 30 was obtained and an embedding dimension of 5 were considered in this study~~. The evidence of the presence of chaos in all the time series data was obtained for all the data used, as indicated by the positive Lyapunov exponent. The results of Tsallis entropy show the variations in the dynamical complexity of the ionosphere, which may be due to geomagnetic storms and other phenomena like changes in the internal irregularities of the ionosphere. The response of the Tsallis entropy to various changes in the ionosphere also shows the deterministic nature of the system. The results of the Tsallis entropy show a lot of similarities with that of the Lyapunov exponents correlating between 0.78 and 0.81, with both results showing a phase transition from higher values in the solstices to lower values during the equinoxial months. The values of Lyapunov exponent were found lower for the days of the months in which storm was recorded relative to the nearest relatively quiet days which agree with previous works by other investigators. A similar pattern of results was obtained for the computed values of Tsallis entropy. The random variations in the values of chaoticity in the detrended TEC describing the internal dynamics of the ionosphere as seen in the result obtained from both Lyapunov exponent and Tsallis entropy depicts the ionosphere as a system with a continuously changing internal dynamics, which shows that the ionosphere is not totally deterministic but also has some elements of stochasticity influencing its dynamical behaviour.

The phase transition in the systems of the ionosphere resulting in the lower values of the chaoticity and dynamical complexity quantifiers during the geomagnetic storms and the equinoxial months is the evidence that the ionosphere can be greatly modified by stochastic drivers like solar wind and other incoming particle systems. The drop in values during equinoxes can be seen as form of semiannual variation, a phenomenon peculiar to low latitude regions.

~~It can also be seen that the results of Tsallis entropy follow the same pattern with Lyapunov exponent, which shows show that both can be use simultaneously and comparatively as measures of chaos and dynamical complexity as the correlation of all the values obtained for both quantities give values between 0.78 and 0.81.~~

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Although the knowledge of being able to characterize the ionospheric behaviour using the two major quantifiers shows their ability to measure level of determinism when used together, the relationship between these two quantifiers calls for more research, in the use of these qualifiers, to enable proper description and characterization of the state of ionosphere. The response of both Tsallis entropy and Lyapunov exponents to changes in the ionosphere shows that the two quantifiers can be used as indices to describe the processes/dynamics of the ionosphere.

Even though we cannot conclude totally until further investigations have been carried out on various properties of the ionosphere describing its dynamics. It can be safely established that this study has created roadmap for the use of the chaoticity and dynamical complexity measures as indices to describe the process/dynamics of the ionosphere.

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Table 1: Coordinates of the GPS stations

Station Name	Geographic Coordinates Long (°E)	Lat(°N)	Dip latitude (°N)
Birnin Kebbi	$4°\ 12'E$	$12°\ 32'N$	$0.62°N$
Torro	$9°\ 04'E$	$10°\ 03'N$	$-0.82°N$
Yola	$12°\ 30'E$	$9°\ 12'N$	$-1.39°N$
Lagos	$3°\ 23'E$	$6°\ 27'N$	$-3.07°N$
Enugu	$7°\ 30'E$	$6°\ 26'N$	$-3.21°N$

Table 2a :

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Results of Surrogate data test for Lyapunov exponent for TEC data for the quietest days of October 2011 at Birnin Kebbi station.

Original Data	Surrogate data
0.1165	0.3921 ± 0.0420
0.0931	0.2029 ± 0.0756
0.1041	0.3860 ± 0.0741
0.0498	0.2891 ± 0.0598
0.1420.	0.3621 ± 0.0504

Table 2b: Results of Surrogate data test for Lyapunov exponent for TEC data for the most disturbed days of October 2011 at Birnin Kebbi station.

Original Data	Surrogate data
0.0579	0.3039 ± 0.0541
0.0502	0.3156 ± 0.0428
0.0786	0.2527 ± 0.0296
0.1795	0.3662 ± 0.0468
0.1038	0.3100 ± 0.0416

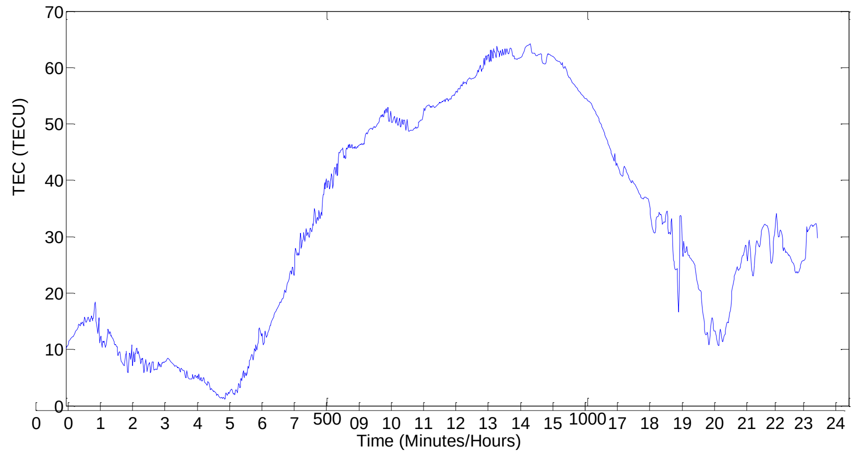

Fig 1. A typical time series plot for TEC measured at Lagos for 20 November 2011

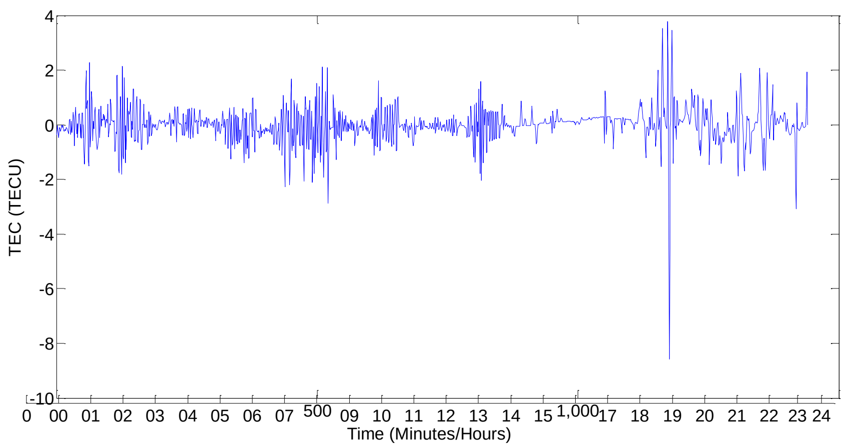

Fig 2.The detrended time series plot for TEC measured at Lagos

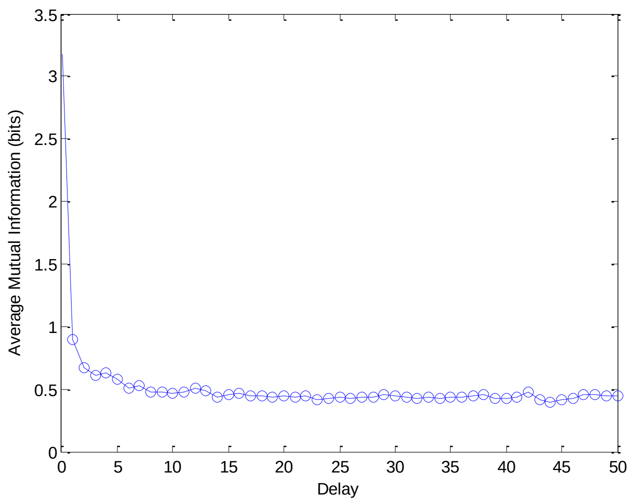

Fig. 3 Average mutual information against time Delay for TEC measured at Yola

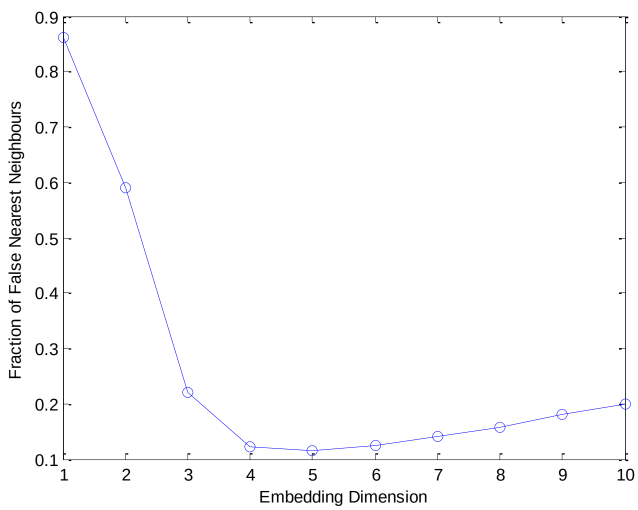

Fig. 4 Fraction of false nearest neighbours against embedding dimension for TEC measured at yola

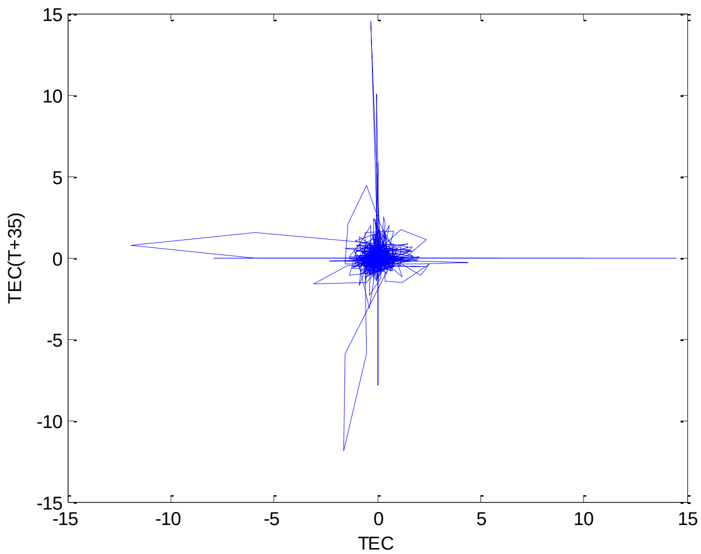

Fig.5 The Delay representation of the phase space reconstruction of the detrended TEC

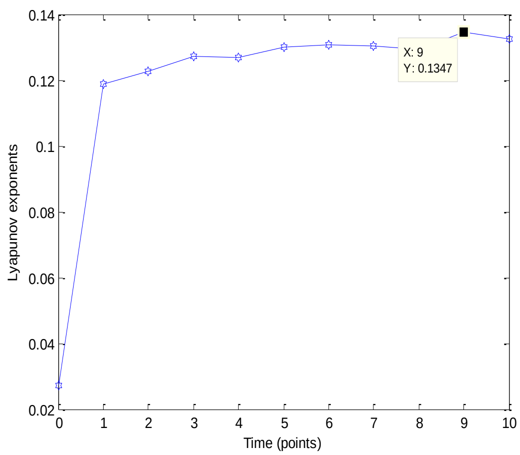

Fig. 6 Lyapunov Exponent computed and its evolution, computed as the state space trajectory scanned with tau=30, m=5 for detrended time series measured at Yola with Largest Lyapunov Exponent equal to 0.1347.

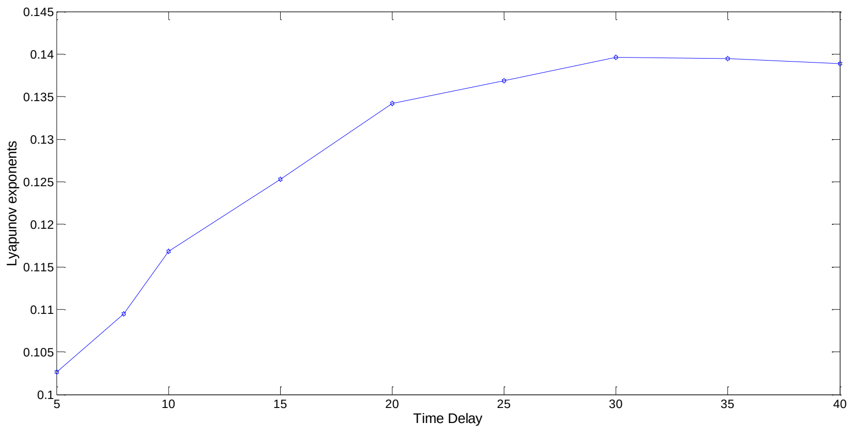

Fig. 6b Lyapunov exponent computed for different time delay with a constant embedding dimension.

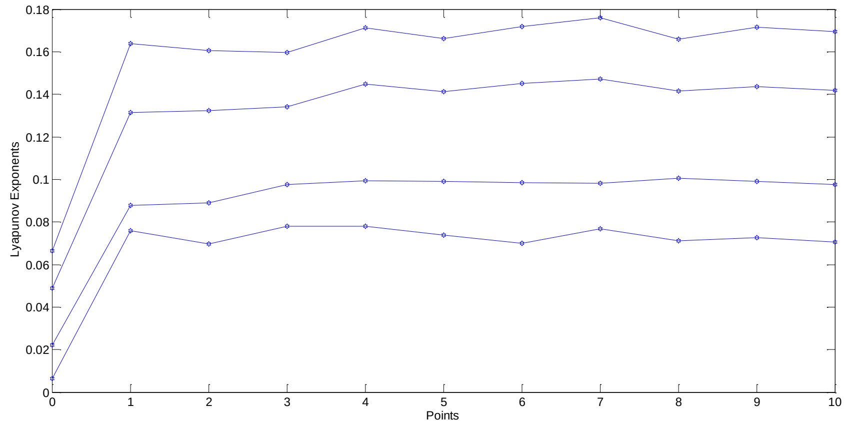

Fig. 6c Lyapunov exponents computed for different embedding dimension at constant time delay

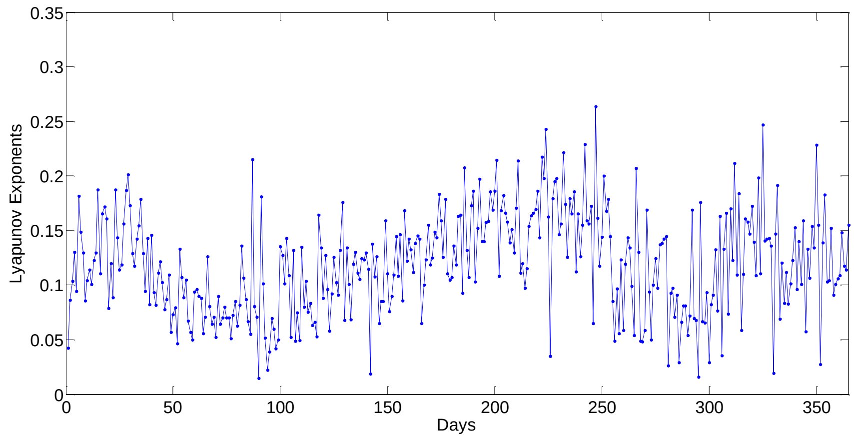

Fig. 7a The transient variations of Lyapunov exponents for 365 days of 2011 for detrended TEC measured at Enugu

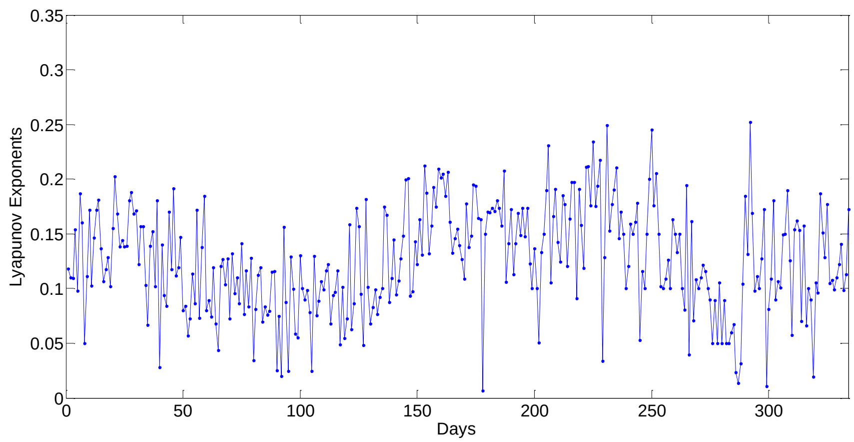

Fig. 7b The transient variations of Lyapunov exponents for 334 days (Jan1 –Nov30) of 2011 for detrended TEC measured at Toro

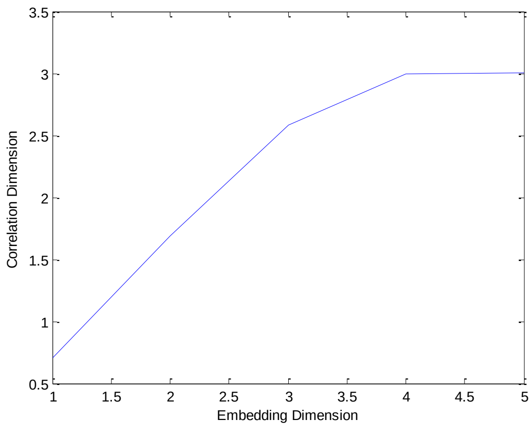

Fig. 8a The correlation dimension of the detrended TEC for the quietest day of October at Birnin Kebbi which saturates at $m \geq 4$ and $\tau = 39$

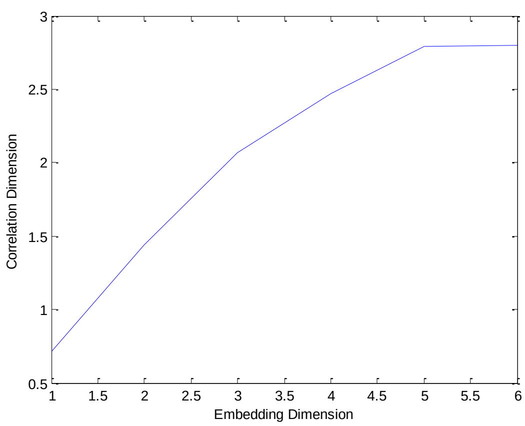

Fig. 8b The correlation dimension of the detrended for the most disturbed day of October at Birnin Kebbi which saturates at $m \geq 5$ and $\tau = 34$

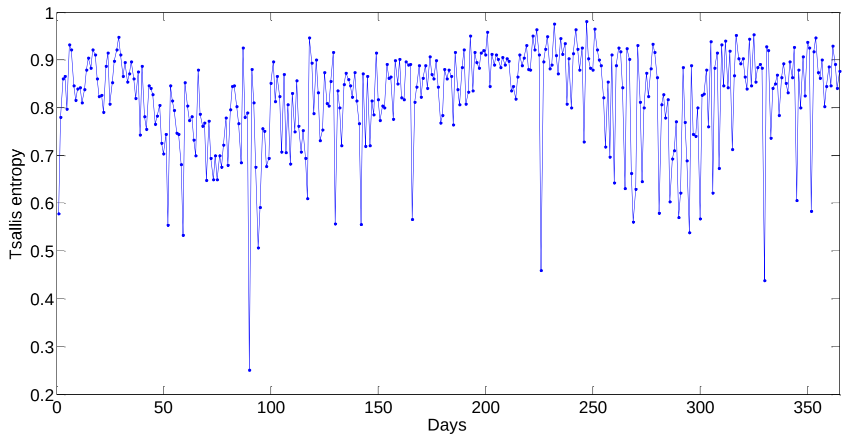

Fig. 9a The transient variations of Tsallis Entropy for 365 days (Jan1 –Nov30) of 2011 for detrended TEC measured at Enugu

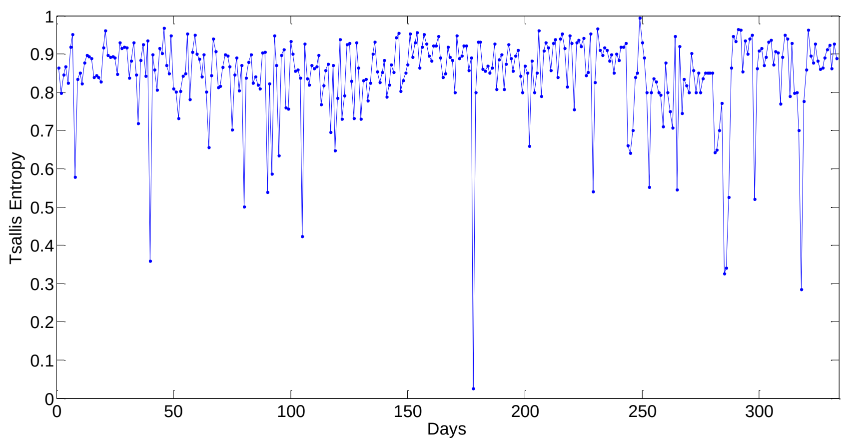

Fig. 9b The transient variations of Tsallis Entropy for 334 days (Jan1 –Nov30) of 2011 for detrended TEC measured at Toro

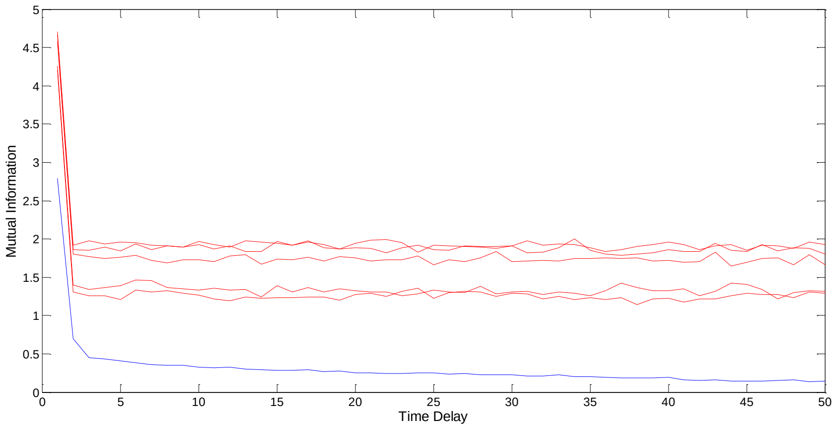

Fig 10 Mutual information plotted against time delay for the original detrended data in (blue curve) with the mutual information for the surrogate data (red curve) for TEC data measured at Lagos for the quietest day of march 2011

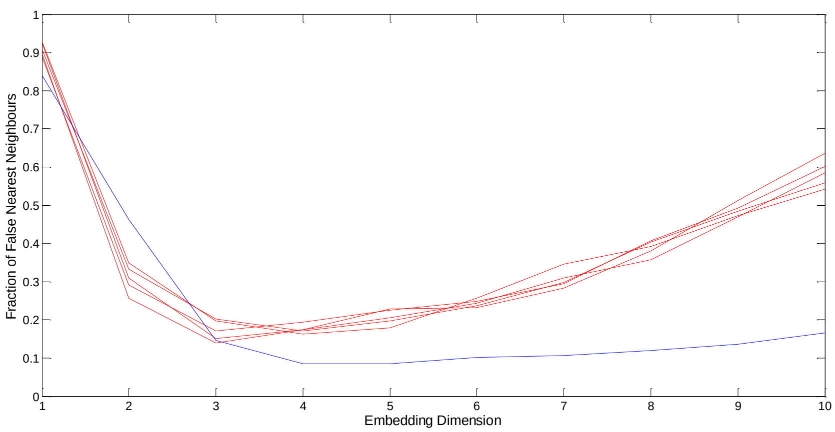

Fig 11 Fraction of false nearest neighbours plotted against time embedding dimension for the original detrended data in (blue curve) with the mutual information for the surrogate data (red curve) for TEC data measured at Lagos for the quietest day of march 20

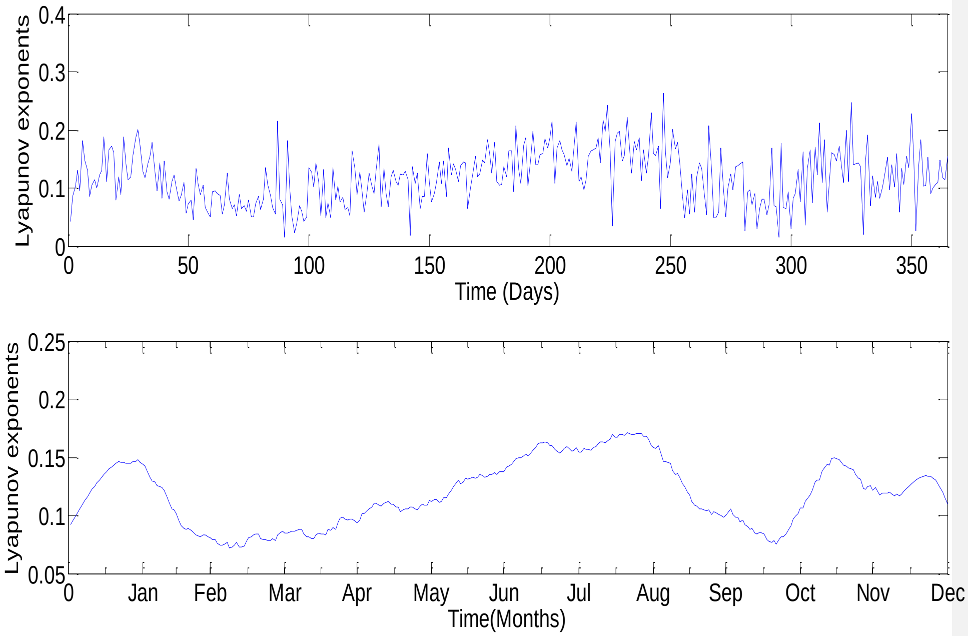

Fig. 12a Daily variation of Lyapunov exponents for TEC measured at the Enugu station for the year 2011 showing the Original data (Upper Panel) and the smoothed Plot of daily variation of Lyapunov exponents for TEC measured at the Enugu station for the year 2011 (Lower panel)

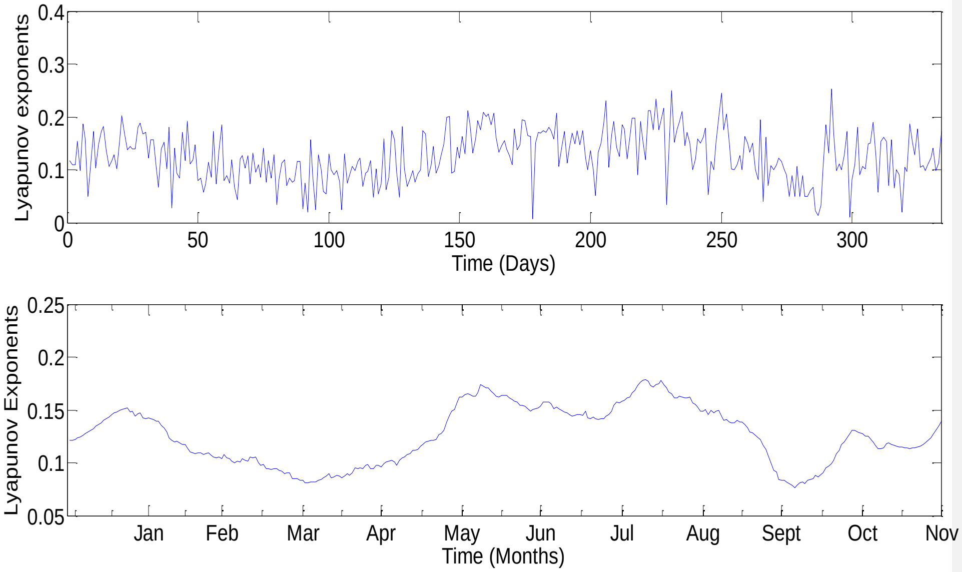

Fig 12b Daily variation of Lyapunov exponents for TEC measured at the Toro station for the year 2011 showing the Original data (Upper Panel) and the smoothed Plot of daily variation of Lyapunov exponents for TEC measured at the Toro station for the year 2011 (Lower panel)

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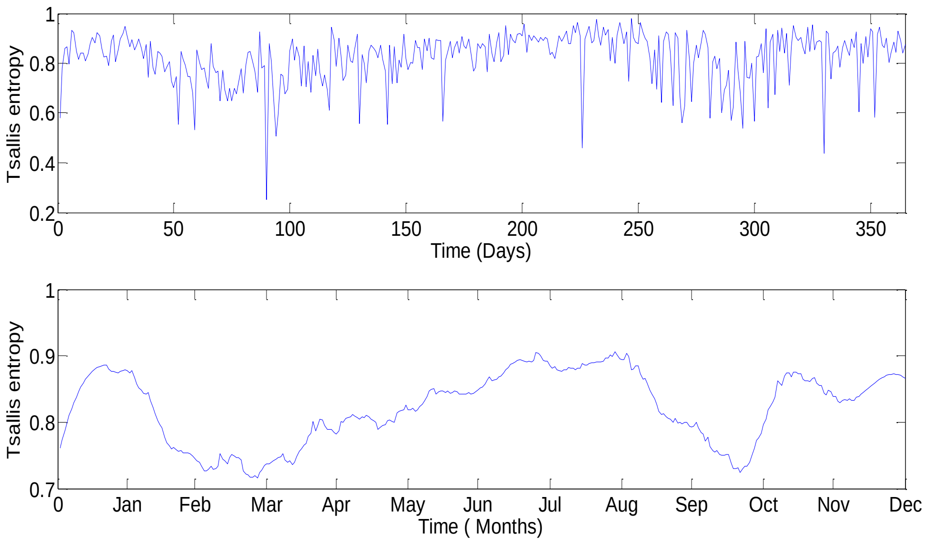

Fig. 13a Daily variation of Tsallis entropy for TEC measured at the Enugu station for the year 2011 showing the Original data (Upper Panel) and the smoothed Plot of daily variation of Lyapunov exponents for TEC measured at the Enugu station for the year 2011 (Lower panel)

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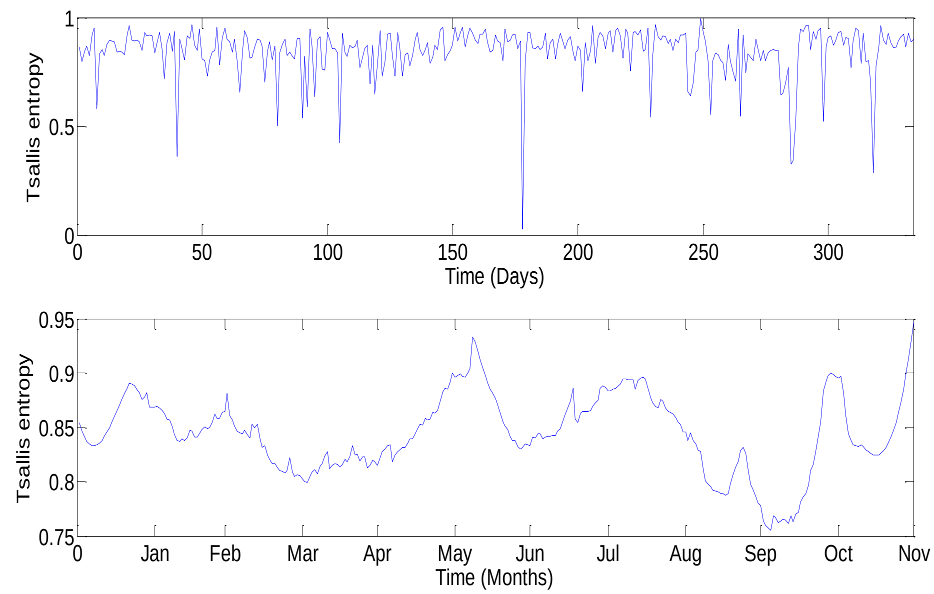

Fig. 13b Daily variation of Tsallis entropy for TEC measured at the Toro station for the year 2011 showing the Original data (Upper Panel) and the smoothed Plot of daily variation of Lyapunov exponents for TEC measured at the Enugu station for the year 2011 (Lower panel)

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