

We have to say many thanks to reviewer for useful and constructive comments which push us to improve the manuscript.

Here is the list of comments and corrections.

1. Reviewer does not agree that "the model is free from the narrowband approximation for surface waves and relatively weak adverse current" (lines 4-5). The frequency (or wavenumber) detuning of their three-modes model has an order ε . Thus, we have a sort of narrow band approximation. The strength of the current should be consistent with the problem scaling when effects of nonlinearity and inhomogeneity are of the same order of magnitude and, thus, cannot be free from the approximation;

Of course, any modeling of Benjamin-Feir instability by its main property considers the narrow band approximation of the order ε . Absolute frequencies difference (see equation 7) for the stationary modulation has the same order for the whole zone of wave-current interaction. But wave number and intrinsic frequencies of waves are no more constant and can change significantly in accordance with current dependant dispersion relations and effects of nonlinearity. Variations can be essential and even of order unit (see Fig.1h) on a long current scale, and so indeed we have the narrow band property, but only locally in space. (Here appear another important question from the 2nd referee: may be due to interaction with current and nonlinearity effects resonance conditions are totally destroyed due to the large detuning?). To clarify this property we add to manuscript the figure describing the typical behavior of phase-shift difference function $\varphi[X] = 2\theta_1 - \theta_0 - \theta_2$ (Figure 1d). Intensity of quasi-resonance energy exchange mostly defined by this function together with wave amplitudes. (see equations (17), (20)). Result looks rather surprising - several strong phases' jumps take a place with corresponding changing of the wave energy fluxes direction. But in any case we see an intensive quasi-resonant energy exchange in the entire interaction zone.

The effects of current and nonlinearity are assumed to have the same order and in this sense, of course, we have some restrictions to current strength. The model considers values of current speed up to the initial group velocity of surface waves and "long enough" space scale to go with the acceptable accuracy and to catch the strongest effects of wave blocking.

Corresponding corrections and explanations are made in the text of paper

"the model is free from the narrowband approximation for surface waves and relatively weak adverse current"

- > the model considers essential variations of the wave numbers and frequencies of interacting waves and wave blocking adverse current

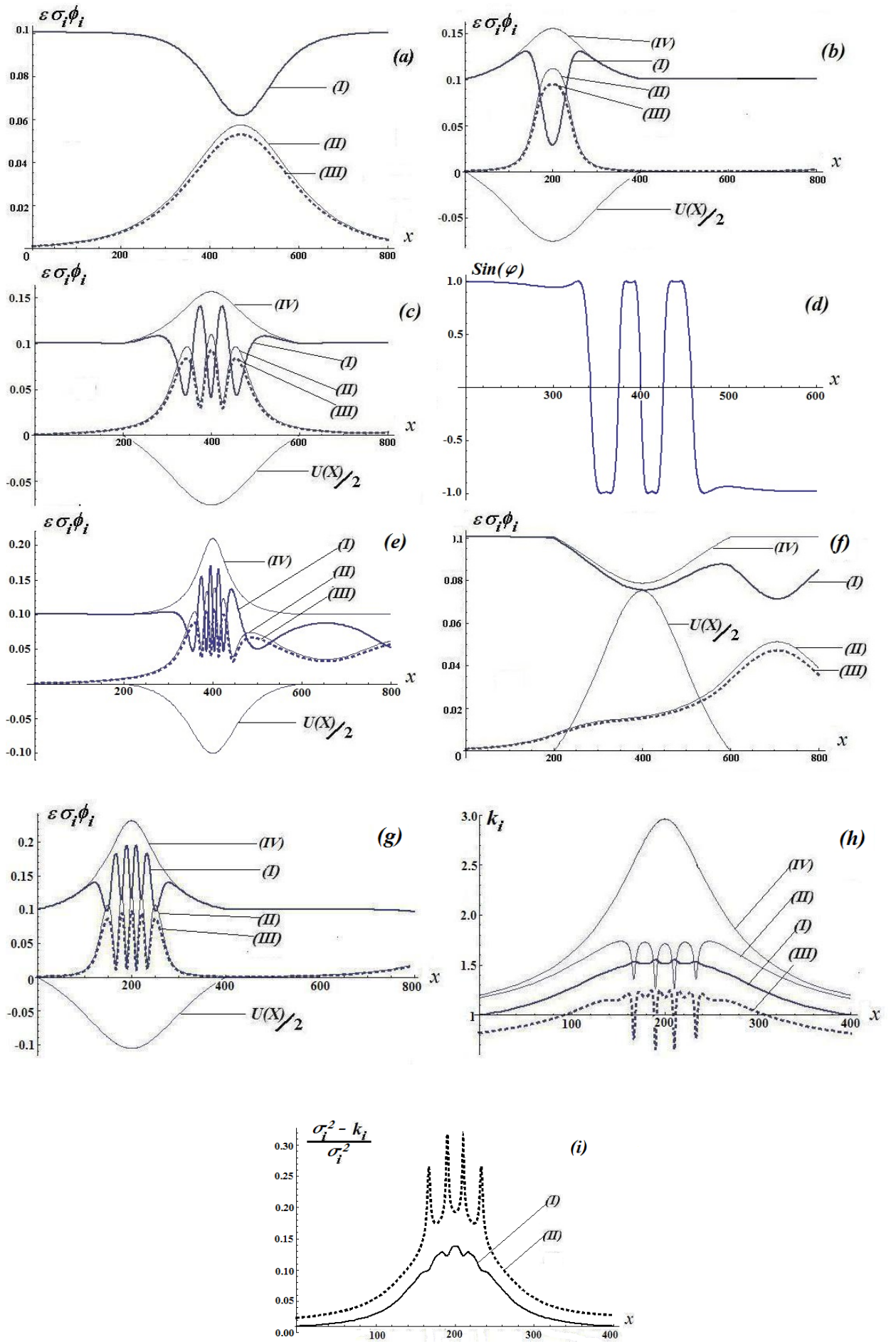


FIG. 1. (a) BF instability without current. (b), (c) Modulation of surface waves by adverse current

$U = U_0 \text{Sech}[\varepsilon^2(x - x_0)]$, ($U_0 = -0.15$); (b) $x_0 = 200$, (c) $x_0 = 400$. (d) phase difference function $\varphi[X] = 2\theta_1[X] - \theta_0[X] - \theta_2[X]$, $\theta_1[0] = 0; \theta_0[0] = \theta_2[0] = -\pi/4$, (e) Modulation of surface waves by adverse current $U = U_0 \text{Sech}[2\varepsilon^2(x - x_0)]$, ($U_0 = -0.2$), (f) Modulation instability for following current ($U_0 = 0.16, x_0 = 400$). (g), (h) Functions of wave amplitude and wave number respectively for $U_0 = -0.2$. (I), (II), (III) Amplitude envelopes of the carrier, superharmonic and subharmonic waves, respectively. (IV) Linear solution for the carrier envelope. The initial steepness of the carrier wave is $\varepsilon = 0.1$. (i) Relative distortion of the linear dispersion relation for the case (g), (I) – carrier, (II) – higher side band.

2. Authors develop their asymptotic approach in primitive variables that leads to rather cumbersome expressions. At the same time, the resulting equations should have evident properties of symmetry in indices of satellites 0; 2. In absence of current ($U = 0$) it has to give, in particular, the Manley-Rowe relations that are equivalent to the momentum conservation. These relations give a good basis for qualitative analysis of the effect of current inhomogeneity. Eqs. (16,17,20), unfortunately, do not emphasize this key feature of the system.

Following reviewer recommendations we rewrite equations (17), (20) in a more clear and compact form of wave action law:

$$\left\{ \begin{aligned} [\phi_0^2 \sigma_0]_T + [(U(X) + \frac{1}{2\sigma_0})\phi_0^2 \sigma_0]_X &= \varepsilon \phi_1^2 \phi_2 \phi_0 \sigma_1^3 \sigma_2^2 (2\sigma_1^3 - 2\sigma_1^2 \sigma_2 + 2\sigma_1 \sigma_2^2 - \sigma_2^3) \text{Sin}[\varphi]; \\ [\phi_2^2 \sigma_2]_T + [(U(X) + \frac{1}{2\sigma_2})\phi_2^2 \sigma_2]_X &= \varepsilon \phi_1^2 \phi_2 \phi_0 \sigma_0^2 \sigma_1^3 (2\sigma_1^3 - 2\sigma_0 \sigma_1^2 + 2\sigma_0^2 \sigma_1 - \sigma_0^3) \text{Sin}[\varphi]; \\ [\phi_1^2 \sigma_1]_T + [(U(X) + \frac{1}{2\sigma_1})\phi_1^2 \sigma_1]_X &= -\varepsilon \phi_1^2 \phi_2 \phi_0 \sigma_0 \sigma_1^2 \sigma_2 (\sigma_0^4 - \sigma_0^3 \sigma_1 - \sigma_0 \sigma_1 (\sigma_1 - \sigma_2)^2 + \\ &+ \sigma_0^2 (\sigma_1^2 - \sigma_1 \sigma_2 + 2\sigma_2^2) - \sigma_2 (\sigma_1^3 - \sigma_1^2 \sigma_2 + \sigma_1 \sigma_2^2 - \sigma_2^3)) \text{Sin}[\varphi]; \end{aligned} \right. \quad (17)$$

$$\left\{ \begin{aligned} [(U + \frac{1}{2\sigma_0})\phi_0^2 \sigma_0]_X &= \varepsilon \phi_1^2 \phi_2 \phi_0 \sigma_1^3 \sigma_2^2 (2\sigma_1^3 - 2\sigma_1^2 \sigma_2 + 2\sigma_1 \sigma_2^2 - \sigma_2^3) \text{Sin}[\varphi] \\ [(U + \frac{1}{2\sigma_2})\phi_2^2 \sigma_2]_X &= \varepsilon \phi_1^2 \phi_2 \phi_0 \sigma_1^3 \sigma_0^2 (2\sigma_1^3 - 2\sigma_0 \sigma_1^2 + 2\sigma_0^2 \sigma_1 - \sigma_0^3) \text{Sin}[\varphi] \\ [(U + \frac{1}{2\sigma_1})\phi_1^2 \sigma_1]_X &= -\varepsilon \phi_1^2 \phi_2 \phi_0 \sigma_0 \sigma_1^2 \sigma_2 (\sigma_0^4 - \sigma_0^3 \sigma_1 - \sigma_0 \sigma_1 (\sigma_1 - \sigma_2)^2 + \\ &+ \sigma_0^2 (\sigma_1^2 - \sigma_1 \sigma_2 + 2\sigma_2^2) - \sigma_2 (\sigma_1^3 - \sigma_1^2 \sigma_2 + \sigma_1 \sigma_2^2 - \sigma_2^3)) \text{Sin}[\varphi] \end{aligned} \right. \quad (20)$$

Derived system has a strong symmetry with respect to indexes 0 and 2 (it was one of the checking procedures for the validity of the final equations)

To perform the qualitative analysis of the problem, we suggest the law of wave action conservation flux in a slowly moving media as analogue of the three Manley-Rowe dependent integrals:

$$\begin{cases} \left(U + \frac{1}{2\sigma_2} \right) \phi_2^2 \sigma_2 + \left(U + \frac{1}{2\sigma_0} \right) \phi_0^2 \sigma_0 + \left(U + \frac{1}{2\sigma_1} \right) \phi_1^2 \sigma_1 = const; \\ \frac{1}{2} \left(U + \frac{1}{2\sigma_1} \right) \phi_1^2 \sigma_1 + \left(U + \frac{1}{2\sigma_0} \right) \phi_0^2 \sigma_0 = const; \\ \frac{1}{2} \left(U + \frac{1}{2\sigma_1} \right) \phi_1^2 \sigma_1 + \left(U + \frac{1}{2\sigma_2} \right) \phi_2^2 \sigma_2 = const; \\ \left(U + \frac{1}{2\sigma_0} \right) \phi_0^2 \sigma_0 - \left(U + \frac{1}{2\sigma_2} \right) \phi_2^2 \sigma_2 = const. \end{cases}$$

These integrals follow from the system (20) with acceptable accuracy $O(\varepsilon^4)$ for the stationary regime of modulation. The second and third relations here clearly show that the wave action flux of the side bands can grow up at the expense of the main carrier wave flux. The last relationship manifests the almost identical behavior of the main sidebands for the problem of their generation due to Benjamin-Feir instability.

The obtained system of equations (16), (20) in the absence of current is similar to classical Zakharov equations for discrete wave interactions (Mei, Stiassnie and You, 2009 . Mei et al. Theory and Applications of Ocean Surface Waves. World Scientific, 2009, 14.9.1-14.9.3). Corresponding references are added to the text. The main property in the presence of current is the variability of interaction coefficients. It is also answer to some of the similar comments of the referees 2nd and 4th.

3. In sect.4 authors introduce semi-empirical functions of wave dissipation and breaking. The extension of the conservative system (16-18) to the non-conservative counterpart (21-22) looks strange. Additional terms contain terms of different orders in ε . Additionally, denominators of these terms contain small differences of wavenumbers (order of ε). This extension requires to be re-written in more consistent way or additional comments.

We employ the adjusted dissipative model of Tulin and Li (1996) and Huang et al. (2011) to describe the effect of breaking on the dynamics of the water wave. The sinks of energy and momentum terms for each of the waves are calculated in accordance with the dissipative Schrodinger model for the complex amplitude $A \sim \sum \phi_i e^{i\theta_i} ::$

$$A_T + C_g A_x + i \frac{C_g}{4k} A_{xx} + \frac{i}{2} \omega k^2 |A|^2 A = \left(-\frac{DA}{g|A|^2} - 4i\gamma A \int \frac{\omega^2 D dX}{g|A|^2} \right) H \left[\frac{|A_x|}{A_s} - 1 \right]$$

where $D \sim gD_b |A|^4$, $D_b = O(10^{-1})$, $\gamma = O(10^{-1})$ - constants of proportionality taken from the field observations, g – gravity acceleration, H is the Heaviside unit step function, and A_s is the threshold value of the characteristic steepness $A_x = \varepsilon \sum \sigma_i \phi_i k_i$. Right side part leads to additional terms in the governing modulation equations (16), (17), (20). The terms $\varepsilon / (k_i - k_j)$ appear due to integration procedure and have an order of unit. The singularity was not detected in numerical simulations.

Wave breaking leads to permanent (not temporal) frequency downshifting at a rate controlled by breaking process. A crucial aspect here is the cooperation of dissipation and near-neighbor energy transfer in the discretized spectrum acting together.

Authors present simulations for parameters of previous experimental studies and show strong effects of current on modulational instability for very special cases. An important question is a root to these special and, somewhat, extreme cases. A dependence of the effect on parameters of the current inhomogeneity can be presented to show a gradual transition from weak inhomogeneity to the extreme cases. May be additional figures can make the presentation of the results more clear and attractive.

We add some more figures with additional examples of wave interactions to clarify its properties.

The numerical simulations for initially high steepness waves ($\varepsilon = 0.25$) propagation with wave breaking dissipation is presented in Fig. 3(a-c). We calculate the amplitudes of surface waves on linearly increasing opposing current $U(x) = -U_0 x$ with different strength U_0 . Most unstable regime was tested for frequency space $\Delta\omega_{\pm} / \omega_1 \sim \varepsilon$ and most effective initial phases $\theta_1(0) = 0, \theta_0(0) = \theta_2(0) = -\pi/4$

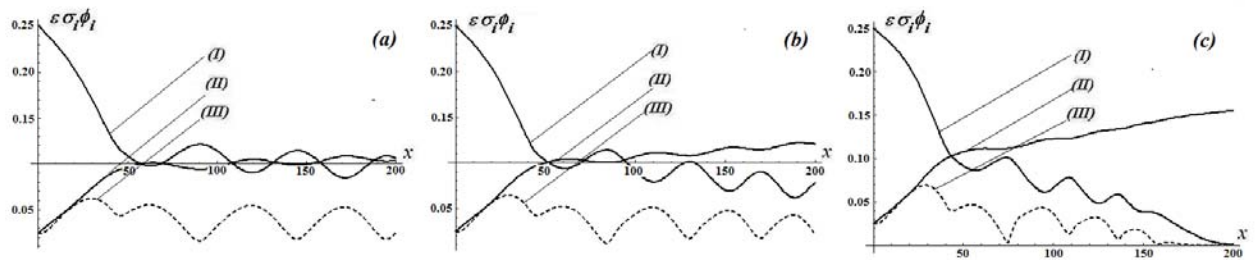


FIG. 3. Modulation of surface waves by the adverse current $U = U_0 x$. (a) $U_0 = -2.5 \cdot 10^{-4}$; (b) $U_0 = -5 \cdot 10^{-4}$, (c) $U_0 = -10^{-3}$. (I), (II), (III) - amplitude envelopes of the carrier, subharmonic and superharmonic waves, respectively. Initial wave steepness $\varepsilon = 0.25$. Dissipation parameters $D_b = 0.1, \gamma = 0.5$

A very weak opposite current $U_0 = 2.5 \cdot 10^{-4}$ (Fig.3(a)) has a pure impact on wave behavior: it is finally results in almost bichromatic wave train with two dominant waves: carrier and lower side band. Frequency downshift here is not clearly seen. Two times stronger current case with $U_0 = 5 \cdot 10^{-4}$ is presented in Fig. 3(b). We note some tendency to final energy downshift to the lower side band. Really strong permanent downshift with total domination of the lower side band is seen for two times more strong current $U_0 = 10^{-3}$.

One more example of wave modulation with not symmetrical behavior we present at Figure 1e. (Here is also answer to one of the comments of the 2nd referee) The modulation equations permit symmetrical solutions for the symmetrical current function, but, of course, here is no any special obligations. Outside of interaction zone we can have different kinds of nonlinear periodic waves depending from the boundary conditions and constant Stokes wave is only one of the possibilities. The symmetrical behavior is typical for a sufficiently long scale current. We add one more Figure 1.e with the same wave initial characteristics as for Figure 1c and two time's shorter space scale of the current. After interaction zone we see three wave system with comparable amplitudes and periodic energy transfers.

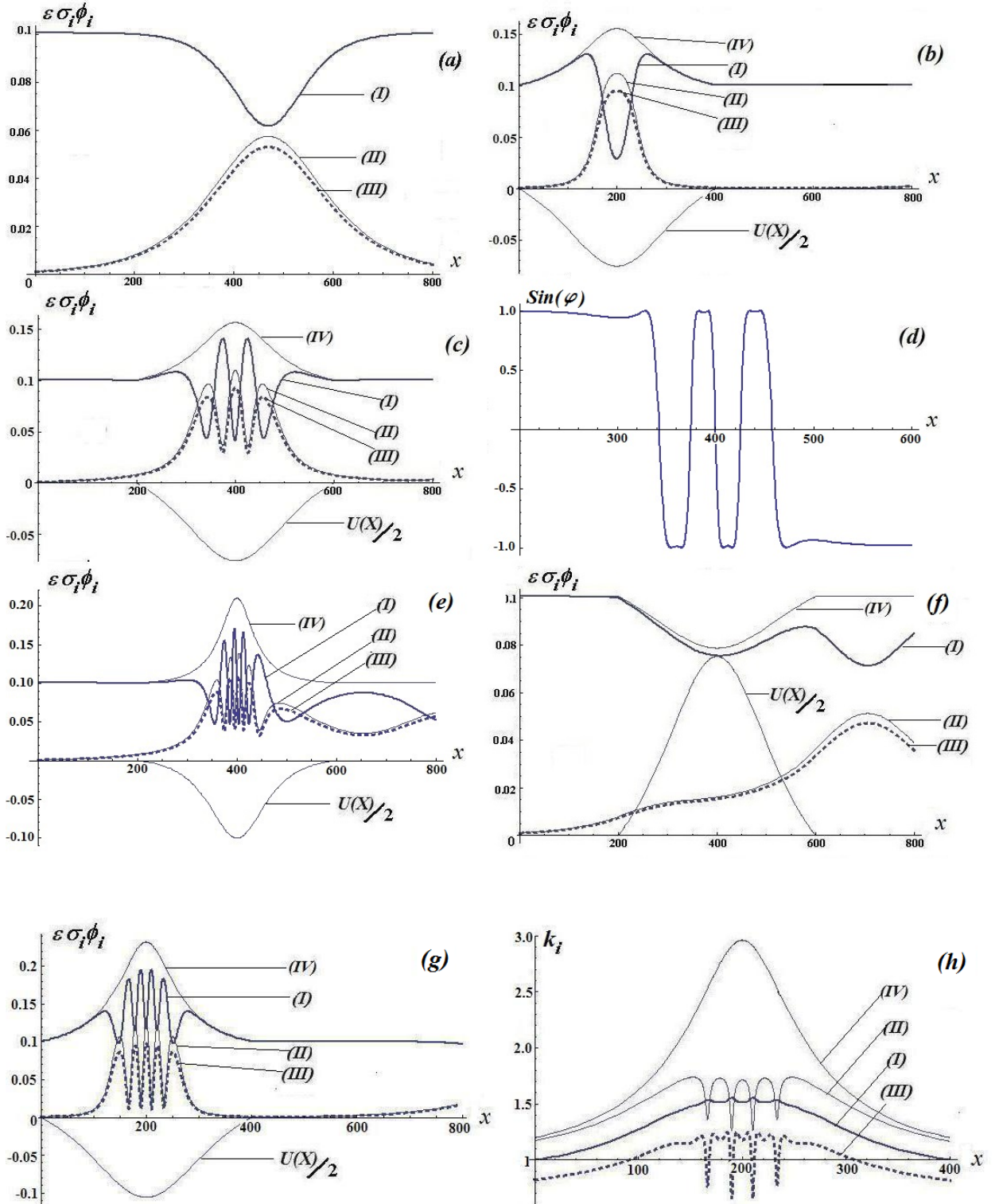
To give an idea about the strength of nonlinearity we present one more Figure (1i) with distortion of the linear dispersion relation for different modes. As one can see the effect of nonlinearity for the carrier (at maximum is about 10%) is much less compare to side bands (at peak is more than 30 %). The main impact of nonlinearity comes from amplitude Stokes dispersion.

The answers to comments of 2nd, 3rd and 4th referees are coming soon.

We appreciate 2nd referee for the useful comments and questions which help us to go inside the considered problems.

1. The obtained system of equations (20) in the limit of a zero (or constant) current should tend to the classic theory for a resonant wave quartet (e.g. Mei et al, *Theory and Applications of Ocean Surface Waves*. World Scientific, 2009, §14.7). Was this limit verified?

The obtained system of equations (20) in the absence of current is similar to classical Zakharov equations for discrete wave interactions (Mei, You and Stiassnie, 2009 . *Theory and Applications of Ocean Surface Waves*. World Scientific, 2009, 14.9.1-14.9.3). Corresponding references are added to the text.



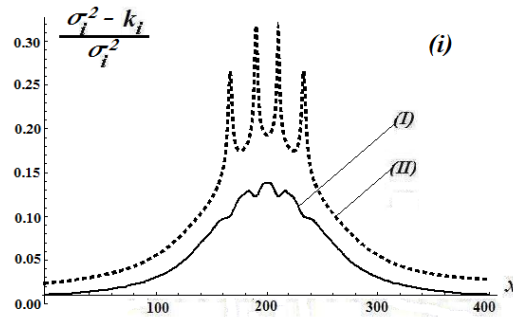


FIG. 1. (a) BF instability without current. (b), (c) Modulation of surface waves by adverse current $U = U_0 \text{Sech}[\varepsilon^2(x - x_0)]$, ($U_0 = -0.15$); (b) $x_0 = 200$, (c) $x_0 = 400$. (d) phase difference function $\varphi[X] = 2\theta_1[X] - \theta_0[X] - \theta_2[X]$, $\theta_1[0] = 0; \theta_0[0] = \theta_2[0] = -\pi/4$, (e) Modulation of surface waves by adverse current $U = U_0 \text{Sech}[2\varepsilon^2(x - x_0)]$, ($U_0 = -0.2$), (f) Modulation instability for following current ($U_0 = 0.16, x_0 = 400$). (g), (h) Functions of wave amplitude and wave number respectively for $U_0 = -0.2$. (I), (II), (III) Amplitude envelopes of the carrier, superharmonic and subharmonic waves, respectively. (IV) Linear solution for the carrier envelope. The initial steepness of the carrier wave is $\varepsilon = 0.1$. (i) Relative distortion of the linear dispersion relation for the case (g), (I) – carrier, (II) – higher side band.

2. Let us focus on the stationary boundary problem (Sec. 3). The interacting waves are assumed to be in resonance along entire Ox , though their local wavenumbers vary according to (19) (and some nonlinear corrections to σ_j).

Therefore the waves naturally get detuned, what should destroy the description (the modal approach is applied in Shrira & Slunyaev, *J. Fluid Mech.*, 738, 65-104 (2014) to overpass a similar obstacle). Am I mistaken?

This is a really important question. Absolute frequencies for the stationary modulation satisfy to quasi-resonance conditions (7) for the entire region of interaction. But that is may be not the case for the local wave numbers and intrinsic frequencies - they are substantially variable due to current effects and nonlinearity. Here evidently appears one more critical question: may be due to interaction with current and nonlinearity effects the almost resonance conditions are totally destroyed due to large detuning? (Shrira & Slunyaev, *J. Fluid Mech.*, 738, 65-104 (2014))

To clarify this property we add to manuscript the figure describing the typical behavior of phase-shift difference function $\varphi[X] = 2\theta_1 - \theta_0 - \theta_2$ (Figure 1d) for the wave modulation presented at Figure 1c. Intensity of nonlinear energy transfer mostly defined by this function together with wave amplitudes. (see equations (17), (20)). Result looks for us rather surprising - several strong phases' jumps take a place with corresponding changing of the wave energy fluxes direction. But in any case we see an intensive quasi-resonant energy exchange in the entire interaction zone. Quasi-resonant conditions are satisfied locally in space with a relatively small detuning value. Qualitatively similar behavior of phase-shift function we found also for other regimes of wave modulation.

3. The authors claim that the present theory and the linear solution for the carrier envelope on variable current give different estimations for the wave maximum (Fig. 1b). At the same time, the maxima are significantly closer in Fig. 1c. It is interesting to know how significant is the difference between the developed theory and the analysis of the modulational instability of the current-modified nonlinear Schrodinger equation (derived in the cited work by

Onorato et al.). The comparison between the theories and laboratory measurements (Fig. 2) does not lead to a decisive conclusion, which theory is better. In this respect the paper by C. van Duin (J. Fluid Mech., 399, 237-249, 1999) may be relevant.

We recalculate the amplification of waves on adverse current presented on Figures 2, a,b in accordance with conditions of Toffoli et al. (2013) and Ma et al. (2013) experiments. Waves are generated in a still water and then undergo a current quickly raised to a constant value $-U$. (not gradually increased opposite current through entire tank as in previous simulations)

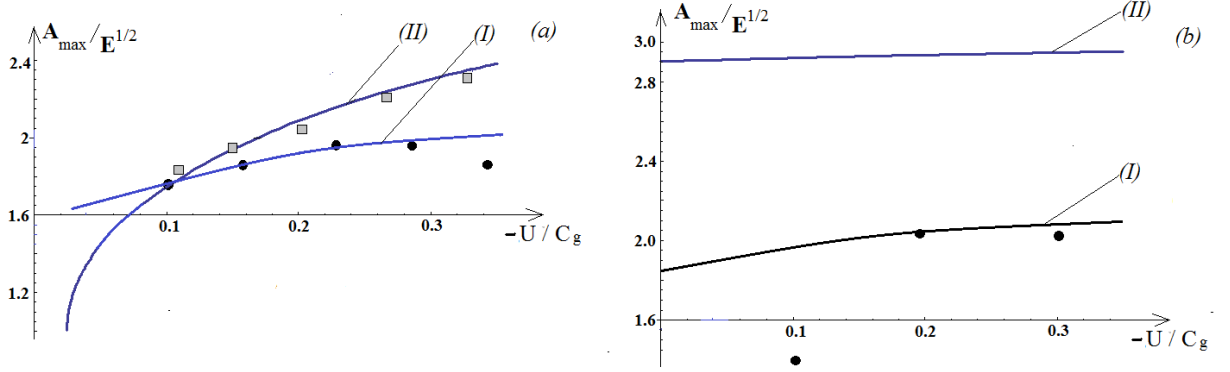


FIG. 2. Nondimensional maximum wave amplitude as a function of U/C_g , where C_g is the group velocity of the carrier wave and $E^{1/2}$ is the local standard deviation of the wave envelope.

(a) Experiments conducted by Toffoli et al. (2013) for carrier wave of period $T = 0.8s$ (wavelength $\lambda \cong 1m$), initial steepness $k_1 a_1 = 0.063$, and frequency difference $\Delta\omega/\omega_1 = 1/11$. Solid dots show measurements made using a flume at Tokyo University and squares show results obtained at Plymouth University. Line (I) shows the resonance model prediction while line (II) shows the prediction made using Eq. (2) (Toffoli et al. (2013)).

(b) Case T11 in Ma et al. (2013) for carrier-wave frequency $\omega_1 = 1Hz$, initial steepness $k_1 a_1 = 0.115$, and frequency difference $\Delta\omega/\omega_1 = 0.44a_1 k_1$. Solid dots show measurements. Line (I) shows the resonance model prediction, while line (II) shows the prediction made using Eq. (15) (Toffoli et al., 2013).

Our simulations confirm that initially stable waves in experiments of Toffoli et al. (2013) undergo a modulationally unstable process and wave amplification in the presence of adverse current. Maximum amplification reasonably corresponds to results of experiments in Tokyo University Tank for moderate strength of current. Maximum of nonlinear focusing in dependence on the value of current is weaker compare to the model of Toffoli (2013).

Experiments of Ma et al. (2013) (Figure 2b) show that the development of the modulational instability for a gentle waves and relatively weak adverse current ($U/C_g \sim -0.1$, see the first experimental point) is limited due to the presence of dissipation. Our resonance-model simulations are in a good agreement with the experimental values for the moderate values of adverse current $U/C_g \sim -0.2 - 0.4$. Results of Toffoli et al. (2013) notably overestimate the maximum of wave amplification.

4. *The variation of sideband wavenumbers due to nonlinearity (according to authors' description, – gaps on the corresponding curves in Fig. 1f) – are of order $O(1)$ of the carrier wavenumber. At the same time, the value of kI is just slightly altered. Does this mean that the situation is too much nonlinear? Do the dependences look more realistic for smaller waves?*

To give an idea about the strength of nonlinearity we present one more Figure (1i) with distortion of the linear dispersion relation for different modes. As one can see the effect of nonlinearity for the carrier (at maximum is about 10%) is much less compare to side bands (at peak is more than 30 %). The main impact of nonlinearity comes from amplitude Stokes dispersion.

5. I am puzzled by the curves in Fig. 1a,b,c,e.

5.1) *Two spatial scales seem to exist: of the nonlinearity, and of the current. According to authors' choice they are of same orders, but may be aliquant. Why there is always an integer number of oscillations under the current profile? The solutions seem to be perfectly symmetric. Why?*

The modulation equations permit symmetrical solutions for the symmetrical current function, but, of course, here is no any special obligations. Outside of interaction zone we may have different kinds of nonlinear periodic waves depending from the boundary conditions and constant Stokes wave is only one of the possibilities. The symmetrical behavior is typical for a sufficiently long scale current. We add one more Figure 1.e with the same wave initial characteristics as for Figure 1c and two time's shorter space scale of the current. After interaction zone we see three wave system with comparable amplitudes and periodic energy transfers.

5.2) *Do the authors have an idea, why the location of the current maximum (cf. Fig. 1b and Fig. 1c) results in such big difference between the solutions?*

As one can see from Figures 1b -1c the initial stage of wave-current interaction is characterized by the dominant process - absorbing of energy by waves where all three waves grow simultaneously. Preliminary growth of side band modes (Figure 1c) leads to more deep modulated regime. Increasing of wave steepness in turn accelerates instability and finally these two dominate processes alternate. Correspondingly, the triggering of this complicate process is essentially depending from the displacement of the current maximum.

5.3) *In the course of modulational growth the superharmonic attains larger amplitude than the subharmonic. This contradicts the classical result, which is opposite (i.e. Tanaka, Wave Motion, 12, 559-568, 1990, or the recent study by Slunyaev & Shrira, J. Fluid Mech., 735, 203-248, 2013).*

Frequency upshift and not downshift first was received for the three-wave calculations by Stiassnie & Shemer (1987) using Zakharov equations. They predicted that at peak modulation the upper sideband amplitude becomes slightly bigger than that of the lower. This effect is unavoidable "price" for using three wave's weakly nonlinear conservative interaction model. (This conclusion immediately follows from the Mainly-Rower conservation integrals). (By the way, it looks like we see some confusion in the cited book of C.C. Mei et al. - Figure 14.6 really show upshift and not downshift, see Stiassnie & Shemer (1987)). The experimental evidence contradicts this prediction. The spectral downshift has been predicted by computations made by the Dysthe equations (Lo & Mei 1985; Trulsen & Dysthe 1990; Hara & Mei 1991) for a much more

number of excited waves, the same prediction was also made by simulations of fully nonlinear equations (i.e. Tanaka, Wave Motion, 12, 559-568, 1990, Slunyaev & Shrira, J. Fluid Mech., 735, 203-248, 2013).

The principle aspect here seems to be the temporal character of slight frequency shift - at the end of the modulation loop the system revert to the almost initial state with some energy spreading to higher frequencies. Situation is principally different for relatively high initial wave steepness - at the peak of modulation the system loose the energy due to breaking mostly at the expense of super harmonic and higher frequency modes. So we can observe permanent frequency downshift with final dominating of the lower subharmonic and wave system does not revert to its initial state. This process can be described more or less adequately by the three wave's dynamical model including dissipation effects.

Corresponding references are added to the text

Minor remarks.

– Capture to Fig. 2. What is “SD”? The spectral widths for panels (a) and (b) are given in different manners, what may lead to confusion.

SD=Standart Deviation of wave envelope.

The value of the breaking parameter γ is not specified.

Empirical parameter was assumed $\gamma = 0.7$

We appreciate 3rd referee for the well-disposed review and try to improve the manuscript in accordance with comments made.

1. My major objection to the manuscript is that the results of the comparison between the model proposed and the experiments are rather inconclusive. Apparently their model should be superior to all the others; however, for example in Fig. 2a, the model does not seem to fit well the experiments. The reason of this discrepancy is not explained in the text. Also in Fig 2b, the first two experimental points are well off the curve obtained from the model. It is quite difficult to state safely that the developed model is better than the others. I would suggest to discuss more in deep the comparison between experiments and the model.

We recalculate the amplification of waves on adverse current presented on Figures 2, a,b in accordance with conditions of Toffoli et al. (2013) and Ma et al. (2013) experiments. Waves are generated in a still water and then undergo a current quickly raised to a constant value $-U$. (not gradually increased opposite current through entire tank as in previous simulations

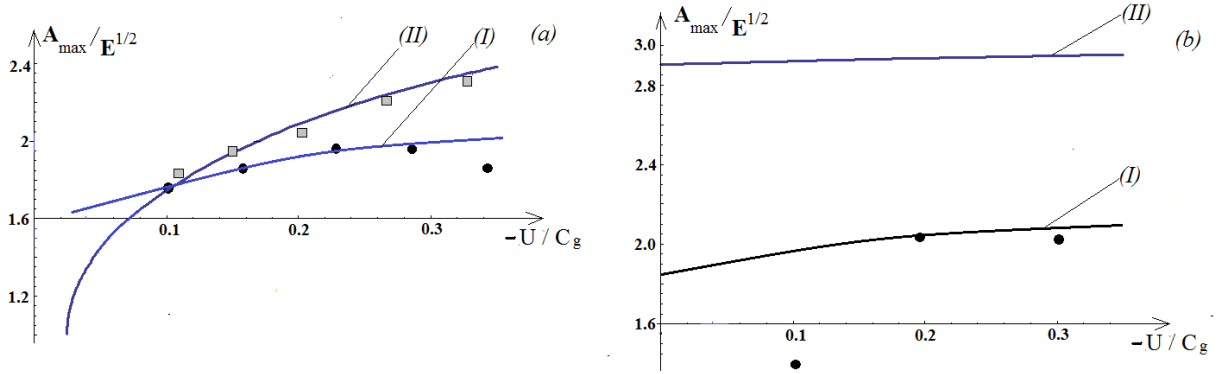


FIG. 2. Nondimensional maximum wave amplitude as a function of U/C_g , where C_g is the group velocity of the carrier wave and $E^{1/2}$ is the local standard deviation of the wave envelope.

(a) Experiments conducted by Toffoli et al. (2013) for carrier wave of period $T = 0.8s$ (wavelength $\lambda \cong 1m$), initial steepness $k_1 a_1 = 0.063$, and frequency difference $\Delta\omega/\omega_1 = 1/11$. Solid dots show measurements made using a flume at Tokyo University and squares show results obtained at Plymouth University. Line (I) shows the quasi-resonance model prediction while line (II) shows the prediction made using Eq. (2) (Toffoli et al. (2013)).

(b) Case T11 in Ma et al. (2013) for carrier-wave frequency $\omega_1 = 1Hz$, initial steepness $k_1 a_1 = 0.115$, and frequency difference $\Delta\omega/\omega_1 = 0.44a_1 k_1$. Solid dots show measurements. Line (I) shows the quasi-resonance model prediction, while line (II) shows the prediction made using Eq. (15) (Toffoli et al., 2013).

Our simulations confirm that initially stable waves in experiments of Toffoli et al. (2013) undergo a modulationally unstable process and wave amplification in the presence of adverse current. Maximum amplification reasonably corresponds to results of experiments in Tokyo University Tank for moderate strength of current. Maximum of nonlinear focusing in dependence on the value of current is weaker compare to the modeling results of Toffoli (2013).

Experiments of Ma et al. (2013) (Figure 2b) deals with initially unstable waves. Results show that development of the modulational instability for a gentle waves in the presence of relatively weak adverse current ($U/C_g \sim -0.1$, see the first experimental point) is limited due to the presence of viscous dissipation. Our resonance-model

simulations are in a good agreement with the experimental values for larger values of adverse current $U/C_g \sim -0.2 - 0.4$. Results of Toffoli et al. (2013) notably overestimate the maximum of wave amplification.

The characteristic spatial scale used in developing the BF instability of the Stokes wave is lc/ε^2 , where $lc = 2\pi/kc$ is the typical wavelength of surface waves (Benjamin and Feir, 1967). We consider long-scale slowly varying current $U(x)$ with horizontal length scale L of the same order: $L = O(lc/\varepsilon^2)$. iv. It is assumed that the $U(x)$ dependence is due to the inhomogeneity of the bottom profile $h(x)$, which is sufficiently deep so that the deep-water regime for surface waves is ensured; i.e., $\exp(2kch) \ll 1$. The characteristic current length L at which the function $U(x)$ varies noticeably is assumed to be much larger than the depth of the fluid, $h(x) \ll L$. Under these conditions, $U(x)h(x)$ is approximately constant, and the vertical component of the steady velocity field on the surface $z = \eta(x)$ can be neglected. This velocity field is directed along a tangent, and the slope of the tangent in the cases considered is negligibly small; i.e., $\eta'(x) \ll 1$. Correspondingly, it follows from the Bernoulli time-independent equation that the surface displacement induced by the current is small (Ruban, 2012). Such a situation can occur, for example, near river mouths or in tidal/ebb currents.

2. Why do the authors name the model as “resonant”? The modulational instability in one horizontal direction is not a resonant process but a quasi-resonance. Does the presence of a current make it exactly resonant?

We absolutely agree with this comment: the quasi-resonant process with some detuning from conditions of strong resonance takes a place and without current and in the presence of current. Taking into account the distortion from exact resonance conditions is critically important to predict the energy exchange properties and nonlinear dispersive properties of interactive waves. Corresponding corrections are made in the text of manuscript.

3. Paragraph (iv page 181) is taken entirely from Ruban (2012); this is not a major problem, but I just do not understand the reason of this choice.

We are so sorry to reproduce the explanation for “classical” shallow water approximation for a large scale stationary horizontal current made by Ruban (2012). The reason for this choice was physical evidence and practical applicability of assumed approximation. We shortened the entire piece of text and slightly changed the context.

We appreciate 4th referee for highlighting the significant questions which help us to clarify the main results of research.

1. *It should be stressed in the manuscript that the 3-wave system analysis can only be relevant at the initial stages of evolution, as long as the sidebands are sufficiently small and thus no significant new harmonics are generated due to nonlinear near-resonant interactions. It is well known that addition of even a single new harmonic to an initially 3 wave system prevents exact Fermi-Pasta-Ulam recurrence. The FPU recurrence thus cannot be expected to occur in reality. Nevertheless, this simplified wave system may be useful if reservations regarding the limited validity of results are understood and clearly spelled out.*

The evolution of the wave spectrum in the absence of breaking includes energy exchange between the carrier wave and two main resonant side-bands and spreading of the energy to higher frequencies. Inclusion of higher frequency free waves in the Zakharov, modified Schrodinger or Dythe equations is crucial, since the asymmetry of the lower and the upper side-band amplitudes at peak modulation in non-breaking case results from that. Such a conclusion can be made regarding to developing of modulation instability in a calm water.

The developing of modulation instability in the presence of significant adverse current is different. Experimental results of Chavla and Kirby (2002) and Ma et al. (2010) clearly show that energy spectrum is mostly concentrated in the main triad of waves and high frequency discretized energy spreading is depressed due to the short wave blocking by the strong enough adverse current.

The relatively high initial wave steepness leads to wave breaking dissipation with discriminatory energy loss from the carrier and higher side-band modes (Tulin and Waseda, 1999). Even those waves which do not blocked lose a considerable amount of energy due to wave breaking on the strong opposite current. The permanent frequency downshift with final dominating of the lower subharmonic takes a place and wave system does not revert to its initial state.

3-wave dynamical model analysis in the presence of significant opposite current therefore can be relevant not only at the initial stages but also at the further stages of wave evolution.

Corresponding discussion added to text of paper.

2. *As mentioned by the 2nd reviewer, the evolution of the 3-wave system beyond the initial exponential growth stage has been considered before. The main motivation for limiting the analysis to 3 waves only in those works was the availability of a closed analytical solution in the framework of the Zakharov equation (see the book by Mei et al., Stiassnie and Shemer 1987, 2005 and Shemer 2009). The possibility to apply similar analytical approach in the presence of the current should be examined and discussed in the paper. The current investigation considers spatial evolution, whereas the temporal variation was apparently studied in the earlier publications mentioned above. The advantages (if any) of the adopted approach also have to be discussed. The present formulation has the 3rd order accuracy and thus should not be essentially different from the Zakharov equation (at least in the absence of the current). The differences between the two formulations have to be clarified and an effort made to carry out quantitative comparison of results when possible.*

The obtained system of equations (20) in the absence of current is similar to classical Zakharov equations for discrete wave interactions (Mei, You and Stiassnie, 2009 . Theory and Applications of Ocean Surface Waves. World Scientific, 2009, 14.9.1-14.9.3). Corresponding references are added to the text.

Following reviewer recommendations we rewrite equations (17), (20) in a more clear and compact form of wave action law:

$$\begin{cases} [\phi_0^2 \sigma_0]_T + [(U(X) + \frac{1}{2\sigma_0})\phi_0^2 \sigma_0]_X = \varepsilon \phi_1^2 \phi_2 \phi_0 \sigma_1^3 \sigma_2^2 (2\sigma_1^3 - 2\sigma_1^2 \sigma_2 + 2\sigma_1 \sigma_2^2 - \sigma_2^3) \sin[\varphi]; \\ [\phi_2^2 \sigma_2]_T + [(U(X) + \frac{1}{2\sigma_2})\phi_2^2 \sigma_2]_X = \varepsilon \phi_1^2 \phi_2 \phi_0 \sigma_0^2 \sigma_1^3 (2\sigma_1^3 - 2\sigma_0 \sigma_1^2 + 2\sigma_0^2 \sigma_1 - \sigma_0^3) \sin[\varphi]; \\ [\phi_1^2 \sigma_1]_T + [(U(X) + \frac{1}{2\sigma_1})\phi_1^2 \sigma_1]_X = -\varepsilon \phi_1^2 \phi_2 \phi_0 \sigma_0 \sigma_1^2 \sigma_2 (\sigma_0^4 - \sigma_0^3 \sigma_1 - \sigma_0 \sigma_1 (\sigma_1 - \sigma_2)^2 + \\ + \sigma_0^2 (\sigma_1^2 - \sigma_1 \sigma_2 + 2\sigma_2^2) - \sigma_2 (\sigma_1^3 - \sigma_1^2 \sigma_2 + \sigma_1 \sigma_2^2 - \sigma_2^3)) \sin[\varphi]; \end{cases} \quad (17)$$

$$\begin{cases} [(U + \frac{1}{2\sigma_0})\phi_0^2 \sigma_0]_X = \varepsilon \phi_1^2 \phi_2 \phi_0 \sigma_1^3 \sigma_2^2 (2\sigma_1^3 - 2\sigma_1^2 \sigma_2 + 2\sigma_1 \sigma_2^2 - \sigma_2^3) \sin[\varphi] \\ [(U + \frac{1}{2\sigma_2})\phi_2^2 \sigma_2]_X = \varepsilon \phi_1^2 \phi_2 \phi_0 \sigma_0^2 \sigma_1^3 (2\sigma_1^3 - 2\sigma_0 \sigma_1^2 + 2\sigma_0^2 \sigma_1 - \sigma_0^3) \sin[\varphi] \\ [(U + \frac{1}{2\sigma_1})\phi_1^2 \sigma_1]_X = -\varepsilon \phi_1^2 \phi_2 \phi_0 \sigma_0 \sigma_1^2 \sigma_2 (\sigma_0^4 - \sigma_0^3 \sigma_1 - \sigma_0 \sigma_1 (\sigma_1 - \sigma_2)^2 + \\ + \sigma_0^2 (\sigma_1^2 - \sigma_1 \sigma_2 + 2\sigma_2^2) - \sigma_2 (\sigma_1^3 - \sigma_1^2 \sigma_2 + \sigma_1 \sigma_2^2 - \sigma_2^3)) \sin[\varphi] \end{cases} \quad (20)$$

The main property of the derived modulation equations is the variability of interaction coefficients in the presence of variable current.

To perform the qualitative analysis of the stationary problem, we suggest the law of wave action conservation flux in a slowly moving media as analogue of the three Manley-Rowe dependent integrals:

$$\begin{cases} \left(U + \frac{1}{2\sigma_2} \right) \phi_2^2 \sigma_2 + \left(U + \frac{1}{2\sigma_0} \right) \phi_0^2 \sigma_0 + \left(U + \frac{1}{2\sigma_1} \right) \phi_1^2 \sigma_1 = const; \\ \frac{1}{2} \left(U + \frac{1}{2\sigma_1} \right) \phi_1^2 \sigma_1 + \left(U + \frac{1}{2\sigma_0} \right) \phi_0^2 \sigma_0 = const; \\ \frac{1}{2} \left(U + \frac{1}{2\sigma_1} \right) \phi_1^2 \sigma_1 + \left(U + \frac{1}{2\sigma_2} \right) \phi_2^2 \sigma_2 = const; \\ \left(U + \frac{1}{2\sigma_0} \right) \phi_0^2 \sigma_0 - \left(U + \frac{1}{2\sigma_2} \right) \phi_2^2 \sigma_2 = const. \end{cases}$$

These integrals follow from the system (20) with acceptable accuracy $O(\varepsilon^4)$ for the stationary regime of modulation. The second and third relations here clearly show that the wave action flux of the side bands can grow up at the expense of the main carrier wave flux. The last relationship manifests the almost identical behavior of the main sidebands for the problem of their generation due to Benjamin-Feir instability.

3. *The manuscript is prepared quite sloppily and some references (for example, Moreira and Peregrine, among others, or Hwung et al 2010) do not appear in the list.*

Reference list is checked and updated.

R. M. Moreira and D. H. Peregrine, (2012) Nonlinear interactions between deep-water waves and currents
Journal of Fluid Mechanics, Vol. 691, pp 1- 25.

W. Chiang, H. Hwung (2012) Large transient waves generated through modulational instability in deep water
Journal of Hydrodynamics, 22(5), pp. 114-119. DOI: 10.1016/S1001-6058(09)60179-7

4. *I failed to understand the small slope approximation on line 15 p. 1811.*

Small slope approximation follows from the stationary shallow water model for a large scale slowly varying current.

IV - th assumption of the model is rewritten.

5. *The initial conditions in the simulations are not defined. As follows from the previous studies, the evolution pattern strongly depends not only on the frequencies of the sidebands, but also on their initial amplitudes and phases. I found no mention of these quantities in the manuscript, and subsequently there is no attempt to discuss their relative importance.*

The development of side band instability highly depends from amplitudes and phases of initially imposed side bands. Increasing of initial amplitudes of side bands sharply accelerate the instability. Initial phase shift between side bands and carrier in our simulations was always taken to $-\pi/4$ corresponding to the maximum growth condition predicted by Benjamin & Feir (1967).

Boundary conditions for sideband amplitudes and phases are specified in the text.

6. *The dissipation due to breaking is introduced into the model equation. The parameters used are not specified, and the important details of the dissipation model used in the simulations are missing. Moreover, I question the importance of dissipation in the framework of this study. The variation of the amplitude of various harmonics in the spectrum in the process of breaking was studied in some recent studies (see, e.g. Perlin et al. Ann. Rev. Fluid Mech. 2013 and references therein) and was found to be hardly detectable. As the details of the dissipation model are missing, it remains unclear to what extent accounting for dissipation in the present work indeed affects the results. On the other hand, even prior to breaking the 3-wave approximation adopted in the study apparently ceases to be even approximately valid, as new harmonics inevitably emerge due to nonlinearity. This phenomenon most probably is much more significant than dissipation. It would be highly desirable to have some spectral information from experiments about the wave field, both as initially generated and at advanced stages of evolution. In any case, the relative significance of various factors that lead to poor agreement with available measurements has to be discussed.*

We employ the adjusted dissipative model of Tulin and Li (1996) and Huang et al. (2011) to describe the effect of breaking on the dynamics of the water wave. The sinks of energy and momentum terms for each of the waves are calculated in

accordance with the dissipative Schrodinger model for the complex amplitude $A \sim \sum \phi_i e^{i\theta_i} ::$

$$A_T + C_g A_X + i \frac{C_g}{4k} A_{XX} + \frac{i}{2} \omega k^2 |A|^2 A = \left(-\frac{DA}{g|A|^2} - 4i\gamma A \int \frac{\omega^2 D dX}{g|A|^2} \right) H \left[\frac{|A_X|}{A_s} - 1 \right]$$

where $D \sim g D_b |A|^4$, $D_b = O(10^{-1})$, $\gamma = O(10^{-1})$ - constants of proportionality taken from the field observations, g - gravity acceleration, H is the Heaviside unit step function, and A_s is the threshold value of the characteristic steepness

$A_X = \varepsilon \sum \sigma_i \phi_i k_i$. Right side part leads to additional terms in the governing modulation equations (16), (17), (20).

Wave breaking leads to permanent (not temporal) frequency downshifting at a rate controlled by breaking process. A crucial aspect here is the cooperation of dissipation and near-neighbor energy transfer in the discretized spectrum acting together.

We add some more figures with additional examples of wave interactions accompanied by breaking dissipation to clarify its properties.

The numerical simulations for initially high steepness waves ($\varepsilon = 0.25$) propagation with wave breaking dissipation is

presented in Fig. 3(a-c). We calculate the amplitudes of surface waves on linearly increasing opposing current $U(x) = -U_0 x$ with different strength U_0 . Most unstable regime was tested for frequency space $\Delta\omega_{\pm} / \omega_1 \sim \varepsilon$ and most effective initial phases $\theta_1(0) = 0, \theta_0(0) = \theta_2(0) = -\pi/4$

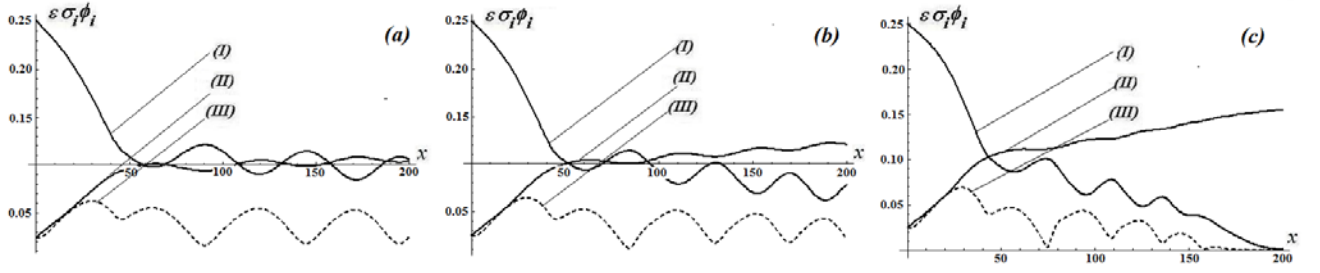


FIG. 3. Modulation of surface waves by the adverse current $U = U_0 x$. (a) $U_0 = -2.5 \cdot 10^{-4}$; (b) $U_0 = -5 \cdot 10^{-4}$, (c) $U_0 = -10^{-3}$. (I), (II), (III) - amplitude envelopes of the carrier, subharmonic and superharmonic waves, respectively. Initial wave steepness $\varepsilon = 0.25$. Dissipation parameters $D_b = 0.1, \gamma = 0.5$

A very weak opposite current $U_0 = 2.5 \cdot 10^{-4}$ (Fig.3(a)) has a pure impact on wave behavior: it is finally results in almost bichromatic wave train with two dominant waves: carrier and lower side band. Frequency downshift here is not clearly seen. Two times stronger current case with $U_0 = 5 \cdot 10^{-4}$ is presented in Fig. 3(b). We note some tendency to final energy downshift to the lower side band. Really strong permanent downshift with total domination of the lower side band is seen for two times more strong current $U_0 = 10^{-3}$.